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## EQUATION OF STATE FOR A TWO-DIMENSIONAL COULOMB GAS

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*This work develops the cluster approach proposed by L.A. Bulavin and M.N. Malomuzh to the description of the phase diagram for a two-dimensional Coulomb gas. We restrict ourselves by the ensemble of the simplest clusters – dipole pairs. The effective interaction potential of dipole pair conserving a two particle configuration integral is constructed. In order to reflect more completely the long-range interaction in the system, the third virial coefficient is taken into account. The phase diagram of the Coulomb gas is analyzed on the basis of the generalized van der Waals equation of state, whose parameters are some functions of the temperature and the density. The position of the critical point is determined in different approximations. It is shown that this problem is essentially non perturbative.*

*Keywords:* two-dimensional Coulomb gas, van der Waals equation, critical point.

### 1. Introduction

The two-dimensional Coulomb gas is one of the interesting objects in theoretical physics. It is a system that consists of charged hard disks in the two-dimensional space, which interact through a logarithmic law.

Figure 1 shows the phase diagram in the dimensionless density  $\tilde{n}$  and dimensionless temperature  $\tilde{T}$  coordinates obtained in work [1] by a computer simulation using the scaling Monte Carlo method. The main elements in this diagram are the insulator/conductor transition line (1) and the “vapor-liquid” coexistence curve (2), as well as the critical point.

The “vapor-liquid” coexistence curve separates two phases: an insulator and a plasma with low and high densities, respectively.

The insulator/conductor transition line separates two phases. The first phase called a dielectric consists of electrically neutral clusters. Due to the Coulomb interaction, the discs with opposite charges attract and form dipole pairs, quadrupole and hexagonal clusters, and higher order clusters. The second phase

is conductive. This can be explained by the fact that the interaction between particles in different clusters becomes significant, as the density increases, and the clusters become unstable. They start to decay into cations and anions consisting of clusters of lower order than those, which exist in the dielectric phase. To describe such a transition, the approach based on the isomorphism of a Coulomb gas and the xy model of magnetic [2, 3] can be used. In the area of high densities, the system can be considered as an ideal plasma [4, 5].

In the investigation of that part of the diagram, where clusters are stable, the significant progress had been achieved in works [6–8], where the interaction potentials for dipole pairs and quadrupoles were constructed. The rotating clusters were approximately considered to be hard disks, and the interaction potentials were averaged, accordingly. Next, the authors constructed the equation of state of the system. On this basis, they determined a binodal and a spinodal and found the position of the critical point, which is close to the results of computer experiments. However, the potentials of the dipole-dipole, dipole-quadrupole, and quadrupole-quadrupole interactions were not completely satisfactory. The main

problem is that the effective potentials were constructed by averaging the real interaction potential on a circle of unit radius. As a result, the obtained configuration integral differs from the real one.

The goal of this paper is to improve the constructive approach developed by L.A. Bulavin and M.N. Malomuzh within the cluster approach [6–8]. In our work, the main attention will be paid to the determination of the critical point. In connection with this, we note that the critical point (CP) is out of the dielectric phase considered by us as an ensemble of dipole pairs. However, the density of disintegrated pairs near the CP is relatively small (see [1, 4]). Therefore, we conclude that the phase transition in a vicinity of the CP can be satisfactorily described with the help of the ensemble of electroneutral dipole pairs. The additional influence of quadrupoles and clusters of higher order will be investigated separately.

We are planning to construct the effective interaction potential of dipole pairs, which conserves the configuration integral in the two-particle approximation. This will be done in two ways: 1) using perturbation theory (approach used in works [6–8]); 2) via integration. We will construct a generalized van der Waals equation in the dipole approximation based on the virial expansion and study the temperature and density dependences of its coefficients. We will also show the importance of the consideration of interactions through the third particle. Eventually, the position of the critical point will be found.

