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POLARIZED RADIATION INTERFEROMETRY RESEARCH OF THERMOELASTICITY IN SILICON CRYSTAL WITH THE USE OF MODULATION POLARIMETRY

By registering the polarization state of probing radiation transmitted through a silicon crystal wafer, the optical anisotropy induced in the crystal by the heat flow from an external contact heater has been studied. The modulation polarimetry technique is used, the high resolution of which allowed the measurements to be made under conditions of the low temperature gradient and, hence, temperature-independent coefficients. A superposition of two- and multibeam interferences of circularly polarized radiation is detected. It is shown that the corresponding parameters provide information on the magnitude of mechanical stresses (dielectric anisotropy), as well as on some optical coefficients.

Keywords: modulation polarimetry, thermoelasticity, photoelasticity, polarized radiation interference, mechanical stress.

1. Introduction

The physics of thermoelasticity is a domain in the mechanics of deformed elastic bodies dealing with the relations between the space-time distributions of stress (deformation) and temperature (the medium composition) [1]. Plenty of methods are used for the registration of this phenomenon: ultrasonic [2], Raman spectroscopy [3,4], X-ray [5] and neutron [6,7] diffraction, magnetic [8] ones, as well as methods based on holographic interferometry [9–11]. At the same time, the optical polarization method [12] continues to be used for the research of the kinetics and the dynamics of thermomechanic stresses in a solid. This method is based on the phenomenon of the two-beam interference of polarized radiation. In due time, it was modified by applying the polarization modulation technique [13] and became rather extended. The essential advantage of the method consists in its applicability to objects of different nature, because the thermoelasticity phenomenon is inherent in almost all substances. This is confirmed by the experimental data on stresses and deformations in metals [14, 15] and ceramics [16–18]. Therefore, there are the grounds to suppose that detecting the optical responses of a medium on the action of external thermal sources by combining the polarized interference phenomenon and the modulation polarimetry method will make it possible to obtain extended information on various medium parameters; first of all, structural, thermal, relaxation, and optical properties.

Therefore, this work is aimed at studying the features of the thermoelasticity phenomenon by applying the polarized radiation interference technique. Numerous relevant publications testify that this phenomenon is not less extended in the Nature than the non-polarized interference. At the same time, the scope of its application in practice is narrower, which has its own reasons. The matter is that this method is characterized by a spatial inhomogeneity of the wave polarization rather than the energy, as it occurs in the case of non-polarized interference. The detection of the polarization state inhomogeneity needs the application of additional analytical elements, e.g., a linear polarizer. In this case, the polarization state is analyzed by a polarization modulation. The combination of interferometry and modulation polarimetry provides a high detecting sensitivity to stresses (the optical anisotropy), which should favor the discovery of specific features in the thermoelasticity phenomenon in the case of weak temperature fields and, as a consequence, when the coefficients are temperature-independent.

2. Formalism

In the case of a uniaxial crystal with its optical axis oriented along the OY one, the corresponding tensor of dielectric permittivity is determined by two components, $\varepsilon_x = \varepsilon_z$ and ε_y, and the complex refractive indices are $\tilde{n}_x = \sqrt{\varepsilon_x}$ and $\tilde{n}_y = \sqrt{\varepsilon_y}$, respectively. Let a light wave with the frequency ω and the amplitude $\mathbf{E} = (E_x, E_y, 0)$ be directed to the specimen surface $z = 0$. For every wave, one can use the Fresnel formulas [19] for an anisotropic wafer with the refractive indices $\tilde{n}_x$ and $\tilde{n}_y$ in order to describe its reflection and transmission coefficients. For $\tilde{n}_x$, those formulas look like (for $\tilde{n}_y$, the corresponding expressions are analogous)

$$r_x = \frac{r_{x12}(1 + e^{i\delta_x})}{1 + r_{x12}^2 e^{i\delta x}}, \quad t_x = \frac{(1 - r_{x12}^2)}{1 + r_{x12}^2 e^{i\delta_x}}\, e^{i\delta_x/2}, \qquad (1)$$

