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V.I. ROMANENKO,<sup>1</sup> A.V. ROMANENKO,<sup>2</sup> L.P. YATSENKO<sup>1</sup><sup>1</sup>Institute of Physics, Nat. Acad. of Sci. of Ukraine  
(46, Nauky Ave., Kyiv 03680, Ukraine; e-mail: vr@iop.kiev.ua)<sup>2</sup>Taras Shevchenko National University of Kyiv  
(4, Academician Glushkov Ave., Kyiv 03022, Ukraine; e-mail: alexrm@univ.kiev.ua)**AN OPTICAL TRAP FOR ATOMS  
ON THE BASIS OF COUNTER-PROPAGATING  
BICHROMATIC LIGHT WAVES**PACS 37.10.Gh, 37.10.Mn,  
37.10.Pq

*It has been shown that the field of counter-propagating bichromatic light waves can form a one-dimension trap for atoms. The confinement of an ensemble of atoms in the trap and their simultaneous cooling to a temperature close to the Doppler cooling limit can be achieved without using auxiliary fields, by properly choosing the detuning of monochromatic components from the atomic transition frequency. The specific numerical simulation is carried out for sodium atoms. A possible application of such a trap for the confinement and the cooling of molecules with an almost diagonal array of Franck-Condon factors is discussed.*

*Keywords:* trap for atoms and molecules, laser cooling, Monte Carlo wave function construction.

**1. Introduction**

Optical traps for atoms where the latter are additionally cooled down are one of the basic tools in modern optics. In particular, the magneto-optical trap [1] is widely used for this purpose, where continuous laser radiation together with a magnetic field is applied to confine and simultaneously to cool down atoms. As was shown in a number of papers [2–6], the trap for atoms can be formed by the field of pulsed lasers as well. Moreover, if the pulse parameters are properly chosen, it is possible to reach simultaneously the confinement and the cooling of atoms in such traps [7–9]. Traps on the basis of light pulses can be used to confine and to cool down not only atoms, but also molecules with the matrix close to the diagonal one, when the interaction between a molecule and the field can be described well by a scheme with two or three energy levels [9].

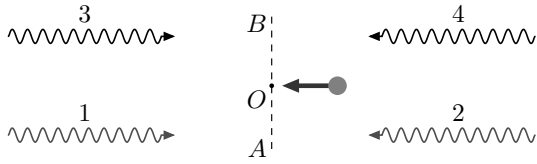
Traps formed by light pulses are based on the spatial nonuniformity of an electromagnetic field, which results in the formation of a force directed to a certain point on a large spatial interval. This force brakes atoms at their removal from this point and, hence, forms a trap. In the field generated by sequences of counter-propagating light pulses, the point, where

those pulses “meet” [2], is a trap center. On the different sides from this point, the sequences of pulse-atom interactions are different, and this fact results in the formation of a force directed to the trap center. The spatial period of the field in the trap formed by the sequences of counter-propagating pulses is equal to the distance between the pulses propagating in the same direction. Besides atoms and molecules, the trap on the basis of light pulses can retain nanoparticles as well.

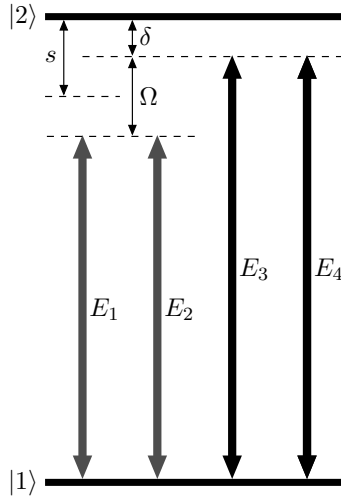
The large-scale spatial structure of the field generated by the sequences of counter-propagating pulses is closely related to their spectrum: to the set of frequencies and the phases of spectral components. Evidently, at least two spectral components are required to form such a structure. Therefore, a trap for atoms may expectedly be constructed by the field of counter-propagating bichromatic waves. If the frequency difference between two components of a bichromatic wave is  $\Omega$ , the period of spatial modulation of this field equals  $L = 2\pi c/\Omega$ . In the case of bichromatic field, if comparing it with the case of short light pulses, which have a wide spectrum, one may expect a reduction of the probability for undesirable resonances to emerge at the interaction of the field with atoms and molecules, as well as a simplification in the implementation of experimental light traps.

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**Fig. 1.** Schematic diagram of the trap. The combined action of counter-propagating waves 1, 2 and 3, 4 stimulates an atom to move near the trap center  $O$



**Fig. 2.** Schematic diagram of the interaction between atoms or molecules and light. The difference of frequencies of the waves 2, 4 and 1, 3 is equal  $\Omega$ , their detunings from the resonance frequency of the atomic transition are  $\delta_3 = \delta_4 = \delta$  and  $\delta_1 = \delta_2 = \delta + \Omega$ , the parameter  $s = \delta + \Omega/2$

The possibility to form a trap for atoms by the field of running counter-propagating bichromatic waves, which can also be considered as a bichromatic field of two collinear standing waves, was indicated for the first time in work [12]. It should be noted that the bichromatic field has already been used for a long time to brake atoms for their further cooling and confinement, and numerous publications are aimed just at this aspect [10–20]. Recent researches devoted to the monochromation of atomic beams with a bichromatic field at a low rate of spontaneous radiation emission from the excited state [21, 22] open new prospects for the application of bichromatic fields to atomic physics, in particular, to the control over the motion of molecules and their cooling.

Below, we consider the interaction of atoms with a bichromatic field of two standing waves near the point, where the antinodes of both waves coincide,

i.e. their phase difference equals zero. Near this point, the force of light pressure that acts on the atom is proportional to the phase difference [10, 20]. Since the latter linearly depends on the coordinate, a trap for atoms is formed. We will demonstrate that atoms in the trap can be simultaneously confined and cooled, as it takes place in a trap formed by the sequences of counter-propagating pulses [7–9].

The structure of the present work is as follows. The trap model is described in the next section, and the master equations are presented in Section 3. Section 4 is devoted to the calculation of the atomic wave function by the Monte Carlo method. In Section 5, an analytical expression for the force of light pressure acting on the atom in a weak field is obtained. Section 6 gives description of the numerical calculation procedure. The corresponding results are presented and discussed in Section 7. Short conclusions are formulated in Section 8.

## 2. Trap Model

A one-dimensional trap for atoms is formed by counter-propagating monochromatic waves with the frequencies close to the frequency  $\omega_0$  of the transition between the ground,  $|1\rangle$ , and excited,  $|2\rangle$ , states of the atom (Figs. 1 and 2). An atom (a circle in Fig. 1) near the point  $O$  is subjected to the action of a bichromatic field generating by two standing waves; each of them can be considered as a superposition of two counter-propagating monochromatic waves (1, 2 and 3, 4). Under the action of the light pressure force, the atom moves near the plane  $AB$  that passes through the point  $O$  and is oriented perpendicularly to the wave vectors of the monochromatic waves that form the trap.

