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MICROWAVE DEVICE ON THE BASIS OF A BARREL RESONATOR FOR DETERMINING THE AVERAGE DENSITY AND THE DENSITY PROFILE IN PLASMA FORMATIONS

A microwave resonator device for measuring the plasma density is described. A method is proposed for determining the radial distribution function of the plasma density by measuring a shift of resonance frequencies for two oscillation modes. According to experimental results, the device is suitable for measuring the plasma density in the range 10^9–10^{11} cm^{-3}.

Keywords: microwave resonator, plasma, radial distribution function, resonance frequency, plasma density.

1. Introduction

Researches in plasma physics and plasma applications are carried out in a rather wide interval of the plasma density ranging from 10^9 cm^{-3} to 10^{15} cm^{-3} and even more. Every subrange of the plasma density demands its own measuring methods and devices. A diagnostic kit for carrying out experiments in a density interval of 10^{13}–10^{15} cm^{-3} is almost determined [1–7]. At the same time, for a low-density interval of 10^9–10^{11} cm^{-3}, the kit of means and methods for plasma diagnostics still remains incomplete. In most cases, bearing in mind the low values of plasma density and the small linear dimensions of plasma formations, the latter are usually studied with the help of probes. The work of the latter is based on a direct contact with plasma, which results in the contamination of plasma with particles of a probe material. In many cases, this circumstance gives rise to the necessity of an additional probe calibration. Hence, there emerges the impetus to develop research methods allowing the noncontact measurements of plasma densities in the interval of 10^9–10^{11} cm^{-3} to be carried out in plasma formations and flows in a volume with magnetic field and beyond it with regard for the physical and engineering features of experimental installations and devices. The tasks of this kind, as will be shown below, can be solved with the help of microwave methods of plasma diagnostics.

Plasma with the density indicated above (10^9–10^{11} cm^{-3}) is studied in plasma installations of various types with the help of resonator methods [8–16]. The role of a vibrator is played by the plasma chamber or its section excited at high-mode oscillations. Such methods have a rather simple measurement procedure and allow the density to be measured in large volumes, provided that the plasma frequency is much lower than that of a microwave signal. Among the resonator methods, those based on the application of barrel resonators [9–11] have a number of advantages, because they are characterized by more rarefied frequency spectra in comparison with those for closed resonators and by insignificant radiation losses from the end faces. There is a known way [9] to measure the radial distribution of the plasma density with the help

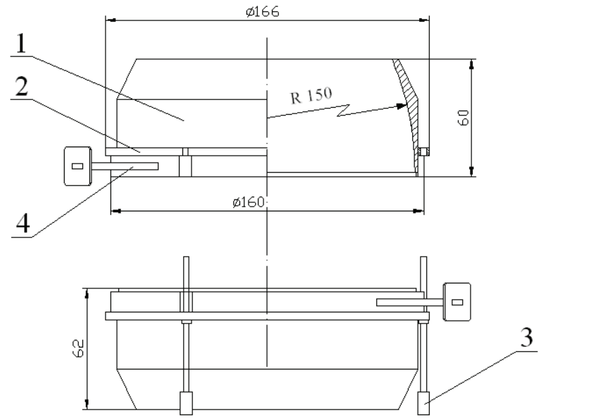

Fig. 1. Schematic diagram of a two-section barrel microwave resonator: resonator section (*1*), gripping ring (*2*), compression bolt (*3*), demountable microwave input (*4*)

of an open barrel resonator, in which seven oscillation peaks with different caustics were excited with the use of a system of apertures providing a connection with the exciting and reception waveguides. However, owing to technical difficulties at the device tuning and an inconvenience of the mathematical processing of obtained signals, the method turned out difficult to be used in practice. Therefore, the following basic requirements to the microwave device created on the basis of a barrel resonator for the determination of the average density and profile in plasma formations were formulated: 1) the device has to allow measurements of the plasma density in the interval from 10^8 to 10^{11} cm^{-3} within a time period of 5–10 ms or in the stationary regime; 2) in order to exclude the influence of an external magnetic field, the operation frequency ω should satisfy the condition $\omega \gg \omega_{\rm He}$, where $\omega_{\rm He}$ is the electron cyclotron frequency at the magnetic field maximum; and 3) the resonator device size has to be as short as possible and, at the same time, to possess low radiation losses from its end faces.