## 2. Free Energy of the System of Dipole Pairs

Consider the system consisting of dipole pairs. Its free energy has the standard structure [9]:

$$F = F_{\text{id}} - T \ln Q_{\text{conf}}, \quad (1)$$

where  $F_{\text{id}}$  is a free energy of an ideal gas,  $Q_{\text{conf}}$  is a configuration integral.

Here, we use the following approximation: rotating dipole pairs are replaced by effective disks. We suppose that such discs are impenetrable, so the repulsive potential is close to the hard sphere potential  $\Phi_{\text{HS}}$ :

$$\Phi_{\text{HS}}(r_{ij}) = \begin{cases} \infty, & r_{ij} \leq \sigma_d, \\ 0, & r_{ij} > \sigma_d, \end{cases}$$

where  $r_{ij}$  is the distance between the centers of effective discs,  $\sigma_d = 2\sigma$  is the diameter of an effective disc,

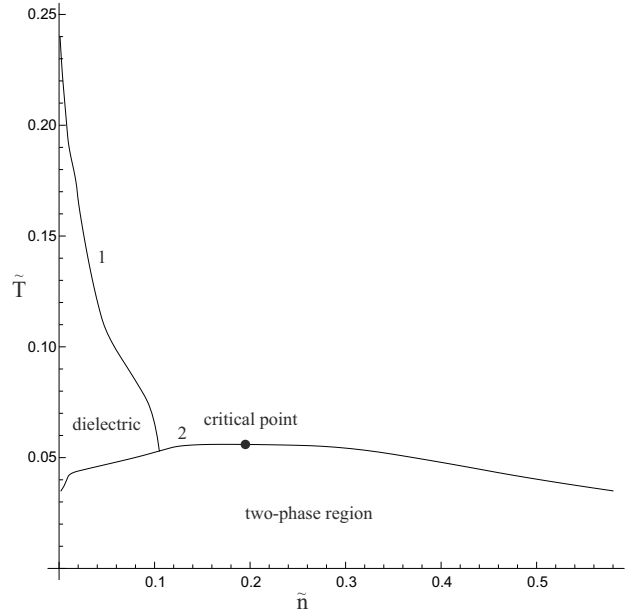


Fig. 1. Phase diagram of a two-dimensional Coulomb gas

and  $\sigma$  is the diameter of a charged disk. In addition, the effective discs attract each other. The attractive potential  $\Phi_a(r_{ij})$  is the averaged potential  $U_{ij}$  of interaction of two dipoles:

$$U_{ij} = \frac{d^2}{r_{ij}^2} \left( \mathbf{e}_i \mathbf{e}_j - 2 \frac{(\mathbf{e}_i \mathbf{r}_{ij})(\mathbf{e}_j \mathbf{r}_{ij})}{r_{ij}^2} \right), \quad (2)$$

where  $d = q\sigma_0$  is the dipole moment, and  $\mathbf{e}_i$  and  $\mathbf{e}_j$  are unit vectors defining the orientation of the dipoles. We will describe  $\Phi_a$  in more details in what follows.

Thus, the interaction potential of two effective discs has the structure:

$$\Phi(r_{ij}) = \Phi_{\text{HS}}(r_{ij}) + \Phi_a(r_{ij}). \quad (3)$$

Introducing the function  $f(r_{ij}) = \exp(-\beta\Phi_a(r_{ij}))$ , the configuration integral can be represented in the form of a virial expansion:

$$Q_{\text{conf}} = \frac{1}{S^N} \int \exp \left( -\beta \sum_{1 \leq i < j \leq N} \Phi_{\text{HS}}(r_{ij}) \right) \times \\ \times \left( 1 + \sum_{1 \leq i < j \leq N} f(r_{ij}) + \sum_{1 \leq i < j \leq N} f(r_{ij}) \sum_{j < k < l \leq N} f(r_{kl}) + \dots \right) dS_N. \quad (4)$$

It is easy to see that the contribution of hard spheres to  $Q_{\text{conf}}$  is given by

$$Q_{\text{HS}} = \frac{1}{S^N} \int_N \exp \left( -\beta \sum_{1 \leq i < j \leq N} \Phi_{\text{HS}}(r_{ij}) \right) dS_N = (1 - n_d S_{\text{exc}})^N, \quad (5)$$

where  $n_d = \frac{N}{S}$  is the density of dipoles, and  $S_{\text{exc}} = 2S_0 = 2\pi\sigma^2$  is the excluded volume per particle.