## 3. Master Equations

In the general case, the spatial and temporal dependences of the electric field strength of monochromatic light waves that act on an atom or a molecule in the trap look like

$$\mathbf{E}_n = \frac{1}{2} \mathbf{e} E_{0n} \exp(i\omega_n t \mp ik_n z + i\varphi_n) + \text{c.c.}, \quad (1)$$

where the sign “ $-$ ” corresponds to odd  $n$ , and the sign “ $+$ ” to even ones;  $\mathbf{e}$  is the unit polarization vector; and  $\omega_n$  the frequency of a monochromatic wave. In the case where standing waves with identical amplitudes

are formed (just this case is considered here),

$$E_{01} = E_{02} = E_{03} = E_{04} = E_0, \quad (2)$$

$$\omega_2 = \omega_1, \quad \omega_4 = \omega_3. \quad (3)$$

The detunings  $\delta_n$  of the frequencies  $\omega_n$  ( $n = 1, \dots, 4$ ) from the resonance frequency of the atomic transition  $\omega_0$  equal

$$\begin{aligned} \delta_n = \omega_0 - \omega_n = \delta, \quad n = 3, 4, \\ \delta_n = \omega_0 - \omega_n = \delta + \Omega, \quad n = 1, 2, \end{aligned} \quad (4)$$

where  $\Omega = \omega_3 - \omega_1 \ll \omega_0$  is the difference between the standing wave frequencies. The parameter  $s = \delta + \Omega/2$  (see Fig. 2) is equal to the detuning of the arithmetic mean of the monochromatic wave frequencies from the atomic transition frequency.

The fields formed by the counter-propagating waves can be written in the form

$$\begin{aligned} \mathbf{E}_{12} = \mathbf{E}_1 + \mathbf{E}_2 = 2E_{01}\mathbf{e} \cos \left[ \omega_1 t + \frac{1}{2}(\varphi_1 + \varphi_2) \right] \times \\ \times \cos \left[ k_1 z + \frac{1}{2}(\varphi_2 - \varphi_1) \right], \end{aligned} \quad (5)$$

$$\begin{aligned} \mathbf{E}_{34} = \mathbf{E}_3 + \mathbf{E}_4 = 2E_{03}\mathbf{e} \cos \left[ \omega_3 t + \frac{1}{2}(\varphi_3 + \varphi_4) \right] \times \\ \times \cos \left[ k_3 z + \frac{1}{2}(\varphi_4 - \varphi_3) \right], \end{aligned} \quad (6)$$

and the total field acting on the atom equals

$$\mathbf{E} = \mathbf{E}_{12} + \mathbf{E}_{34}. \quad (7)$$

A representation of the field in the form of a superposition of counter-propagating amplitude-modulated waves has also been used since the early studies of the mechanical action of a bichromatic field on atoms [10, 12]. This representation allows an analogy to be drawn between the bichromatic field and the field generated by a sequence of counter-propagating pulses. This analogy forms a basis for the explanation of the force of stimulated light pressure, which can considerably exceed the force of light pressure on the atom in a single running wave [23]. In the case

$$E_{01} = E_{02} = E_{03} = E_{04} = E_0, \quad (8)$$

which was analyzed in work [10, 12], those counter-propagating waves look like

$$\mathbf{E}_{13} = \mathbf{E}_1 + \mathbf{E}_3 = 2E_0\mathbf{e} \cos \left[ \omega t - kz + \frac{1}{2}(\varphi_1 + \varphi_3) \right] \times$$

$$\times \cos \left[ \frac{1}{2}\Omega t - \frac{1}{2}\Delta kz + \frac{1}{2}(\varphi_3 - \varphi_1) \right], \quad (9)$$

$$\begin{aligned} \mathbf{E}_{24} = \mathbf{E}_2 + \mathbf{E}_4 = 2E_0\mathbf{e} \cos \left[ \omega t + kz + \frac{1}{2}(\varphi_2 + \varphi_4) \right] \times \\ \times \cos \left[ \frac{1}{2}\Omega t + \frac{1}{2}\Delta kz + \frac{1}{2}(\varphi_4 - \varphi_2) \right], \end{aligned} \quad (10)$$

where

$$\omega = \frac{1}{2}(\omega_1 + \omega_3) = \frac{1}{2}(\omega_2 + \omega_4), \quad (11)$$

$$\Omega = \omega_3 - \omega_1 = \omega_4 - \omega_2, \quad (12)$$

$$k = \frac{1}{2}(k_1 + k_3) = \frac{1}{2}(k_2 + k_4), \quad (13)$$

$$\Delta k = k_3 - k_1 = k_4 - k_2 = \Omega/c. \quad (14)$$

Let us select wave phases  $\varphi_n$  such that the maxima in the standing waves  $\mathbf{E}_{12}$  and  $\mathbf{E}_{34}$  would be observed at  $z = 0$  and  $t = 0$ . It is valid if

$$\varphi_1 = \varphi_2 = \varphi_3 = \varphi_4 = 0. \quad (15)$$

Then, by analogy with the trap on the basis of the sequences of counter-propagating pulses, we may expect that, under certain conditions, the bichromatic field will form a trap for atoms in a vicinity of the coordinate origin  $z = 0$ .

The force of light pressure acting on the atom is determined by the expression [1, 24]

$$F = (\varrho_{12}\mathbf{d}_{21} + \varrho_{21}\mathbf{d}_{12}) \frac{\partial \mathbf{E}}{\partial z}, \quad (16)$$

where  $\mathbf{d}_{ij}$  ( $i, j = 1, 2$ ) are the matrix elements of the dipole moment, and  $\varrho_{ij}$  the elements of the density matrix  $\varrho$ . Under the action of the force ( $\varrho$ ), the atom moves according to Newton's law

$$\dot{v} = F/m, \quad (17)$$

$$\dot{z} = v, \quad (18)$$

where  $m$  is the mass of the atom, and  $v$  its velocity.

In order to calculate the force, one has to know, besides the field, the atom density matrix as well. It can be determined from the wave function of the atom

$$|\psi\rangle = c_1 |1\rangle + c_2 e^{-i\omega_0 t} |2\rangle, \quad (19)$$

which satisfies the Schrödinger equation

$$i\hbar \frac{d}{dt} |\psi\rangle = H |\psi\rangle. \quad (20)$$

The wave function is constructed with regard for quantum jumps that are responsible for the process of spontaneous radiation by the atom (the Monte Carlo method for the wave function [25]). The elements of the density matrix in Eq. (16) look like

$$\varrho_{12} = c_1 c_2^* e^{i\omega_0 t}, \quad \varrho_{21} = c_2 c_1^* e^{-i\omega_0 t}. \quad (21)$$

After the averaging over the ensemble, the Monte Carlo calculations of the wave function gives the same result as the calculations with the help of the equations for the density matrix [25]. At the same time, unlike the calculation on the basis of the density matrix, the calculation of the wave function by the Monte Carlo method simulates the trajectory of motion for a separate atom. The averaging over the atomic ensemble gives its statistical characteristics: the average velocity of atoms,  $\bar{v}$ , the root-mean-square deviation of their velocities from the average value,  $\Delta v$ , the average coordinate of atoms,  $\bar{z}$ , and the root-mean-square deviation of their coordinates from the average value,  $\Delta z$ . Knowing the parameters  $\Delta v$  and  $\Delta z$ , it is possible to evaluate the temperature of the atomic ensemble and the region of its localization.