2. Barrel Resonator Parameters

The requirements stated at the end of the previous section can be satisfied with the use of a barrel resonator excited at the frequency $f = 38 \times 10^9$ Hz. In this resonator, the critical wavelength equals

$$\lambda_{\rm cr.} = \frac{2\pi a_0}{\varepsilon_{nm}}\left(1 - \frac{z_q^2}{2r_0 a_0}\right), \quad (1)$$

where a_0 is the resonator radius measured at the middle cross-section; r_0 the generatrix radius; z_q the distance from the middle cross-section to the plane, where the given wavelength is critical; and ε_{nm} the n-th root of the Bessel function of the m-th order (m is the azimuthal mode number, and n the radial mode number). From Eq. (1), it follows that oscillations propagating in the middle section of the resonator are reflected from narrower end-face parts, which provides a high radiation Q-factor. The latter equals

$$Q = \frac{1}{2l}\left(\frac{\pi\lambda^3}{a_0}\right)^{1/2}\left(1 + \sqrt{\frac{a_0}{r_0 - a_0}}\right)\exp\left(\frac{\pi l^2}{a_0\lambda}\right), \quad (2)$$

and considerably exceeds the Q-factor associated with losses in the resonator walls. The design features and the resonator dimensions (see Fig. 1) are as follows: the small radius $a_0 = 64$ mm, the length $l = 120$ mm, and the generatrix radius $r_0 = 150$ mm was selected taking into account that radiation losses are minimum at the ratio $r_0/a_0 \approx 2$.

The resonator was fabricated in the form of two sections, which made its arrangement and regulation handy, and provided an additional rarefication in the spectrum of excited oscillations. The resonator was excited, and the signal was output from it through two apertures 1.8 mm in diameter each, which were located near the middle cross-section at a distance of 10 mm from the middle device plane and azimuthally shifted by 90° with respect to each other. Oscillations were input into the resonator and output from it by means of waveguide activators connected to the apertures. Unlike the distributed connection method [9] with the help of apertures located along the resonator perimeter, which is intended for the excitation of low-order oscillations, the proposed variant of two-point connection allowed oscillations of high orders to be excited.

It is known [8] that the field structure generated by oscillations of the TM and TE types in the transverse cross-section of a barrel resonator is close to that generated by oscillations with the same name in a cylindrical resonator, which are described by the expression

$$E = E_0 J_m\left(\varepsilon_{m,n}\frac{r}{a_0}\right)\cos m\varphi e^{-\beta z^2}, \quad (3)$$

where E is the electric field strength in the barrel resonator volume; E_0 the field strength at the resonator axis or a normalizing factor; r, z, and φ are the cylindrical coordinates; J_m is the Bessel function; and β is a certain constant, the numerical value of which is determined by the resonator geometry and the oscillation type. Taking into account that the resonator is short and assuming the density in the plasma flow (formation) to be constant over the whole resonator length, the shift of the resonator frequency in the presence of plasma can be presented in the form

$$\frac{\Delta f}{f} = \frac{1}{2}\frac{\bar{N}}{N_{\text{cr.}}}\frac{V_p \int_{V_{\text{res.}}} E^2 N dV}{\int_{V_p} N dV \int_{V_{\text{res.}}} E^2 dV} = \\ = \frac{1}{4}\frac{\bar{N}}{N_{\text{cr.}}}\frac{a_2^2 \int_0^{a_0} N J_m^2\left(\varepsilon_{m,n}\frac{r}{a_0}\right) r dr}{\int_0^{a_2} N r dr \int_0^{a_0} J_m^2\left(\varepsilon_{m,n}\frac{r}{a_0}\right) r dr}, \tag{4}$$

where $\bar{N}$ is the average plasma density, $N_{\text{cr.}}$ the critical plasma density, V_p the volume occupied by plasma, and V_{res} the resonator volume. The radial and azimuthal mode numbers (m and n, respectively) are mutually related by the relation

$$n = \frac{m}{\pi}\left\{\text{tg}\arccos\left[\frac{m\lambda_{\text{cr.}}}{2\pi a_0\left(1 - \frac{z_q^2}{2r_0 a_0}\right)}\right] - \right. \\ \left. - \arccos\left[\frac{m\lambda_{\text{cr.}}}{\pi a_0\left(1 - \frac{z_q^2}{2r_0 a_0}\right)}\right]\right\}. \tag{5}$$