The contribution linear in  $f$  to  $Q_{\text{conf}}$  is

$$Q^{(f)} = S Q_{\text{HS}} \frac{n^2}{2} (1 - n_d S_{\text{exc}}) \int f(r_{12}) g_{\text{HS}}(r_{12}) dS,$$

where  $g_{\text{HS}}(r_{12}) = \frac{S^2 \int_{N-2} \exp(-\beta \sum \Phi_{\text{HS}}(r_{ij})) dS_{N-2}}{\int_N \exp(-\beta \sum \Phi_{\text{HS}}(r_{ij})) dS_N}$  is the pair correlation function of hard disks.

The contribution quadratic in  $f$  to  $Q_{\text{conf}}$  can be represented similarly as

$$Q^{(f^2)} = S Q_{\text{HS}} \frac{n^3}{2} (1 - n_d S_{\text{exc}}) \times \left( \frac{1}{3} \int_2 f(r_{12}) f(r_{23}) g_{\text{HS}}^{(1,2,3)} dS_2 + S \frac{n}{2} (1 - n_d S_{\text{exc}}) \int_2 f(r_{12}) f(r_{34}) g_{\text{HS}}^{(1,2,3,4)} dS_2 \right),$$

where  $g_{\text{HS}}^{(1,2,3)}$  is the triple correlation function, and  $g_{\text{HS}}^{(1,2,3,4)}$  is the four-correlation function.

In the approximation quadratic in  $f$ , the free energy takes the form

$$F = -NT \left( \ln \frac{e}{n} + f'(T) - \ln(1 - n_d S_{\text{exc}}) - \frac{n}{2} (1 - n_d S_{\text{exc}}) I_1[f_{12}] - \frac{n^2}{6} (1 - n_d S_{\text{exc}}) I_2[f_{12}, f_{23}] \right), \quad (6)$$

where

$$I_1[f_{12}] = \int f_{12} g_{\text{HS}}(r_{12}) dS, \quad (7)$$

$$I_2[f_{12}, f_{23}] = \int_2 g_{\text{HS}}^{(1,2,3)} f_{12} f_{23} dS_2.$$

In this paper, we assume that the double and triple correlation functions are approximately equal to 1 on

the basis that the system under study is sufficiently rarefied.

Using the standard formula  $P = -\frac{n_d}{S} \frac{\partial F}{\partial n_d}$ , we obtain the equation

$$P = \frac{n_d T}{1 - b n_d} - a n_d^2, \quad (8)$$

which has the same form as the van der Waals equation. Here,  $b = 2\pi\sigma^2$ ,  $a = a_2 + a_3$ ,

$$a_2(T) = -\pi \int_{\sigma_d}^{\infty} \Phi_a(r) r dr,$$

$$a_3(T, n_d) = -\pi \int_{\sigma_d}^{\infty} \Phi_a(r) r dr \left( 4S_0 + \frac{1}{4T} \int_{\sigma_d}^{\infty} \Phi_a(r) r dr \right) n_d. \quad (9)$$

The function  $a_2(T)$  is a standard coefficient of the van der Waals equation (unlike the classical case, it depends on the temperature). The emergence of  $a_3(T, n_d)$  is a result of taking the third virial coefficient into account.