#### 4. Wave Function of the Atom

The Hamiltonian in the Schrödinger equation (20) for the calculation of a wave function using the Monte Carlo method looks like

$$H = H_0 + H_{\text{int}} + H_{\text{rel}}, \quad (22)$$

where the term

$$H_0 = \hbar\omega_0 |2\rangle\langle 2| \quad (23)$$

describes the atom in the absence of the field and the relaxation, the term

$$H_{\text{int}} = -\mathbf{d}_{12}|1\rangle\langle 2|\mathbf{E}(t) - \mathbf{d}_{21}|2\rangle\langle 1|\mathbf{E}(t) \quad (24)$$

describes the interaction between the atom and the field, and the term

$$H_{\text{rel}} = -\frac{i\hbar}{2}\gamma|2\rangle\langle 2| \quad (25)$$

describes the spontaneous radiation emission of the atom in the excited state accompanied by its transition into the state  $|1\rangle$  at the rate  $\gamma$ .

Hamiltonian (22) is non-Hermitian because of term (25), so that the square of the wave-function absolute value changes in time. When the Monte Carlo method is used to model the wave function, the normalization of the latter is performed after every short time step. In addition, after each time step, the condition of the quantum jump [25], i.e. the emission of a photon by the atom in the excited state and its transition into the state  $|1\rangle$ , has to be checked.

Below, the Monte Carlo method of the first accuracy order (see work [25]) is described. The methods of the second and fourth accuracy orders were described in work [26].

Let the atom be described by the wave function  $|\psi(t)\rangle$  at the time moment  $t$ . The wave function  $|\psi(t + \Delta t)\rangle$  at the moment  $t + \Delta t$  can be found in two stages.

1. From the Schrödinger equation (20), it follows that, after a sufficiently short interval  $\Delta t$ , the wave function  $|\psi(t)\rangle$  becomes equal to

$$|\psi^{(1)}(t + \Delta t)\rangle = \left(1 - \frac{i\Delta t}{\hbar}H\right) |\psi(t)\rangle. \quad (26)$$

Since Hamiltonian (22) is non-Hermitian, the function  $\psi^{(1)}(t + \Delta t)$  is not normalized to unity. From Eqs. (22) and (26), it follows that

$$\langle\psi^{(1)}(t + \Delta t)|\psi^{(1)}(t + \Delta t)\rangle = 1 - \Delta P, \quad (27)$$

where

$$\Delta P = \frac{i\Delta t}{\hbar} \langle\psi(t)|H - H^\dagger|\psi(t)\rangle = \gamma|\Delta t|c_2|^2. \quad (28)$$

2. At the second stage, let us take the possibility of a quantum jump into account. If the value of the random variable  $\epsilon$  uniformly distributed between zero and unity is larger than  $\Delta P$  (it is so in the majority of cases, because  $\Delta P \ll 1$ ), the jump does not occur, and the wave function at the moment  $t + \Delta t$  equals

$$|\psi(t + \Delta t)\rangle = \frac{|\psi^{(1)}(t + \Delta t)\rangle}{\sqrt{1 - \Delta P}}, \quad \Delta P < \epsilon. \quad (29)$$

At the same time, if  $\epsilon < \Delta P$ , the jump takes place, and the atom transits into the state  $|1\rangle$ .

Now, let us obtain equations for the amplitudes  $c_n$  of states  $|n\rangle$  ( $n = 1, 2$ ). The substitution of Eqs. (19) and (22) into Eq. (20) gives the following equations

for  $c_1$  and  $c_2$ :

$$\begin{aligned} i\hbar \frac{d}{dt} c_1 &= -\mathbf{d}_{12} \mathbf{E} c_2 e^{-i\omega_0 t}, \\ i\hbar \frac{d}{dt} c_2 &= -\mathbf{d}_{21} \mathbf{E} c_1 e^{i\omega_0 t} - \frac{1}{2} \gamma c_2. \end{aligned} \quad (30)$$

In the rotating wave approximation (i.e. rapidly oscillating terms  $\sim e^{\pm 2i\omega_0 t}$  are neglected), Eqs. (30) and the relation

$$\mathbf{E} = \sum_{n=1}^4 \mathbf{E}_n \quad (31)$$

yield

$$\begin{aligned} \frac{d}{dt} c_1 &= -\frac{i}{2} \Omega_0 \sum_{n=1}^4 e^{(-1)^n i k_n z - i \delta_n t} c_2, \\ \frac{d}{dt} c_2 &= -\frac{i}{2} \Omega_0^* \sum_{n=1}^4 e^{(-1)^{n+1} i k_n z + i \delta_n t} c_1 - \frac{1}{2} \gamma c_2, \end{aligned} \quad (32)$$

where the Rabi frequency of monochromatic waves is introduced,

$$\Omega_0 = -\mathbf{d}_{12} \mathbf{e} E_0 / \hbar, \quad (33)$$

and Eq. (15) is made allowance for. Without any loss of generality, the parameter  $\Omega_0$  can be regarded as a real-valued number.

The found wave function makes it possible, using Eqs. (16) and (21), to calculate the force of light pressure on the atom and to describe its motion by simultaneously integrating the Schrödinger equation and Newton's equations of motion. After the averaging of expression (16) for the force over a time interval that considerably exceeds the time of rapid oscillations with the characteristic time  $2\pi/\omega_0$ , but, at the same time, is short enough for the averaged force of light pressure to be practically independent of the time of the averaging, we obtain

$$\begin{aligned} F &= \hbar \sum_{n=1}^4 (-1)^{n+1} k_n \text{Im} [c_1 c_2^* \Omega_n^* e^{i\delta_n t} \times \\ &\times e^{i(-1)^{n+1} k_n z}] (|c_1|^2 + |c_2|^2)^{-1}. \end{aligned} \quad (34)$$

The normalization in Eq. (34) is not required, if the method of the first accuracy order was used, when integrating the Schrödinger equation (20) simultaneously with Eqs. (17) and (18). However, in the case where the integration of the Schrödinger equation and

the equations of motion within the method of the fourth accuracy order is combined with the Monte Carlo method of the first accuracy order, such a normalization for the wave function is necessary, because the transient time points are used.

## 5. Force of Light

### Pressure on the Atom in a Weak Field

In order to explain the operating mechanism of the trap, let us determine the force of light pressure acting on the atom in a weak field on the basis of the equations for the density matrix. The analysis of the expression obtained for the force will allow us to understand the physical basis of the trap functioning with regard for the fluctuation variation of the velocity due to the momentum diffusion (a random variation of the atomic momentum) in the process of spontaneous radiation.

The force of light pressure can be determined proceeding from expression (16) and the equation for the density matrix

$$i\hbar \frac{\partial}{\partial t} \varrho_{jk} = \sum_l (H_{jl} \varrho_{lk} - \varrho_{jl} H_{lk}) + i\hbar \sum_{l,m} \Gamma_{jk,lm} \varrho_{lm}, \quad (35)$$

where the Hamiltonian  $H$  equals

$$H = \hbar\omega_0 |2\rangle\langle 2| - \mathbf{d}_{12} |1\rangle\langle 2| \mathbf{E}(t) - \mathbf{d}_{21} |2\rangle\langle 1| \mathbf{E}(t). \quad (36)$$

In contrast to the description of the atom by the wave function, we cannot include relaxation processes into the Hamiltonian. The relaxation in Eq. (35) is described by the term containing the matrix  $\Gamma$ . The nonzero matrix elements of the latter equal

$$\begin{aligned} \Gamma_{12,12} &= \Gamma_{21,21} = -\gamma/2, \\ \Gamma_{11,22} &= -\Gamma_{22,22} = \gamma. \end{aligned} \quad (37)$$