where r_x is the total amplitude of a reflected wave, $\delta_x = 2\,\omega\, d\, \tilde{n}_x/c$ is the phase angle, ω the frequency, d the specimen thickness, c the speed of light; $r_{x12} = (1 - \tilde{n}_x)/(1 + \tilde{n}_x)$ is the reflection coefficient for the air-crystal interface; and t_x the total amplitude of a transmitted wave. The vector amplitudes of reflected, $\mathbf{E}_r$, and transmitted, $\mathbf{E}_t$, waves look like

$$\mathbf{E}_r = r_\perp \mathbf{E}_{ix} + r_\parallel \mathbf{E}_{iy}, \quad \mathbf{E}_t = t_\perp \mathbf{E}_{ix} + t_\parallel \mathbf{E}_{iy}, \qquad (2)$$

where the subscript i marks the first ($i = 1$) and second ($i = 2$) surfaces of the specimen. The complex refractive index $\tilde{n}$ is expressed in terms of the refractive index n and the absorption index k in the form $\tilde{n} = n + ik$.

At a uniaxial crystal deformation, the refractive indices along the squeezing direction, n_y, and perpendicularly to it, n_x, change differently. As a model, let us adopt the linear dependences of the components of the dielectric permittivity tensor on the pressure:

$$\varepsilon_y = \varepsilon_0 + a\sigma, \quad \varepsilon_x = \varepsilon_0 - a\mu\sigma, \qquad (3)$$

where $\varepsilon_0 = (n_0 + ik_0)^2$, n_0 and k_0 are the refractive and absorption, respectively, indices in the absence of a pressure, μ is Poisson's ratio, and $a = a' + ia''$ is a complex parameter depending on the light frequency. In the case of weak absorption ($k_0 \ll n_0$), we obtain, in the linear approximation,

$$\begin{aligned} n_y &= n_0 + C\sigma, \quad k_y = k_0 + C\sigma, \\ n_x &= n_0 - \mu C\sigma, \quad k_x = k_0 - \mu C\sigma, \end{aligned} \qquad (4)$$

where (n_y, k_y) and (n_x, k_x) are the refractive and absorption indices in the directions parallel (y) and perpendicular (x) to the squeezing one.

The expressions obtained for the electric fields of orthogonal wave components transmitted through the specimen should be substituted into expressions for the Stokes parameters $S = [I, Q, U, V]$ [20]. In our experiment, only one of them, which describes a circularly polarized transmitted wave, has a practical value,

$$V_t = 2E_x^t\, E_y^t\, \sin\Delta. \qquad (5)$$

Here, the parameter Δ stands for the phase difference acquired by the wave along the specimen thickness,

$$\Delta = \delta_x - \delta_y = \frac{2\pi(n_x - n_y)d}{\lambda}. \qquad (6)$$

Within the interval of the Hooke law validity, this phase difference linearly depends on the stress magnitude, $n_x - n_y \sim \sigma$, and transforms the initial radiation state into an elliptically polarized one, which, as a consequence, gives rise to the emergence of a circularly polarized component in its composition.

3. Experimental Technique

In order to study the thermoelasticity phenomenon using the method of polarized radiation interference, silicon was selected on the basis of the following reasons. A considerable magnitude of silicon refractive index $n = 3.51$ provides an enhanced reflection coefficient ($r \approx 0.35$), which favors the multibeam interference. In addition, the low adsorption coefficient for undoped crystals ($\alpha \approx 1$ cm^{-1}) [21] satisfies the transparency condition. A specimen in the form of a plane-parallel wafer $15 \times 15 \times 2$ mm^3 in dimensions with perfect z surfaces rested by its own weight on a pyroceramic substrate with a nickel film resistor. The temperature gradient in the specimen was created by

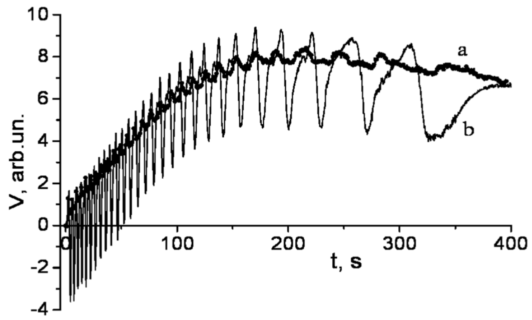

Fig. 1. Intensity of circularly polarized radiation transmitted through the specimen in the course of its heating: two-beam interference (a) and superposition of two- and multibeam interferences (b)