### 3. Effective Interaction Potential of Dipole Pairs

Let us construct an effective potential. For this goal, we will use the condition that it conserves the form of the configuration integral in the two-particle approximation:

$$Q_2 = \int dS_1 \int dS_2 \int \frac{d\alpha_1}{2\pi} \int \frac{d\alpha_2}{2\pi} e^{-\beta U_{12}} = \int dS_1 \int dS_2 e^{-\beta \Phi_a}.$$

From whence, it follows that  $e^{-\beta \Phi_a}$  satisfies the equation

$$e^{-\beta \Phi_a} = \langle e^{-\beta U_{12}} \rangle_0, \quad (10)$$

where the angular brackets are determined by the relation

$$\langle \dots \rangle_0 = \int \frac{d\alpha_1}{2\pi} \int \frac{d\alpha_2}{2\pi}. \quad (11)$$

The bare interaction energy  $U_{12}$  of two dipole pairs as a function of the angles  $\alpha_1$  and  $\alpha_2$  has structure

$$U_{12}(r_{12}, \alpha_1, \alpha_2) = -\frac{d^2}{r_{12}^2} \cos(\alpha_1 + \alpha_2),$$

where  $\alpha_1$  and  $\alpha_2$  are the angles between the dipole moments  $\mathbf{d}_1$ ,  $\mathbf{d}_2$  and the vector  $\mathbf{r}_{12}$ .

Due to the cosine in the exponent, the integration in formula (11) leads to the modified Bessel function of the first kind  $I_0(x)$ :

$$\Phi_a(\tilde{r}_{12}) = -T_0\tilde{T} \ln \left( I_0 \left( \frac{1}{4} \frac{1}{\tilde{T}} \frac{1}{\tilde{r}_{12}^2} \right) \right), \quad (12)$$

where  $\tilde{r}_{12} = \frac{r_{12}}{\sigma_d}$ ,  $\tilde{T} = \frac{T}{T_0}$ , and  $T_0$  is the characteristic temperature, at which a dipole pair decays into charged discs.

Let us compare our results to the results obtained in works [6–8]. To do that along with the exact potential  $\Phi_a(\tilde{r}_{12})$ , we will find the effective potential with help of perturbation theory.

The average value can be found within perturbation theory by expanding the expression on the right-hand side of (10) in a series and then rewriting it in the form of an exponential function. Thus, the effective potential can be expressed through the average values of interaction energy:

$$\Phi_a = \sum_{i=0}^{\infty} \Phi_a^{(i)}, \quad (13)$$

where

$$\Phi_a^{(0)} = -\frac{1}{2}\beta \langle U_{12}^2 \rangle_0,$$

$$\Phi_a^{(1)} = -\frac{1}{8}\beta^4 \left( \frac{1}{3} \langle U_{12}^4 \rangle_0 - \langle U_{12}^2 \rangle_0^2 \right),$$

$$\Phi_a^{(2)} = -\frac{1}{48}\beta^6 \left( \frac{1}{15} \langle U_{12}^6 \rangle_0 - \langle U_{12}^2 \rangle_0 \langle U_{12}^4 \rangle_0 + 2 \langle U_{12}^2 \rangle_0^3 \right),$$

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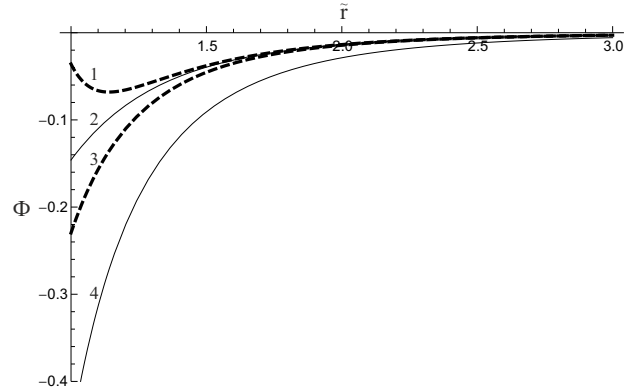
The potential  $\Phi^{(BM)}$  used in works [6–8] is an analog of the main contribution  $\Phi_a^{(0)}$ :

$$\Phi^{(BM)} = 2\Phi_a^{(0)}.$$

The coefficient 2, which appears due to the conservation of the configuration integral, significantly changes not only the potential, but also the virial coefficients and the van der Waals equation.