In view of Eqs. (36) and (37), Eq. (35) implies that

$$\begin{aligned} i\hbar \frac{\partial}{\partial t} \varrho_{11} &= (\mathbf{d}_{12} \varrho_{21} - \mathbf{d}_{21} \varrho_{12}) \mathbf{E}(t) + i\hbar \gamma \varrho_{22}, \\ i\hbar \frac{\partial}{\partial t} \varrho_{22} &= (\mathbf{d}_{21} \varrho_{12} - \mathbf{d}_{12} \varrho_{21}) \mathbf{E}(t) - i\hbar \gamma \varrho_{22}, \\ i\hbar \frac{\partial}{\partial t} \varrho_{12} &= (\varrho_{11} - \varrho_{22}) \mathbf{d}_{12} \mathbf{E}(t) - i\hbar \frac{1}{2} \gamma \varrho_{12} - \hbar\omega_0 \varrho_{12}, \\ i\hbar \frac{\partial}{\partial t} \varrho_{21} &= (\varrho_{22} - \varrho_{11}) \mathbf{d}_{21} \mathbf{E}(t) - i\hbar \frac{1}{2} \gamma \varrho_{21} + \hbar\omega_0 \varrho_{21}. \end{aligned} \quad (38)$$

Let us extract rapidly oscillating multipliers in the non-diagonal elements of the density matrix:

$$\varrho_{12} = \sigma_{12} e^{i\omega_0 t}, \quad \varrho_{21} = \sigma_{21} e^{-i\omega_0 t}. \quad (39)$$

For field (7) in the rotating wave approximation, Eq. (38) yields

$$\begin{aligned} \frac{\partial}{\partial t} \varrho_{11} &= \frac{i}{2} \sigma_{12} \Omega_0 \sum_{n=1}^4 e^{i\delta_n t - (-1)^n i k_n z} - \\ &- \frac{i}{2} \sigma_{21} \Omega_0 \sum_{n=1}^4 e^{-i\delta_n t + (-1)^n i k_n z} + \gamma \varrho_{22}, \\ \frac{\partial}{\partial t} \sigma_{12} &= \frac{i}{2} (\varrho_{11} - \varrho_{22}) \Omega_0 \times \\ &\times \sum_{n=1}^4 e^{-i\delta_n t + (-1)^n i k_n z} - \frac{\gamma}{2} \varrho_{22}, \\ \sigma_{21} &= \sigma_{12}^*, \quad \varrho_{11} + \varrho_{22} = 1, \end{aligned} \quad (40)$$

where the Rabi frequency  $\Omega_0$  is defined by formula (33). In the same approximation, the force of light pressure is equal to

$$F = \hbar \sum_{n=1}^4 (-1)^{n+1} k_n \Omega_0 \operatorname{Im} \sigma_{12} e^{i\delta_n t - (-1)^n i k_n z}. \quad (41)$$

In calculations, we consider that the condition [24]

$$\frac{\hbar^2 k^2}{2m} \ll \hbar \gamma, \quad (42)$$

is satisfied. It provides that the force of light pressure is formed earlier than a variation of the atomic velocity considerably affects its magnitude (the heavy atom approximation).

Let us first calculate the force of light pressure on very slow atoms ( $kv \ll \gamma$ ). Since the frequencies of waves (1,2) and (3,4) are identical in pairs,

$$\delta_1 = \delta_2 = \delta + s, \quad \delta_3 = \delta_4 = -\delta + s. \quad (43)$$

Calculations to the fourth accuracy order of perturbation theory in the field give the following force of light pressure averaged over the coordinate (within the limits  $\Delta z \ll c/\Omega$ ) and time:

$$F_4 = - \frac{32\hbar k \Omega \Omega_0^4 \gamma^2 (\gamma^2 + \Omega^2 + 4s^2) \sin 2\Delta k z}{[(\Omega + 2s)^2 + \gamma^2]^2 [(\Omega - 2s)^2 + \gamma^2]^2}. \quad (44)$$

It is evident that the condition necessary for the formation of a trap is satisfied: at a very low velocity of the atom, the force of light pressure is directed to the trap at any  $s$ .

The force of light pressure at an arbitrary velocity of the atom will be determined in the case  $s = 0$ . In

the fourth order of perturbation theory (the second order in the field gives a zero contribution), we have

$$F_4 = - \frac{32\hbar k \Omega \Omega_0^4 \gamma^2 [\gamma^2 + \Omega^2 - 4(kv)^2] \sin \Delta k z}{[(\Omega + 2kv)^2 + \gamma^2]^2 [(\Omega - 2kv)^2 + \gamma^2]^2}. \quad (45)$$

From this expression, it follows that, until the velocity of the atom remains to be within the interval  $-v_c < v < v_c$ , where

$$v_c = \frac{1}{2k} \sqrt{\gamma^2 + \Omega^2}, \quad (46)$$

the force of light pressure on the atom is directed toward the trap center. If atom's velocity goes beyond the indicated limits (for example, if the velocity fluctuates owing to random spontaneous radiation), the force of light pressure pushes the atom out from the trap.

The motion of the atom is periodic if only force (45) is taken into account. However, it does not mean that this force can hold an ensemble of atoms in the trap. To simulate the motion of atoms in the light field, the knowledge of a light pressure force is not enough. This parameter allows one to calculate only the acceleration of the center of mass of an ensemble of atoms with identical initial conditions and its velocity. The velocities and the coordinates of separate atoms are different owing to the process of momentum diffusion associated with the stochasticity of spontaneous radiation. In due time, the velocities of some atoms will exceed  $v_c$ , and those atoms will leave the trap. Hence, it is impossible to hold atoms in the trap if  $s = 0$ .

A factor that counteracts the diffusion process in the momentum space can be a velocity-dependent force that decelerates atoms. Let us analyze the component of the second order in the force field of light pressure,

$$\begin{aligned} F_2 &= \frac{\hbar k \gamma \Omega_0^2}{(\Omega + 2kv + 2s)^2 + \gamma^2} - \frac{\hbar k \gamma \Omega_0^2}{(\Omega - 2kv + 2s)^2 + \gamma^2} + \\ &+ \frac{\hbar k \gamma \Omega_0^2}{(\Omega - 2kv - 2s)^2 + \gamma^2} - \frac{\hbar k \gamma \Omega_0^2}{(\Omega + 2kv - 2s)^2 + \gamma^2}. \end{aligned} \quad (47)$$

It is distinct from zero at  $s \neq 0$  and  $v \neq 0$ .

At a low velocity of atom motion ( $v \ll \gamma/k$ ), the force  $F_2$  linearly depends on the velocity:

$$F_2 = \kappa v, \quad (48)$$

where

$$\kappa = -\frac{8\hbar k^2 \gamma \Omega_0^2 (\Omega + 2s)}{[(\Omega + 2s)^2 + \gamma^2]^2} + \frac{8\hbar k^2 \gamma \Omega_0^2 (\Omega - 2s)}{[(\Omega - 2s)^2 + \gamma^2]^2}. \quad (49)$$

The friction force, which decelerates the atom, arises if  $\kappa < 0$ . An example of the dependence  $\kappa(s)$  is shown in Fig. 3. The value  $s = -\Omega/2$  corresponds to the resonance of the frequency of waves 1 and 2 with the atomic transition frequency, and  $s = \Omega/2$  to the corresponding resonance for the frequency of waves 3 and 4. The friction force arises, if  $-\varpi < s < 0$  or  $s > \varpi$ , where

$$\varpi = \frac{1}{2} \sqrt{2\Omega \sqrt{\Omega^2 + \gamma^2} - \Omega^2 - \gamma^2}. \quad (50)$$

Making allowance for the components of the second and fourth orders in the field, the force of light pressure can be written in the form

$$F = F_2 + F_4. \quad (51)$$

Here, the first term (of the second order in the field) does not depend on the atomic coordinate and, at properly chosen parameters of the interaction between the atom and the field, results in a deceleration of the atom. The second term (of the fourth order in the field) is responsible for a force that acts on slow atoms in the direction toward the trap center.