Hence, for the known resonator dimensions and wavelength of excited oscillations, there are a limited number of the pairs of integers m and n that satisfy the excitation condition for the given type of oscillations in the resonator. The type of oscillations in the resonator was determined using the method of small perturbations with the help of an absorbing ball 1 mm in diameter that moved along either the resonator axis or the resonator radius [12]. The Q-factor of the system, which was determined on the basis of the resonance curve half-width, was found to equal $Q \sim 10$. In practice, two types of oscillations were reliably excited in the resonator at the frequencies $f_1 = 38.42$ GHz ($\lambda_1 = 0.7808$ cm) and $f_2 = 38.106$ GHz ($\lambda_2 = 0.7873$ cm). The amplitudes of other observed resonances were substantially smaller, approximately by an order of magnitude in comparison with the amplitudes of indicated modes.

Measurements were carried out consecutively for one oscillation type, which was obtained owing to the spectrum rarefication specific for the barrel resonator and to the circumstance that the resonator design in the form of two sections allowed the excitation of modes with longitudinal currents to be confined. Since practically a single resonance was excited in the tuning band with the half-width $\delta f = 150$ kHz, the measurement of the resonance frequency shift in the interval $\delta f = 0.2 \div 100$ MHz became possible for every oscillation type.

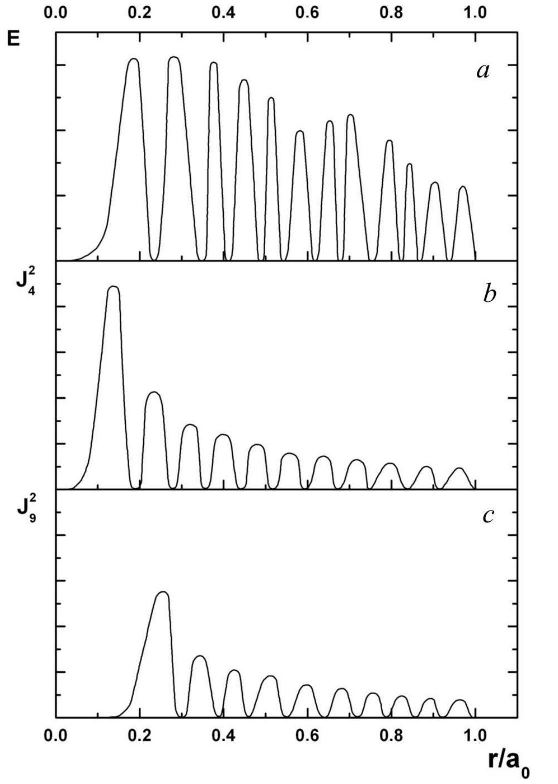

Fig. 2. Radial distribution of microwave fields in the resonator: integrated distribution (a), oscillations of type $E_{4.11}$ (b), oscillations of type $E_{9.9}$ (c)

The observed types of oscillations were identified by directly measuring the field distribution for each resonance excited by the method described above and by comparing it with formula (5). It was found that the observed oscillations correspond to the field distributions $E \sim f_{4.11}$ and $E \sim f_{9.9}$, respectively. For the plasma research, the radial field distribution plays a crucial role. It is shown in Fig. 2. The measurement of the field distribution along the resonator axis testified that the field was concentrated near the equatorial plane.

3. Determination of the Average Density and the Density Profile in the Plasma Formation

From expression (4) describing the frequency shift of the given resonance type, it follows that the ratio between the frequency shifts for oscillations of two types equals

$$\frac{\Delta f_1}{\Delta f_2} = \frac{A_f^{(1)}}{A_f^{(2)}} = \frac{\int_0^{a_2} N(r) J_{m_1}^2 r dr \int_0^{a_0} J_{m_2}^2 r dr}{\int_0^{a_0} J_{m_1}^2 r dr \int_0^{a_2} N(r) J_{m_2}^2 r dr}. \quad (6)$$