It is not difficult to see that the effective potential is a series in the inverse distance to the fourth power:

$$\begin{aligned} \Phi_a^{(0)}(\tilde{r}_{12}) &= -\frac{1}{64} \frac{T_0}{\tilde{T}} \frac{1}{\tilde{r}_{12}^4}, \\ \Phi_a^{(1)}(\tilde{r}_{12}) &= \frac{1}{16384} \frac{T_0}{\tilde{T}^3} \frac{1}{\tilde{r}_{12}^8}, \\ &\dots, \end{aligned} \quad (14)$$



**Fig. 2.** Comparative behavior of different contributions to the interaction potential:  $3 - \Phi_a^{(0)}$ ,  $1 - \Phi_a^{(0)} + \Phi_a^{(1)}$ ,  $4 - \Phi^{(BM)}$ ,  $2 - \Phi_a$

where  $\tilde{T} = \frac{T}{T_0}$ . Series (14) coincides with a power series expansion for the exact expression (12) for the interparticle potential  $\Phi_a$ .

As it should be, the behavior of  $\Phi_a^{(0)} + \Phi_a^{(2)}$  and  $\Phi_a$  for short distances differ considerably (see Fig. 2, where the potential  $\Phi^{(BM)}$  is also given for comparison).

Such behavior of  $\Phi_a$  and its different approximations is a precondition for the essential influence on the values of second and third virial coefficients (see below).

#### 4. Equation of State

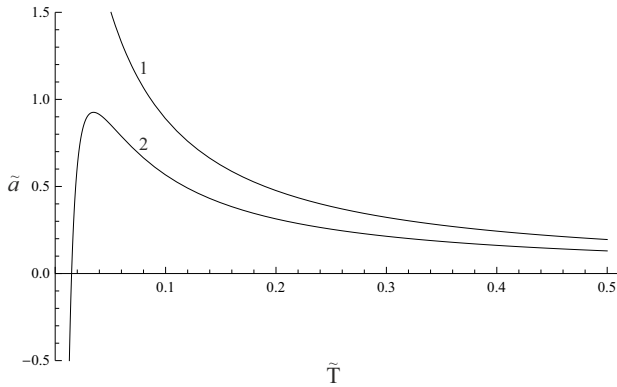
Let us model the equation of state (EoS) of the Coulomb gas by that for the system of dipole pairs. EoS (8) in the dimensionless form has structure

$$\tilde{P} = \frac{\tilde{n}_d \tilde{T}}{1 - \tilde{b} \tilde{n}_d} - \tilde{a} \tilde{n}_d^2, \quad (15)$$

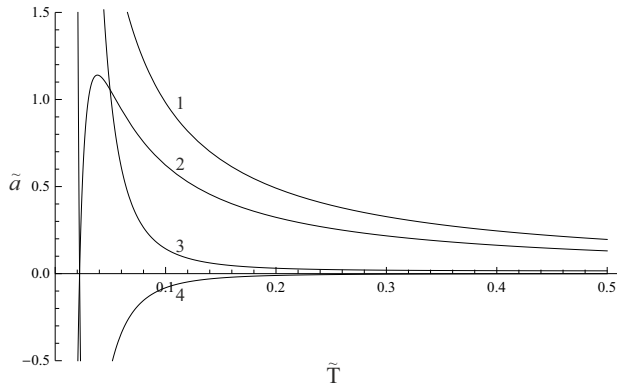
where  $\tilde{P} = \frac{\sigma_d^2}{T_0} P$ ,  $\tilde{n}_d = n_d \sigma_d^2$ ,  $\tilde{b} = 2\pi$ , and  $\tilde{a} = \frac{a}{T_0 \sigma_d^2}$ . We will calculate the coefficient  $\tilde{a}(\tilde{T}) = \tilde{a}_2(\tilde{T}) + \tilde{a}_3(\tilde{T}, \tilde{n}_d)$  in two ways: using the exact effective potential and that obtained with the help of perturbation theory.