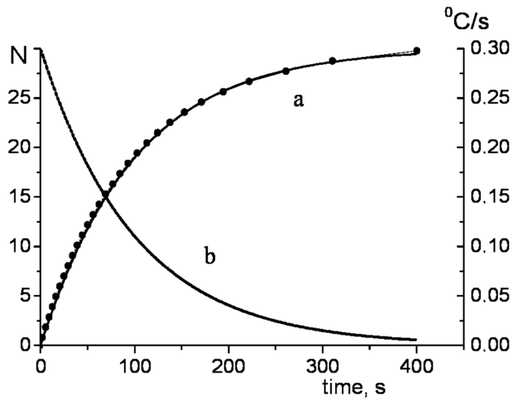

Fig. 2. Time dependence of the oscillation number during the heating of a specimen (point) in comparison with the approximation dependence $N = 30\exp(-t/\tau)$ with $\tau = 100$ s

means of the contact heating of the specimen from its y surface with the use of a heat source 1 W in power, which provided the thermal front close to planar in the specimen. The temperature difference across the specimen was about 10 °C in 400 s after the heater was switched on. The probing radiation of a He–Ne laser ($\lambda = 1.15$ μm) propagated along the z axis, and the field azimuth of a linearly polarized wave amounted to 45° with respect to the thermal flux direction. Measurements were carried out in the air environment, at the atmospheric pressure, and under the natural convection conditions. In this connection, some actions aimed at eliminating the instability of natural air fluxes near the specimen had to be done.

The circular componentamplitude, I_v, was registered using the modulation polarimetry method. Its specific features associated with its adaptation to the registration of thermoelasticity were described in work [22]. In particular, it was shown there that the intensity of the circularly polarized radiation component is determined by the formula

$$I_V = E_x E_y \sin\Delta \sin\Omega t, \qquad (7)$$

where Ω is the polarization modulation frequency (50 kHz). As one can see, at small phase delays, $\Delta < 1$, which took place in our experiment, this parameter is a linear measure of the internal mechanical stress.

The circular component was measured provided that two critical conditions were satisfied. First, in order to increase the spatial resolution, the laser beam was focused into a spot of light as small as possible. Second, the signal detection frequency (a selective amplifier, a lock-in nanovoltmeter, a Ge photodetector) was chosen to be $f = 5$ s^{-1}, which is sufficient for the shape of an oscillating interference signal to be adequately reproduced. The amplitude of the circular radiation component was recalculated into the mechanical uniaxial stresses at certain specimen points, which were registered as time-dependent functions.

4. Measurement Results and Their Discussion

For the further explanation, the meaning of the polarized radiation interference should be elucidated. This term is used for the phenomenon of a phase shift between the orthogonal components of the wave, when the latter propagates in an anisotropic medium, and, as a result, its polarization state changes [23]. This definition is also applicable to a superposition of two or more waves with different polarization states resulting in a certain combination of Stokes parameters.

Those two cases (two- and multibeam interferences) are illustrated in Fig. 1. The time dependences of the same quantity I_V (see Eq. (7)) were measured at the specimen middle point under two different conditions. The time was reckoned from the momentum, when a heater was switched-on. The general form of each of those characteristics exhibits the same kinetics of the mechanical stress induced by a nonlinear gradient of the temperature. However, curve a was obtained in the case where the beam was not maximally focused intentionally. The mechanism of its formation, which reflects the relation between the mechanical stress magnitude and the spatial temperature distribution during the transient period before the stationary heating state was attained, was considered in work [22] in detail. In our case, curve a depicts the amplitude of the circular component of an elliptically polarized wave averaged over the light spot on the temperature gradient. One can see that the almost smooth dependence – in effect, this is a two-beam interference – also contains remnants of the multibeam interference as a result of the internal reflection from both specimen surfaces.

This interference was much more pronounced if the specimen was arranged in the beam focus. In the corresponding curve b in Fig. 1, the multibeam interference amplitude and the parameters of curve b become comparable. Curves a and b differ from each other in that curve b contains an alternating contribution, with an infinitely growing period, made by a circularly polarized radiation resulting from its multiple reflections from the specimen surfaces.