It is possible to choose such a functional dependence $N(r)$ that this equality will be satisfied. In our researches, the radial distribution of the plasma density was taken in the form (7)

$$N(r) = N_{\max}\, e^{-\left(\frac{r}{a_1}\right)^2}, \quad (7)$$

where $N_{\max}$ is the plasma density maximum, and a_1 the radius of a plasma formation. In this case, the form factor equals

$$A_f = \frac{k^2}{1 - e^{-k^2}} \frac{\int_0^{a_2} e^{-\left(\frac{r}{a_1}\right)^2} J_m^2 r dr}{\int_0^{a_0} J_m^2 r dr}, \quad (8)$$

where $k = a_2/a_1$, and a_2 is a diameter, over which the plasma flow (formation) is averaged. In view of Eqs. (7) and (8), relation (6) looks like

$$\frac{\Delta f_1}{\Delta f_2} = \frac{\int_0^{a_2} e^{-\left(\frac{r}{a_1}\right)^2} J_{m_1}^2 r dr \int_0^{a_0} J_{m_2}^2 r dr}{\int_0^{a_0} J_{m_1}^2 r dr \int_0^{a_2} e^{-\left(\frac{r}{a_1}\right)^2} J_{m_2}^2 r dr}. \quad (9)$$

One can see that, by measuring the frequency shifts for various types of oscillations, it is possible, from formula (9), to find the characteristic dimension of plasma. In a similar way, the characteristic dimension for the density distribution function presented in any other form can be determined. Note that the radial distribution of the microwave electric field strength, which is described by a Bessel function of a high order (in the case concerned, these are $J_{4.11}$ and $J_{9.9}$) with the field equal to zero at the resonator axis, is not sensitive to plasma in a certain near-axis region, which is larger if the order of a Bessel function is higher. Taking all that into account, the plasma density was not averaged over the whole volume, but starting from a definite radius b_0. Therefore, the distribution function of plasma density for the mode m takes the form

$$\bar{N}_m = \frac{N_{\max} a_1^2}{b_1^2 - b_0^2} \left[e^{-\left(\frac{b_0}{a_1}\right)^2} - e^{-\left(\frac{a_2}{a_1}\right)^2} \right], \quad (10)$$

where b_0 is the radius of a region, in which the microwave field strength for this type of oscillations is negligibly small. From Eq. (10), it follows that the ratio between the average densities of both oscillation types looks like

$$\frac{\bar{N}_{m_1}}{\bar{N}_{m_2}} = \frac{a_2^2 - b_{01}^2}{a_2^2 - b_{02}^2} \frac{e^{-\left(\frac{b_{01}}{a_1}\right)^2}}{e^{-\left(\frac{b_{02}}{a_1}\right)^2}}. \quad (11)$$

At the excitation of the oscillation type corresponding to $E = J_0$, the average density corresponds to the averaging over the whole region occupied by plasma, i.e.

$$\bar{N} = N_{\max} \frac{a_1^2}{a_2^2} \left[1 - e^{-\left(\frac{a_2}{a_1}\right)^2} \right]. \quad (12)$$

In addition, the value $\bar{N}$ can also be determined in terms of the average density obtained at the excitation of the different oscillation type corresponding to $E = J_m$:

$$\bar{N} = \bar{N}_m \left(1 - \frac{b_0^2}{a_2^2} \right) e^{\left(\frac{b_0}{a_1}\right)^2}, \quad (13)$$

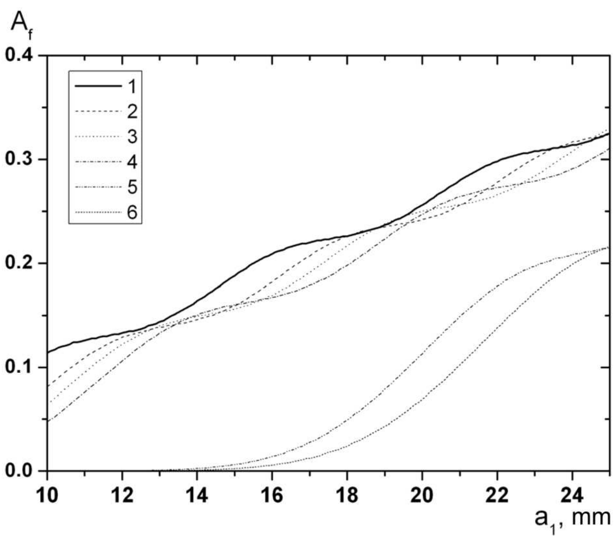

Fig. 3. Form factor versus the characteristic plasma size at the radial density distribution of form (7) for various oscillation types: $E_{3.13}$ (*1*), $E_{4.13}$ (*2*), $E_{4.12}$ (*3*), $E_{4.11}$ (*4*), $E_{9.10}$ (*5*), and $E_{9.9}$ (*6*)