##### 4.1. Behavior of the coefficient $\tilde{a}$ for the exact expression for the effective potential

Let us estimate the relative value of the third virial coefficient. The comparative behavior of the coefficients  $\tilde{a}_2$  and  $\tilde{a}_3$  is shown in Fig. 3.



**Fig. 3.** Temperature dependence of the coefficients: 1 –  $\tilde{a}_2(\tilde{T})$ , 2 –  $\tilde{a}_3(\tilde{T}, \frac{1}{6\pi})$



**Fig. 4.** Temperature dependence of different contributions to the coefficient  $\tilde{a}$ : 1 –  $\tilde{a}_2^{(0)}(\tilde{T})$ , 2 –  $a_3^{(0)}(\tilde{T}, \frac{1}{6\pi})$ , 3 –  $a_2^{(1)}(\tilde{T})$ , 4 –  $a_3^{(1)}(\tilde{T}, \frac{1}{6\pi})$

Here, we assume that the density  $\tilde{n}_d$  equals  $\frac{1}{6\pi}$  (the density coordinate of the critical point in the zeroth approximation).

As we see from Fig. 3, the contribution of the interaction through the third particle is considerable for all temperatures and, therefore, should be taken into account.

#### 4.2. Behavior of the coefficient $\tilde{a}$ for effective potentials found with the help of perturbation theory

Let us approximate series (13) for the effective potential by two contributions:

$$\begin{aligned} \tilde{a}_2 &= \tilde{a}_2^{(0)} + \tilde{a}_2^{(1)}, \\ \tilde{a}_3 &= \tilde{a}_3^{(0)} + \tilde{a}_3^{(1)}, \end{aligned}$$

where indices (0) and (1) correspond to taking the first  $\Phi_a^{(0)}$  and second  $\Phi_a^{(1)}$  contributions into account.

It is not difficult to see that

$$\begin{aligned} a_2^{(0)}(\tilde{T}) &= \frac{\pi}{32} \frac{1}{\tilde{T}}, \\ a_2^{(1)}(\tilde{T}) &= \frac{\pi}{24576} \frac{1}{\tilde{T}^3}, \\ a_3^{(0)}(\tilde{T}, \tilde{n}_d) &= \frac{\pi^2}{8} \frac{\tilde{n}_d}{\tilde{T}} \left(1 - \frac{1}{2048} \frac{1}{\tilde{T}^2}\right), \\ a_3^{(1)}(\tilde{T}, \tilde{n}_d) &= -\frac{\pi^2}{6144} \frac{\tilde{n}_d}{\tilde{T}^3} \left(1 - \frac{1}{2048} \frac{1}{\tilde{T}^2}\right). \end{aligned} \tag{16}$$

The correction term in brackets in expression (16) for  $a_3^{(0)}$  is about ten times less than one (we assume that the density  $\tilde{n}_d$  equals  $\frac{1}{6\pi}$ ). The same is true for the coefficient  $a_3^{(1)}$ .

Note that, in works [6–8], the dependence of the coefficient  $\tilde{a}$  on the density was ignored.

It is worth to note that the use of perturbation theory for finding the positions of the binodal and the critical point is related to the convergency problem of the perturbation procedure. In the zeroth approximation for the interparticle potential  $\Phi_a^{(0)}$ , we get the quite satisfactory coordinates of the critical point. However, the use of the more accurate potential  $\Phi_a^{(0)} + \Phi_a^{(1)}$  leads to such a shift of the critical temperature, which is comparable with the critical temperature in the zeroth approximation. In connection with this, let us consider the behavior of different contributions of virial coefficients in more details.

The temperature dependences of different partial contributions to the second and third virial coefficients are presented in Fig. 4. The comparative behavior of the second and third virial coefficients corresponding to the exact potential and the approximate one is shown in Fig. 5.