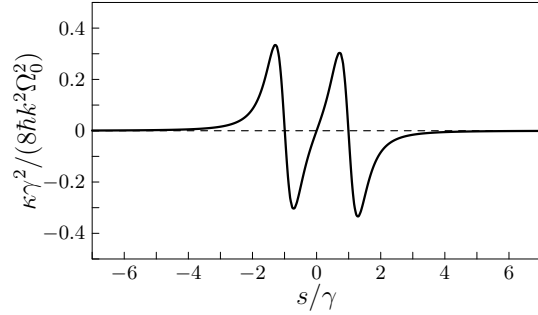
## 6. Numerical Calculation Procedure

While simulating the motion of an atom, Eqs. (17) and (18) for the force and Eqs. (34) and (32) for the probability amplitudes of atomic states were solved simultaneously. In addition, variations of the atomic momentum in the course of spontaneous radiation, as well as fluctuations of the absorption and the stimulated radiation emission resulting in fluctuations of the atomic momentum, were taken into consideration. In our calculations, for the sake of simplicity, we accepted that, in the course of spontaneous radiation, the atomic momentum changes by  $\pm\hbar k$  with an identical probability. When analyzing the Doppler cooling, this model is known [28] to give the following expression for the minimum temperature:

$$T_D = \hbar\gamma/2k_B, \quad (52)$$

where  $k_B$  is the Boltzmann constant.

Expressions (16) and (34) allow one to calculate the force value averaged over the ensemble. For the



**Fig. 3.** Dependence  $\kappa(s)$  for the interaction between the atom and the field of two low-intensity standing waves in the case  $\Omega = 2\gamma$

description of the atomic motion, it is also necessary to consider the stochastic momentum variation that arises owing to the momentum diffusion. In the case of a low laser radiation intensity, when the occupation number of the excited state is insignificant, the force of light pressure and the momentum diffusion coefficient are equal to the sums of corresponding quantities for each of counter-propagating waves [29]. This model was applied by us earlier [9] for the computer simulation of the fluctuation-driven momentum variation at the interaction of atoms with the field of counter-propagating low-intensity pulses.

Let us describe the momentum diffusion of an atom in the field of a running monochromatic wave following book [24]. Let the atomic momentum be equal to  $\mathbf{p}_0$  at the time moment  $t$ . Then, the momentum at the time  $t + \Delta t$  can be expressed by the formula

$$\mathbf{p} = \mathbf{p}_0 + \hbar\mathbf{k}(N_+ - N_-) - \sum_s \hbar\mathbf{k}_s. \quad (53)$$

Here, the second term describes the momentum variation due to the absorption and stimulated radiation of photons with the wave vector  $\mathbf{k}$ . We consider the radiation to propagate along the axis  $z$ . The quantities  $N_+$  and  $N_-$  are the number of photons absorbed and emitted at stimulated radiation, respectively, within the time interval  $\Delta t$ . The third term in Eq. (53) is responsible for the variation of the atomic momentum at spontaneous radiation of photons with the wave vectors  $\mathbf{k}_s$ .

Momentum (53) averaged over the ensemble equals

$$\langle \mathbf{p} \rangle = \langle \mathbf{p}_0 \rangle + \hbar\mathbf{k}(\langle N_+ \rangle - \langle N_- \rangle), \quad (54)$$

where  $\langle \mathbf{p}_0 \rangle$  is the average initial momentum,  $\langle N_+ \rangle$  the average number of absorbed photons, and  $\langle N_- \rangle$  the

average number of photons emitted in the course of stimulated radiation. Spontaneously emitted photons do not change the momentum on the average. Therefore,

$$\left\langle \sum_s \mathbf{k}_s \right\rangle = 0. \quad (55)$$

The difference between Eqs. (53) and (54) gives the momentum fluctuation,

$$\Delta \mathbf{p} = \mathbf{p} - \langle \mathbf{p} \rangle = (\mathbf{p} - \langle \mathbf{p}_0 \rangle) + \hbar \mathbf{k} \Delta N_i - \sum_s \hbar \mathbf{k}_s, \quad (56)$$

where  $\Delta N_i = N_i - \langle N_i \rangle$  is a deviation of the number of photons  $N_i = N_+ - N_-$  that are scattered at stimulated atomic transitions from its average value  $\langle N_i \rangle = \langle N_+ \rangle - \langle N_- \rangle$ .

From Eq. (56), we can find the average square of momentum fluctuations along the axis  $z$ :

$$\langle \Delta p_z^2 \rangle = \langle \Delta p_{0z}^2 \rangle + \hbar^2 k^2 \langle (\Delta N_i)^2 \rangle + \hbar^2 k^2 \langle \cos^2 \theta \rangle \langle N_s \rangle. \quad (57)$$

Here,  $\theta$  is the angle between the direction of spontaneous photon emission and the axis  $z$ , and  $\langle N_s \rangle$  the average number of spontaneously emitted photons. The first term on the right-hand side of Eq. (57) is associated with the initial distribution of atoms over their momenta, the second one with stimulated absorption and radiation processes, and the third one with the spontaneous radiation of photons. Expression (57) was obtained, by assuming the statistical independence of stimulated radiation, absorption, and spontaneous radiation. In addition, the statistical independence of propagation directions for spontaneously emitted photons was taken into account.

The average squared fluctuation of the number of photons scattered at stimulated atomic transitions,  $\langle (\Delta N_i)^2 \rangle$ , will be evaluated by considering the photon scattering as a completely random process. In this case, the probability  $p(N_i)$  for  $N_i$  photons to be scattered at stimulated atomic transitions within the time interval  $\Delta t$  is described by the Poisson distribution

$$p(N_i) = \frac{\langle N_i \rangle^{N_i}}{N_i!} e^{-N_i}. \quad (58)$$

From whence, it follows that

$$\langle (\Delta N_i)^2 \rangle = \langle N_i \rangle. \quad (59)$$

Taking into account that the difference between the numbers of absorbed and stimulated emitted photons is equal to the number of the spontaneously emitted ones ( $\langle N_i \rangle = \langle N_s \rangle$ ), we obtain

$$\langle \Delta p_z^2 \rangle = \langle \Delta p_{0z}^2 \rangle + \hbar^2 k^2 \langle N_s \rangle + \hbar^2 k^2 \langle \cos^2 \theta \rangle \langle N_s \rangle. \quad (60)$$

This equation forms a basis for the computer simulation of the momentum diffusion process in the field of a single running wave. According to it, one atomic momentum variation by  $\pm \hbar k$  due to the stimulated processes is accounted for every random change of the atomic momentum occurring due to spontaneous light radiation.

The described algorithm for the account of the momentum diffusion is valid, as was mentioned above, if the field is weak,  $\Omega_0 \lesssim \gamma$ . In the case of highly intensive counter-propagating waves, which is required to reduce the size of the atomic cloud in a trap, we may consider the momentum diffusion in the stimulated processes in a similar way, bearing in mind that the results obtained have an estimation character. A probable error can be associated with larger fluctuations of the momentum change owing to stimulated radiation processes in comparison with those occurring owing to spontaneous radiation emission. As a result, we can underestimate the atomic cloud size in the trap and the atomic temperature.