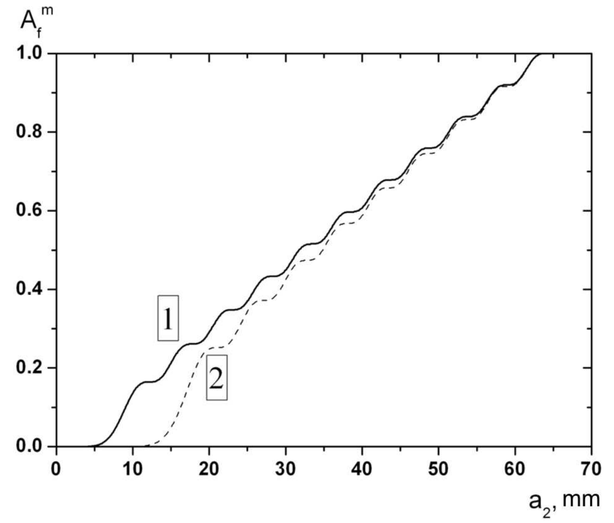

Fig. 4. Form factor versus the plasma radius at the uniform plasma density distribution for oscillation types $E_{4.11}$ (*1*) and $E_{9.9}$ (*2*)

The maximum value of plasma density is related to the measured frequency shift by the formula

$$N_{\max} = \frac{2}{A_f}\frac{\Delta f}{f}N_{\text{cr.}}\frac{a_2^2 - b_0^2}{a_1^2}e^{\left(\frac{b_0}{a_1}\right)^2}. \tag{14}$$

The results of calculations of the integral relations in formula (8) for various Bessel functions and various characteristic plasma dimensions are depicted in Fig. 3. The form factor changes in an interval 10^{-6}–0.216 for oscillations of the types $E_{9.10}$ and $E_{9.9}$ and 0.047–0.329 for oscillations of the types $E_{3.13}$, $E_{4.13}$, $E_{4.12}$, and $E_{4.11}$. With regard for the values of $E(r)$ for every oscillation type, the dimensions of the central region, in which the variation of the plasma density does not result in a change of the resonance frequency, was determined. If the radius of this region is identified as that, at which $E = 0.1E_{\max}$, then $b_0 = 3.4$ mm for $E \sim J_4$ and $b = 9.3$ mm for $E \sim J_9$. At the same time, if the boundary radius corresponds to $E = 0.3E_{\max}$, then $b_0 = 4.8$ mm for $E \sim J_4$ and $b = 11.2$ mm for $E \sim J_9$.

Similar calculations were carried out for the case of uniform radial distribution of the plasma density. In this variant, the diameter of a plasma flow (formation) can be determined from the relation for the frequency shifts of oscillations of two types. The corresponding form factor A_f looks like

$$A_f^{(m)} = \frac{\int_0^{a_2} J_m^2 r dr}{\int_0^{a_0} J_m^2 r dr}, \tag{15}$$

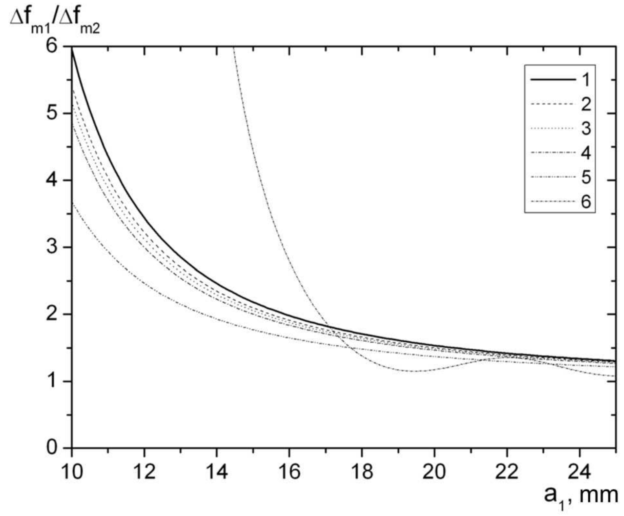

Fig. 5. Dependences of the frequency shift ratio for oscillations of two types on the characteristic plasma size at the radial plasma density distribution of form (7) for oscillations of types $E_{3.12}$ and $E_{E9.9}$ (*1*), $E_{4.13}$ and $E_{9.9}$ (*2*), $E_{4.12}$ and $E_{9.9}$ (*3*), $E_{4.11}$ and $E_{9.9}$ (*4*), $E_{4.11}$ and $E_{9.10}$ (*5*), and for $E_{4.11}$ and $E_{9.9}$ at the uniform radial distribution of the plasma density