Curves (1) and (2) in Fig. 4 correspond to ones (1) and (2) in Fig. 3 (They differ insignificantly at temperatures less than 0.2). The correction terms  $a_2^{(1)}$  and  $a_3^{(1)}$  are significant in the temperature range, which the critical point temperature belongs to.

It is not difficult to see that the values of the functions  $\tilde{a}_2(\tilde{T})$  and  $\tilde{a}_2^{(0)}(\tilde{T}) + \tilde{a}_2^{(1)}(\tilde{T})$ , as well as of  $\tilde{a}_3(\tilde{T}, \frac{1}{6\pi})$  and  $\tilde{a}_3^{(0)}(\tilde{T}, \frac{1}{6\pi}) + \tilde{a}_3^{(1)}(\tilde{T}, \frac{1}{6\pi})$ , are very close at temperatures higher than 0.2 (see Fig. 5). However, in the temperature range, that we are interested in, they differ significantly.

The fact that the coefficients  $\tilde{a}_2$  and  $\tilde{a}_3$  are compatible (in the area of the critical point, the ratio

$\frac{\tilde{a}_2}{\tilde{a}_3}$  approximately equals 0.5 and increases at higher temperatures) makes it impossible to use perturbation theory for refining the position of the critical point obtained in the zeroth approximation. We use numerical methods instead.

### 5. Position of the Critical Point

The position of the critical point is determined by the system of equations:

$$\begin{cases} \frac{\partial \tilde{P}}{\partial \tilde{n}_d} = 0, \\ \frac{\partial^2 \tilde{P}}{\partial \tilde{n}_d^2} = 0. \end{cases} \quad (17)$$

The coordinates of the critical point in the zeroth approximation ( $\tilde{P}_0 = \frac{\tilde{n}_d \tilde{T}}{1 - \tilde{b} \tilde{n}_d} - \tilde{a}_2^{(0)} \tilde{n}_d^2$ ) are:

$$\begin{aligned} \tilde{n}_c^{(0)} &= \frac{1}{3\pi} \approx 0.106, \\ \tilde{T}_c^{(0)} &= \frac{1}{6\sqrt{6}} \approx 0.068, \end{aligned} \quad (18)$$

where  $\tilde{n} = 2\tilde{n}_d$  is the total density of ions in the system equal to the density of dipoles multiplied by 2 (the number of particles in the dipole pair).

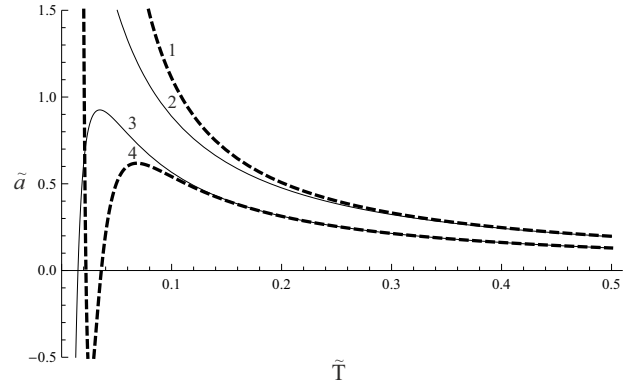
The value  $\tilde{T}_c^{(0)}$  is  $\sqrt{2}$  times less than the critical temperature obtained in [6–8]  $\tilde{T}_c^{\text{BM}} = \frac{1}{6\sqrt{3}}$  for a system consisting of dipole pairs. This is explained by the fact that we use the twice less effective potential (up to the first term), so there are differences in the coefficients of the equation of state, and the critical point coordinates are displaced. It should be noted that it has much better agreement with the value  $\tilde{T}_c^{\text{Comp}} = 0.056$ , obtained as a result of the computer experiment [1].

The following table shows the coordinates of critical points calculated, by using different approximations for the coefficient  $\tilde{a}$ .