Equations (17), (18), and (32) are integrated by the Runge-Kutta method of the fourth order. After every step, a check of whether a quantum jump took place is made, and the wave function is normalized. If the case of quantum jump, the velocity of the atom changes by

$$\Delta v = \hbar k (\epsilon_1 - 0.5) / (M |\epsilon_1 - 0.5|) + \hbar k (\epsilon_2 - 0.5) / (M |\epsilon_2 - 0.5|), \quad (61)$$

where  $\epsilon_{1,2}$  are random numbers uniformly distributed over the interval  $[0,1]$ . One of the terms simulates the momentum fluctuation at spontaneous photon radiation, and the other the momentum fluctuation resulting from fluctuations of the stimulated absorption and radiation processes.

## 7. Results of Numerical Simulation

The motion of the atom in a bichromatic field substantially depends on the field parameters and the initial conditions (atomic velocity and coordinate).



Among the whole set of parameters, we are interested first of all in those that provide the motion of atoms in a narrow interval of coordinates  $z$  in a vicinity of the trap center.

In Subsection 7.1, we simulate the motion of atoms in a weak bichromatic field of counter-propagating waves. The results obtained can be interpreted, by using the formulas for the force of light pressure quoted in Section 5.

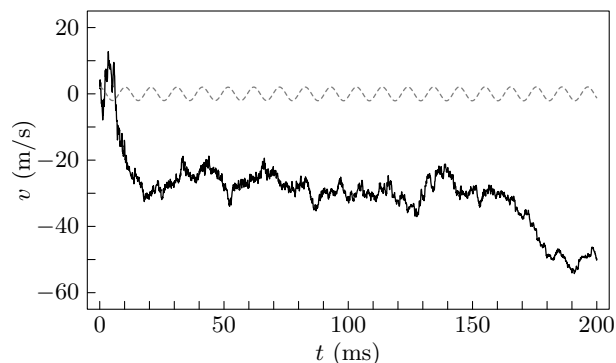
The growth of the intensity of counter-propagating waves gives rise to an increase of the quasielastic force acting on the atom toward the trap center. As a result, the volume occupied by atoms in the trap diminishes, and their concentration increases. Therefore, we also simulate the motion of atoms in a strong field, where  $\Omega_0 \gg \gamma$ . The results of simulation are presented in Subsection 7.2.

Calculations were made for  $^{23}\text{Na}$  atoms, for which a cyclic interaction with the field can be created [1]. The wavelength of the  $3^2S_{1/2} - 3^2P_{3/2}$  transition  $\lambda = 589.16$  nm, the rate of spontaneous radiation  $\gamma = 2\pi \times 10$  MHz, and the Doppler cooling limit for sodium atoms  $T_D = 240$   $\mu\text{K}$  [1].

### 7.1. Low-intensity counter-propagating waves

Consider the motion of atoms in the fields of counter-propagating waves, the intensities of which are close to those saturating the transition, i.e.  $\Omega_0 = \gamma$ . For the interaction between the atom and both standing waves to be significant, let the difference between their frequencies be close to  $\gamma$ , i.e.  $\Omega = 2\gamma$ . The dependence of the friction coefficient  $\kappa$  on the parameter  $s$ , which is equal to the difference between the transition frequency in the atom and the arithmetic mean of the frequency of standing waves, is shown in Fig. 3. The friction force emerges, if  $-0.993\gamma < s < 0$  or  $s > 0.993\gamma$ . This conclusion does not exclude the possibility of a finite atomic motion beyond the indicated intervals or at their limits, since the sign of  $\kappa$  at  $kv \ll \gamma$  testifies only to the deceleration or acceleration of the atom.

If the frequency detunings of the standing waves from the transition frequency in the atom were identical by magnitude but different by sign, i.e. at  $s = 0$ , and the momentum diffusion were absent, the atom would move periodically. Figure 4 illustrates the real motion of the atom (the solid curve) and its motion, if momentum fluctuations (61) at the moment of spon-

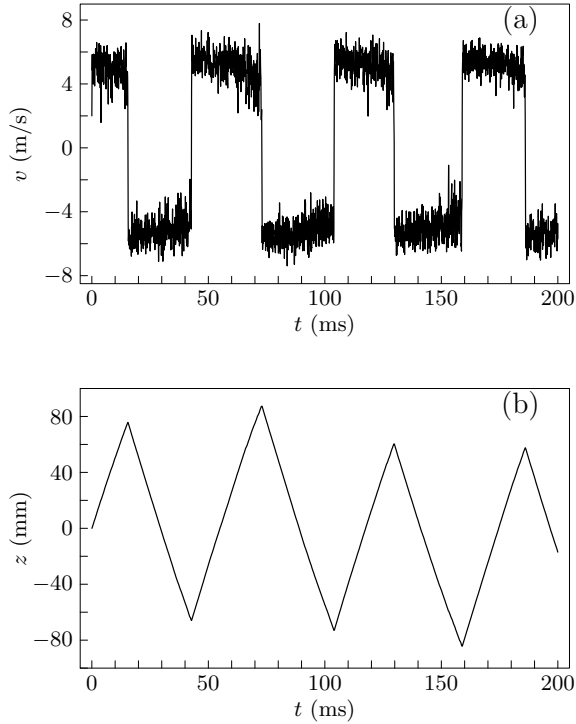


**Fig. 4.** Simulated time dependences of the velocity of  $^{23}\text{Na}$  atom at its interaction with a weak field of bichromatic counter-propagating waves. The initial velocity of the atom  $v_0 = 2$  m/s,  $s = 0$  ( $\delta_1 = \delta_2 = 10$  MHz and  $\delta_3 = \delta_4 = -10$  MHz), the Rabi frequencies of the waves are identical and equal  $\Omega_0 = 2\pi \times 10$  MHz. The process of momentum diffusion is taken into account (the solid curve) or is switched-off (the dashed curve)

taneous radiation have been artificially switched-off (the dashed curve). A comparison of those dependences testifies to a crucial influence of the momentum diffusion on the character of atomic motion in the field of light waves, which results in this case in a considerable growth of the atomic velocity. The period of atomic oscillations in the absence of the momentum diffusion estimated from Fig. 4 (10.3 ms) agrees well with a value of 7 ms obtained from formula (44) at  $s = 0$ , in view of its validity at the limit of the applicability of perturbation theory.

On the basis of the dependence of the friction coefficient  $\kappa$  on  $s$ , which is depicted in Fig. 3, one can see that the finite motion of the atom at a low velocity near the trap center is impossible in the interval  $0 < s < 0.993\gamma$  ( $0 < s < 9.93$  MHz). Really, the calculations show that, at  $s = 2$  MHz, the atom mostly moves at a velocity close to  $\pm 5$  m/s with fluctuations due to the momentum diffusion, and this is the velocity, at which the atom crosses the trap center. As is seen from Fig. 5, the atom changes the direction of its motion some time after crossing the trap center. This is partly promoted by the term  $F_4$ , which is approximately proportional to a deviation from the center, in the light pressure force (51). As a result, the pressure of light on the atom is directed to the trap center.