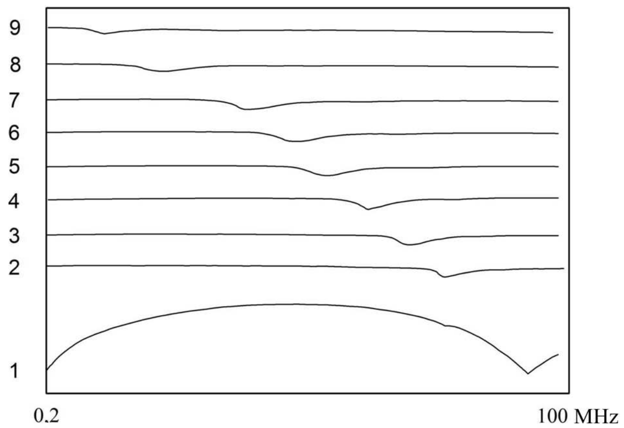

Fig. 6. Characteristic oscillograms describing the resonance shift for oscillations of type $E_{9.9}$: (*1*) variation of the radiation power in the generation zone, (*2–9*) resonance positions at various plasmatron currents I = 0, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, and 1.6 A

and the ratio $\Delta f_{m1}/\Delta f_{m2}$ between the frequency shifts like

$$\frac{\Delta f_{m_1}}{\Delta f_{m_2}} = \frac{\int_0^{a_2} J_{m_1}^2 r dr \int_0^{a_0} J_{m_2}^2 r dr}{\int_0^{a_0} J_{m_1}^2 r dr \int_0^{a_2} J_{m_2}^2 r dr}. \tag{16}$$

The dependence $A_f = f(a_2)$ is plotted in Fig. 4 (the form factor changes from 0 to 1), and the dependen-

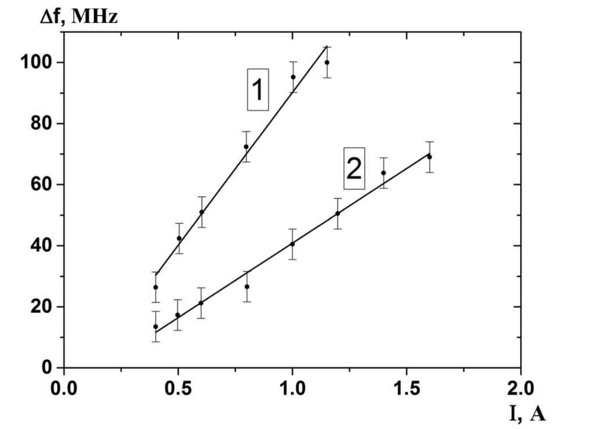

Fig. 7. Dependences of the resonance frequency shift on the plasmatron current for various types of excited oscillations: $E_{4.11}$ (*1*) and $E_{9.9}$ (*2*)

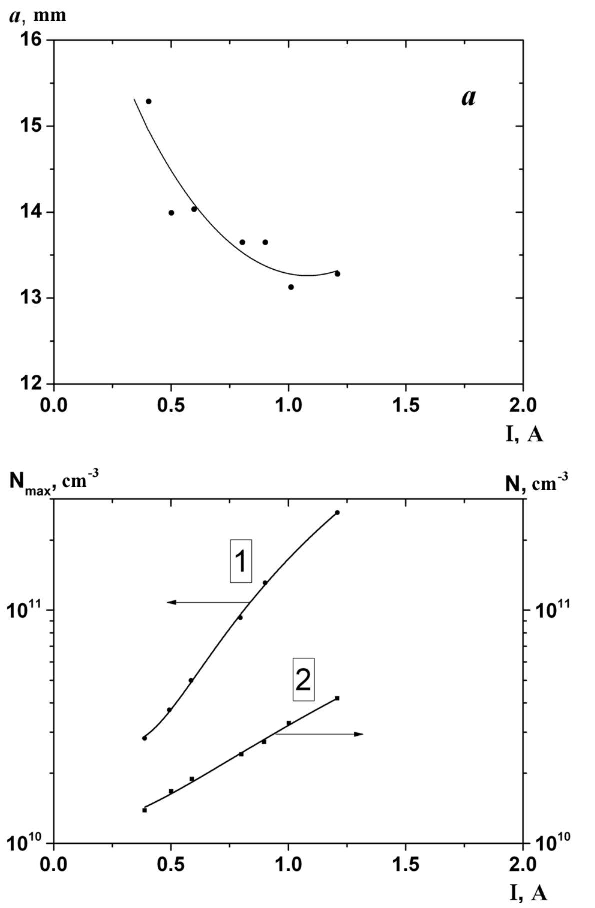

Fig. 8. Dependence of the characteristic plasma size on the plasmatron current (*a*); dependences of the maximum (1) and average (2) plasma densities on the plasmatron current (*b*)