The consideration of the interaction through a third particle, which is expressed in the addition of the coefficient  $\tilde{a}_3$  (or  $\tilde{a}_3^{(i)}$  in the case of perturbation theory), increases the values of both density and temperature. The coefficients with index (1), which correspond to the  $\Phi_a^{(1)}$  term, do not affect the results significantly.

### 6. Discussion of the Results Obtained

This work develops the cluster approach to the description of properties of a Coulomb gas, which was



**Fig. 5.** Comparative behavior of different approximations of the coefficients  $\tilde{a}_2$ : 1 –  $\tilde{a}_2^{(0)}(\tilde{T}) + \tilde{a}_2^{(1)}(\tilde{T})$ , 2 –  $\tilde{a}_2(\tilde{T})$ , and  $\tilde{a}_3$ : 3 –  $\tilde{a}_3(\tilde{T}, \frac{1}{6\pi})$ , 4 –  $\tilde{a}_3^{(0)}(\tilde{T}, \frac{1}{6\pi}) + \tilde{a}_3^{(1)}(\tilde{T}, \frac{1}{6\pi})$

first proposed in works [6–8]. We limited ourselves by the simplest clusters – dipole pairs.

We can use this assumption in a vicinity of the critical point despite the fact that it is out of the dielectric phase. This is because the density of disintegrated pairs is relatively small near the critical point.

We pay attention to the following factors unaccounted in works [6–8]:

1. Exact definition of the average interaction potential.
2. Account for the third virial coefficient, which is very important in systems with long-range interactions.

The effective potential is constructed in such a way that it conserves a configuration integral in the two-particle approximation. This is done in two ways: 1) via integration; 2) using perturbation theory (approach used in works [6–8]).

To find the position of the critical point, we use a generalized van der Waals equation, whose coeffi-

#### Results of calculations of the critical point coordinates

	Accuracy	$\tilde{T}_c$	$\tilde{n}_c$
Perturbation theory	$\tilde{a}_2^{(0)}$	0.068	0.106
	$\tilde{a}_2^{(0)} + \tilde{a}_2^{(1)}$	0.075	0.106
	$\tilde{a}_2^{(0)} + \tilde{a}_2^{(1)} + \tilde{a}_3^{(0)}$	0.101	0.138
	$\tilde{a}_2^{(0)} + \tilde{a}_2^{(1)} + \tilde{a}_3^{(0)} + \tilde{a}_3^{(1)}$	0.098	0.135
Exact	$\tilde{a}_2$	0.061	0.106
	$\tilde{a}_2 + \tilde{a}_3$	0.093	0.139

cients depend significantly on the temperature and the density.

The use of the effective potential that conserves the configuration integral decreases the critical point temperature in the zeroth approximation by  $\sqrt{2}$  times compared to that obtained in works [6–8].

The use of a more accurate averaged interaction potential and taking the interaction through a third particle into account do not allow us to get values close to the experimental ones. It is necessary to consider the presence of quadrupole clusters in the system. This is the topic of the next work.

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#### РІВНЯННЯ СТАНУ ДВОВИМІРНОГО КУЛОНІВСЬКОГО ГАЗУ

#### Р е з ю м е

У роботі розвинуто кластерний підхід, запропонований Л.А. Булавиним та М.М. Маломужем для опису фазової діаграми двовимірного кулонівського газу. Ми обмежуємося ансамблем найпростіших кластерів — дипольних пар. Побудовано ефективний потенціал взаємодії дипольних пар, який зберігає конфігураційний інтеграл у двочастинковому наближенні. Для більш повного врахування дальньої взаємодії в системі враховується третій віріальний коефіцієнт. Фазова діаграма кулонівського газу досліджується на підставі узагальненого рівняння стану Ван-дер-Ваальса, параметри якого є функціями температури та густини. Визначено положення критичної точки в різних наближеннях. Показана незастосовність теорії збурень до даної проблеми.