Now, let us consider the influence of the coordinate-independent component  $F_2$  of the light pressure



**Fig. 5.** Simulated time dependences of the velocity (a) and the coordinate (b) of  $^{23}\text{Na}$  atom at its interaction with a weak field of bichromatic counter-propagating waves. The initial velocity of the atom  $v_0 = 2$  m/s, its initial coordinate  $z_0 = 0$ ,  $s = 2$  MHz ( $\delta_1 = \delta_2 = 12$  MHz and  $\delta_3 = \delta_4 = -8$  MHz), the Rabi frequencies of the waves are identical and equal  $\Omega_0 = 2\pi \times 10$  MHz

force  $F$  (51) on the atom motion (see Fig. 5). From Eq. (47), it follows that, at small  $s > 0$ , the force component  $F_2$  accelerates the atom, if the velocity of the latter  $|v| < \varpi/k$ , where the frequency  $\varpi$  is defined by expression (50). In the case  $|v| > \varpi/k$ , this component of the force acting on the atom decelerates it. At the boundaries of the indicated  $v$ -interval,  $F_2 = 0$ , and, beyond it, the acceleration of the atom transforms into a deceleration. The action of the force  $F_2$  alone would stimulate the atom to move at a velocity of 5.83 m/s, provided the other parameters are the same as in Fig. 5.

One can see from Fig. 5 that the fluctuations of the velocity are large, and sometimes the velocity can change its sign. In this case, the atom is accelerated to the velocity  $\varpi/k$  in the opposite direction. Since  $\varpi/k < v_c$ , the atom moving at the velocity  $\varpi/k$  is subjected to the action of the force  $F_4$  directed toward the trap center, and this force grows as the

atom moves away from it. The simultaneous action of both terms in Eq. (51) can explain the fact that the velocity of an atom (averaged over fluctuations) just after reaching the next coordinate maximum is a little higher than before that: the components  $F_2$  and  $F_4$  act in the same direction in the former case and oppositely to each other in the latter one.

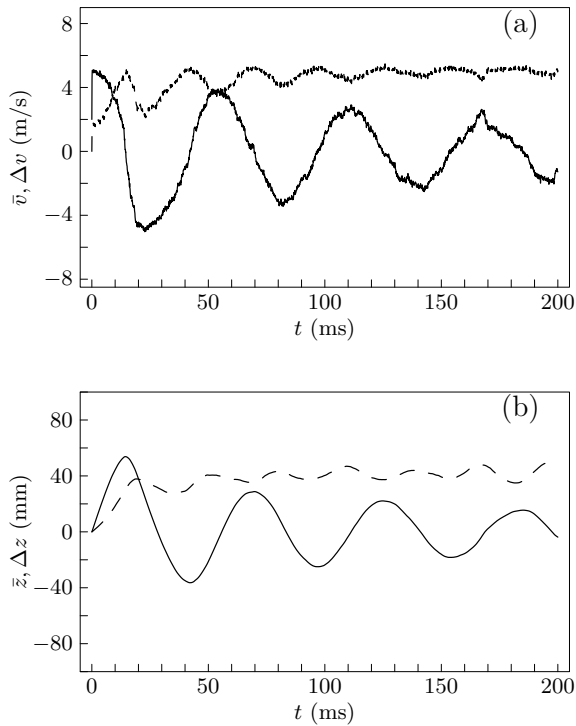
It is known [25] that Monte Carlo calculations of the wave function, after the averaging over the ensemble, give the same result as the calculations using the density matrix method. At the same time, the Monte Carlo method makes it possible not only to find the evolution of the coordinate and the velocity of the center of mass of an atomic ensemble, but, by calculating the squared velocity and coordinate values averaged over the ensemble, to estimate the root-mean-square deviations of the coordinate and the velocity from their average values.

In Fig. 6, we show the time dependences of the average velocity  $\bar{v}$  for 100  $^{23}\text{Na}$  atoms, the position  $\bar{z}$  of their center of mass, and the root-mean-square deviations of the coordinate,  $\Delta z$ , and the velocity,  $\Delta v$ , from their average values calculated for the same field parameters and initial conditions as in Fig. 5. At the beginning, all atoms are in the same state. However, as the time grows, the correlations between velocities and coordinates of atoms decrease owing to the stochasticity inserted by spontaneous radiation. Just this factor can explain the damping of oscillations of the coordinate and the velocity of the center of mass of atoms.

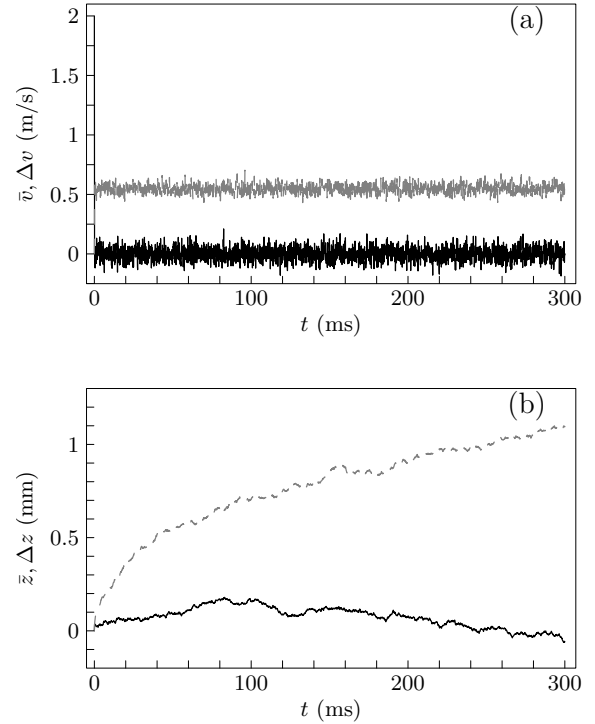
Let us consider the motion of atoms in the trap for such detunings  $s$ , at which the confinement of atoms in the trap and, simultaneously, their cooling can be expected. When selecting the detuning, we proceed from the fact that the friction force and the force returning the atom back to the trap center should be close to maximum. At low velocities, the friction force  $F_2$  is maximum at  $s = -0.72\gamma$  and  $1.27\gamma$  (see Fig. 3). The force  $F_4$  [Eq. (44)], which brings the atom back to the coordinate origin and provides the trap functioning, is maximum at  $s = \pm 0.93\gamma$ . Since the  $F_2$ - and  $F_4$ -maxima do not coincide, the optimum detunings  $s$  that ensure the confinement and the cooling of atoms in the trap at  $\Omega = 2\gamma$  fall within the intervals  $-0.93\gamma < s < -0.72\gamma$  and  $0.93\gamma < s < 1.27\gamma$ .

In Fig. 7, the dependences  $\bar{v}(t)$  and  $\bar{z}(t)$  (panel a), and  $\Delta v(t)$  and  $\Delta z(t)$  (panel b) are shown for a certain, close to the optimal,  $s$ -value from the interval

$0.93\gamma < s < 1.27\gamma$ . After the atoms have been braked (approximately in  $50 \mu\text{s}$ ), their velocity, owing to the momentum diffusion, fluctuates with  $\Delta v \approx 0.55 \text{ m/s}$ , which corresponds to the ensemble temperature  $T = 0.8 \mu\text{K}$ . Rather a high temperature of atomic ensemble in comparison with the Doppler limit of  $240 \mu\text{K}$  is associated with the violation of the condition  $\Omega_0 \ll \gamma$  in order to achieve this limit. The behavior of the dependence  $\Delta z(t)$  brings us to a conclusion that, in 300 ms after the interaction started, the size of atomic clouds still continues to grow, which testifies to an insufficient compensation of the diffusive-like motion of atoms by the light pressure force. In the next section, we will demonstrate that, by increasing the radiation intensity by two orders of magnitude, we can reach the confinement of atomic clouds with  $\Delta z \sim 0.1 \text{ mm}$  already after  $20\text{-}\mu\text{s}$  interaction interval with the field.