ces $(\Delta f_{m1}/\Delta f_{m2}) = f(a_2)$ and $(\Delta f_1/\Delta f_2) = f(a_2)$ in Fig. 5.

Preliminary measurements were carried out with the help of the method described above and the microwave device on an installation "Kristall-2" operating in the normal regime (the working gas was hydrogen, the strength of the toroidal guiding magnetic field equaled 0.1 T, and the plasmatron arc discharge current was varied from 2 to 12 A) [17,18]. The measuring resonator was arranged in the toroidal holding magnetic field. The plasma flow was injected from a plasmatron and propagated along the internal toroid axis. The dependence of the plasma density on the plasmatron discharge current was measured. Microwave measurements were carried out for two resonances spaced by a frequency interval of 314 MHz.

Characteristic oscillograms illustrating the dependence of the resonance frequency shift on the plasmatron current are exhibited in Fig. 6. The resonance smearing is a result of density fluctuations. The dependences of the frequency shift for oscillations of types $E_{4,11}$ and $E_{9.9}$ on the plasmatron current are shown in Fig. 7. The processing of measured frequency shifts with the help of formulas (7), (9)–(12), and (14) allowed the maximum and average plasma densities, as well as the characteristic transverse size $2a_1$ of a plasma formation (see Fig. 8), to be obtained.

The average density values calculated from the frequency shifts Δf_1 and Δf_2 of both oscillation types $E_{4.11}$ and $E_{9.9}$ differ from each other. The average density measured from the frequency shift for mode $E_{4.11}$ differs from the value averaged over the whole volume by 5–10% at the plasmatron arc currents $I > 0.6$ A. At the same time, the same procedure for mode $E_{9.9}$ gives a 1.5–1.7 times difference. The main source of the error is the inaccuracy in the determination of oscillation type, because oscillations with high mode numbers differ slightly by the frequency and the field distribution in the resonator. In this connection, a numerical experiment was carried out to study a deviation of the calculated dependences at the variation of mode numbers. The obtained results testify that these errors (deviations) amount to approximately 10%, which is less than the spread associated with the irreproducibility of experimental conditions.

4. Conclusions

Our researches allowed a dismountable design for the barrel microwave resonator to be proposed, which makes it possible to arrange the resonator in magnetic systems with rather complicated spatial configurations. The described device is intended for the measurements of the average and maximum densities of plasma and its profile at $N/N_{\rm cr.} \ll 1$ in plasma devices equipped or not with a plasma chamber. Earlier, the probe measurements or microwave measurements at longer waves were used for this purpose.

A method is proposed to calculate the spatial distribution of the plasma density, which is based on the measurement of the resonance frequency shifts for oscillations of two types with high mode numbers. The form factors for various oscillation types and the functional dependence of the frequency shift ratio on the characteristic plasma parameter are calculated. The average and maximum plasma densities and the characteristic sizes of the plasma formation at various plasmatron discharge currents are measured on the installation "Kristall-2". The numerical simulation method is used to estimate the error obtained at the determination of the absolute value and the spatial distribution of the plasma density as a function of the error made while determining the field distribution in the resonator.

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НВЧ ПРИСТРІЙ НА ОСНОВІ
БОЧКОПОДІБНОГО РЕЗОНАТОРА
ДЛЯ ВИЗНАЧЕННЯ СЕРЕДНЬОЇ ГУСТИНИ
І ПРОФІЛЯ ПЛАЗМОВОГО УТВОРЕННЯ

Р е з ю м е

Наведено опис НВЧ резонаторного пристрою, призначеного для вимірювання густини плазми. Запропоновано метод визначення функції радіального розподілу густини по вимірюванню зсуву резонансних частот двох типів коливань. Проведені експерименти показали, що пристрій придатний для вимірювання параметрів плазми в діапазоні густини 10^9–10^{11} см$^{-3}$.