Let us mark some features of those characteristics. The curves coincide at their end points, i.e. when the temperature does not vary in time. The asymmetry of the oscillation shape is typical of the spectral dependences for the Fabry–Pérot resonator. The variation of the oscillation period (frequency) reflects the stress kinetics and the corresponding phase at a certain coordinate of the thermal flux. In Fig. 2, this kinetics, $\sigma(t)$, is presented by curve a demonstrating the dependence of the oscillation number on the time. The curve is almost perfectly approximated by the exponential dependence $N = 30\exp(-t/\tau)$, with the best fitting attained at the characteristic parameter value $\tau = 100$ s.

Using Eq. (5) for the simulation, the dependences $\sigma(t)$ were calculated for the cases of two-beam interference (curve a) and the superposition of two- and multibeam interferences (curve b), which are shown in Fig. 3. Curve a is a model one, because it was obtained as an artificial combination of two exponential functions with the values of characteristic parameters measured from the experiment. At the same time, curve b was calculated, by considering the temperature dependences of such wafer parameters as the thickness d, complex refractive index $\tilde{n}$, and stress $\sigma(y)$ at the observation point. The qualitative agreement between the experimental and theoretical results testifies to the validity of the used model approximation, in which the real values of corresponding temperature coefficients were applied [21].

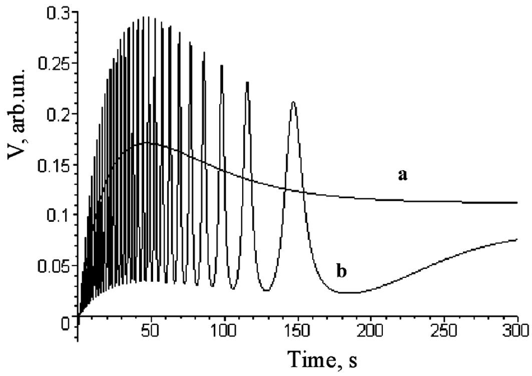

Fig. 3. Computer simulation of two-beam interference (a) and the superposition of two- and multibeam interferences (b) of circularly polarized radiation. The model parameters are $n = 3.6 + 0.0001(T - 273)$, $T = 340 - 20\exp(-t/\tau)$, $d = 0.29915 + 1.385 \times 10^{-5}(T - 273)$, $\sigma = 0.2\exp(-t/\tau) + [0.06 - 0.3\exp(-t/3)]$, $\tau = 40$ s, and $\omega = 1.6 \times 10^{15}$ Hz

Let us consider the behavior of the dependence a in Fig. 2. Here, the ordinate difference ΔN between neighbor points, which is characterized in accordance with Eq. (7) by the identical phase difference, corresponds to longer and longer segments on the abscissa axis. It can be demonstrated that curve b plotted for the ratio $\Delta N/\Delta t$ corresponds to the temperature variation rate at a certain coordinate. This result is not strange because of the invariance of the exponential function with respect to its differentiation and integration. Strange is the discrepancy between the functions obtained in the cases of two- and multibeam interferences, which should be a matter of the further consideration.

5. Conclusions

A method based on the interference of polarized radiation was used to study the thermoelasticity that arises in a silicon crystal with an inhomogeneous temperature field. Unlike the method based on the photoelastic effect, which is, in essence, a two-beam interference of polarized radiation, the variant of multibeam interference was applied. The multibeam interference is registered by the modulation polarimetry technique. It is due to the analytical capabilities of the applied method that the measurements of the circularly polarized radiation component resulting from the elastic birefringence become possible under the conditions of a small temperature difference. The exponential dependence of the characteristic obtained from the multibeam interference parameter testifies to the linearity of differential equations describing the heat transfer and the invariability of corresponding temperature coefficients in the examined interval. The result of a multibeam interference simulation qualitatively agrees with the experimental one for the known temperature dependences of the refractive index and the thickness of the specimen. This fact gives grounds for the phenomenon concerned to be used, while solving the inverse problem of determination of the thermal parameters of a substance.

1. A.D. Kovalenko, *Thermoelasticity. Basic Theory and Applications* (Walters-Noordhoff, Groningen, 1969).
2. T. Iwaoka, S. Yokogama, and Y. Osaka, Jpn. J. Appl. Phys. **24**, 112 (1984).
3. I.I. Vlasov, V.G. Ralchenko, E.D. Obraztsova, A.A. Smolin, and V.I. Konov, Appl. Phys. Lett. **71**, 1789 (1997).
4. B. Eigenmann, B. Scholtes, and E. Macherauch, Mater. Sci. Eng. A **118**, 1 (1989).
5. M.R. Daymond, M.A. Bourke, and R.B. Von Dreele, B. Clausen, and T.J. Lorentzen, J. Appl. Phys. **82**, 1554 (1997).
6. M. Ceretti, C. Braham, J.L. Lebrun, J.P. Bonnafe, M. Perrin, and A. Lodini, Exp. Techniques **20**, No. 3, 14 (1996).
7. E.S. Gorkunov and M.V. Tartachnaya, Zav. Laborat. **59**, 22 (1993).
8. G.N. Chernyshev, A.L. Popov, V.M. Kozintsev, and I.I. Ponomarev, *Residual Stresses in Deformed Solids* (Nauka, Moscow, 1996) (in Russian).
9. M.J. Pechersky, R.F. Miller, and C.S. Vikram, Opt. Eng. **34**, 2964 (1995).
10. C.S. Vikram, M.J. Pechersky, C. Feng, and D. Engelhaupt, Exp. Techniques **20**, No. 6, 27 (1996).
11. A.Ya. Aleksandrov and M.Kh. Akhmetzyanov, *Polarization Optical Methods of Mechanics of Deformable Solids* (Nauka, Moscow, 1973) (in Russian).
12. Yu.I. Sirotin and M.P. Shaskolskaya, *Fundamentals of Crystal Physics* (Imported Publ., Chicago, 1983).
13. B.K. Serdega, *Modulation Polarimetry* (Naukova Dumka, Kyiv, 2011) (in Ukrainian).
14. M. Qian, Chinese J. Acoust. **14**, No. 2, 97 (1995).
15. R.M. Burbelo and M.K. Zhabitenko, in *Progress in Natural Science* (Taylor and Francis, London, 1996), Suppl. Vol. 6, p. 720.
16. R.M. Burbelo, A.L. Gulyaev, L.I. Robur *et al.*, J. Phys. (Paris) C7 **4**, 311 (1994).
17. H. Zhang, S. Gissinger, G. Weides, and U. Netzelmann, J. Phys. (Paris) C7 **4**, 603 (1994).
18. K.L. Muratikov, A.L. Glazov, and D.N. Rose, J.E. Dumar, and G.H. Quay, Pis'ma Zh. Tekhn. Fiz. **26**, No. 5, 44 (1997).
19. R.M.A. Azzam and N.M. Bashara, *Ellipsometry and Polarized Light* (North-Holland, Amsterdam, 1989).
20. M. Born and E. Wolf, *Principles of Optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Light* (Cambridge Univ. Press, Cambridge, 1999).
21. A. Dargys and J. Kundrotas, *Handbook on Physical Properties of Ge, Si, GaAs and InP* (Science and Encyclopedia Publishers, Vilnius, 1994).
22. I.E. Matyash, I.A. Minailova, O.N. Mishchuk, and B.K. Serdega, Fiz. Tverd. Tela **55**, 1003 (2013).
23. S.A. Akhmanov and S.Yu. Nikitin, *Physical Optics* (Clarendon Press, Oxford, 1997).

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ДОСЛІДЖЕННЯ ТЕРМОПРУЖНОСТІ
У КРИСТАЛІ КРЕМНІЮ МЕТОДОМ ІНТЕРФЕРЕНЦІЇ
ПОЛЯРИЗОВАНОГО ВИПРОМІНЮВАННЯ
З ВИКОРИСТАННЯМ МОДУЛЯЦІЙНОЇ
ПОЛЯРИМЕТРІЇ

Р е з ю м е

Методом реєстрації стану поляризації зондуючого випромінювання у пропусканні пластинкою кристалічного кремнію досліджується оптична анізотропія, що зумовлена тепловим потоком від зовнішнього контактного нагріву. У вимірюваннях використано техніку модуляційної поляриметрії, завдяки високій виявній здатності якої дослідження проведено в умовах незначного перепаду температур та температурно незалежних коефіцієнтів. Виявлено суперпозицію інтерференцій циркулярно поляризованого випромінювання у дво- та багатопроменевому варіантах і показано, що із її параметрів можна отримати відомості про величини механічного напруження (діелектричної анізотропії) та визначити деякі оптичні коефіцієнти.