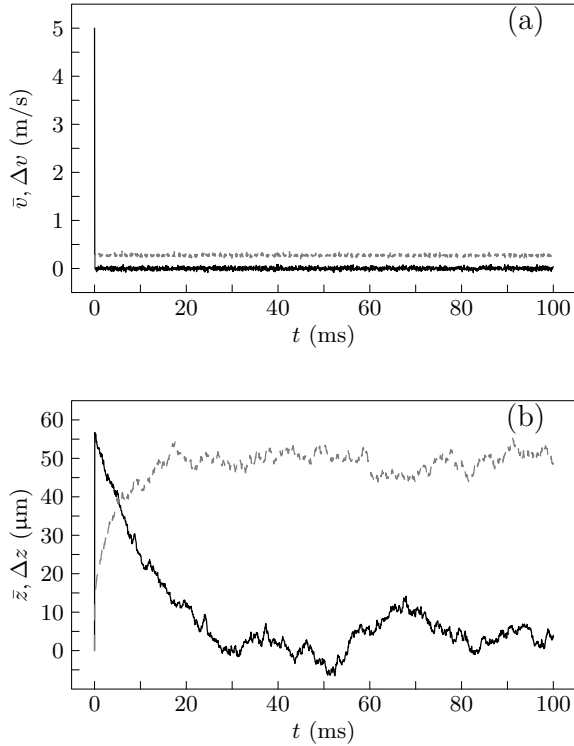
**Fig. 6.** Simulated time dependences of the average velocity  $\bar{v}$ , coordinate  $\bar{z}$  of the center of mass, and root-mean-square deviations of the velocities,  $\Delta v$ , and the coordinates,  $\Delta z$ , of atoms from their average values for an ensemble of 100  $^{23}\text{Na}$  atoms at their interaction with a weak field of bichromatic counter-propagating waves. The initial conditions and parameters of the interaction between the atom and the field are the same as in Fig. 5



**Fig. 7.** Simulated dependences  $\bar{v}(t)$ ,  $\bar{z}(t)$  (solid curves) and  $\Delta v(t)$ ,  $\Delta z(t)$  (dashed curves) for an ensemble of 100  $^{23}\text{Na}$  atoms in the field of bichromatic counter-propagating waves. The initial velocity of the atom  $v_0 = 2 \text{ m/s}$ , its initial coordinate  $z_0 = 0$ ,  $s = 12 \text{ MHz}$  ( $\delta_1 = \delta_2 = 12 \text{ MHz}$  and  $\delta_3 = \delta_4 = 2 \text{ MHz}$ ), the Rabi frequencies of the waves are identical and equal  $\Omega_0 = 2\pi \times 10 \text{ MHz}$

For checking the calculation accuracy of the temperature, we calculated the average value of the velocity for one standing wave with a low intensity ( $\Omega_0 = 0.1\gamma$ ) and at the optimum detuning from the resonance  $\gamma/2$  [28] for 1000 sodium atoms. We obtained  $\Delta v = 30 \pm 1 \text{ cm/s}$ , which corresponds to the temperature limits of  $249 \pm 17 \mu\text{K}$  for the ensemble of atoms. This interval includes the theoretical value of Doppler temperature limit equal to  $240 \mu\text{K}$ .

Having studied the behavior of atoms in the field of weak counter-propagating waves, let us proceed to the case of strong light fields. It should be noted that the Doppler temperature limit (52) was found for the interaction of an atom with a monochromatic standing wave. For weak fields and in the case where the frequency of either wave is close to that of the transition in the atom (see Fig. 7), the atom mainly interacts with this wave, and the inequality



**Fig. 8.** Simulated dependences  $\bar{v}(t)$ ,  $\bar{z}(t)$  and  $\Delta v(t)$ ,  $\Delta z(t)$  for an ensemble of 100  $^{23}\text{Na}$  atoms interacting with a strong field of bichromatic counter-propagating waves. The initial velocity of the atom  $v_0 = 5$  m/s, its initial coordinate  $z_0 = 0$ ,  $s = 10$  MHz ( $\delta_1 = \delta_2 = 110$  MHz and  $\delta_3 = \delta_4 = -90$  MHz), the Rabi frequencies of the waves are identical and equal  $\Omega_0 = 2\pi \times 100$  MHz, the frequency difference of standing waves  $\Omega = 2\pi \times 200$  MHz

$\Omega_0 \ll \gamma$  has to be obeyed in order to reach temperatures close to (52). However, if  $\Omega_0 \gg \gamma$  and  $\Omega \sim \Omega_0$ , the atom strongly interacts with both standing waves. Therefore, estimate (52) obtained for the limiting temperature in the case of a single wave is incorrect. We will see that temperatures close to (52) can be reached in the field of two standing waves and at  $\Omega_0 \gg \gamma$ .

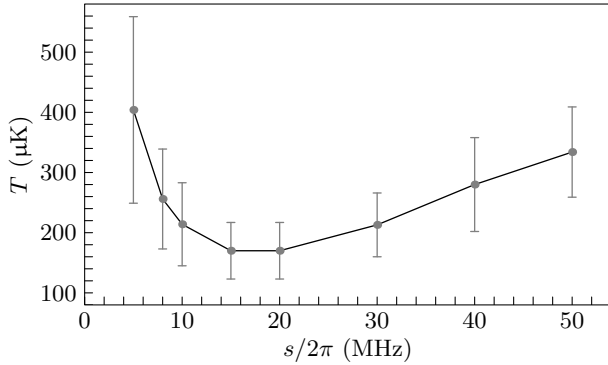
### 7.2. High-intensity counter-propagating waves

Now, let us consider the interaction of sodium atoms with strong fields created by counter-propagating waves, the intensity of which is by two orders of magnitude higher than those chosen for the simulation in the previous section. As is seen from Fig. 8, higher intensities allow the atoms to be localized in a narrow interval of coordinates of about 100  $\mu\text{m}$ . Owing to

the spatial dependence of the light pressure force, the expansion of the atomic cloud terminates in approximately 20 ms. Its size is governed by a competition between the diffusion-like motion resulting from the momentum diffusion, which is associated with fluctuations of spontaneous and stimulated radiation, and the motion under the action of the light pressure force directed to the trap center and proportional to the distance from the center. Atoms with the initial velocity  $v_0 = 5$  m/s are braked by the field at a distance of about 57  $\mu\text{m}$ ; then, their center of mass shifts into a region about  $\pm 10$   $\mu\text{m}$  around the trap center, which unambiguously evidences the action of the light pressure force toward the latter. The braking of an atom lasts for about 30  $\mu\text{s}$ ; afterward, the velocity fluctuates around zero with a root-mean-square deviation of 27.5 cm/s. This velocity fluctuation corresponds to a temperature of the atomic ensemble equal to 209  $\mu\text{K}$ , which is a little lower than the Doppler temperature limit 240  $\mu\text{K}$  (52). Taking into consideration that the number of atoms, over which the averaging is performed, is relatively small,  $N_a = 100$ , and a typical relative accuracy of the average values obtained by the Monte Carlo method,  $f_{\text{rel}} \sim 1/\sqrt{N_a}$ , we come to a conclusion that atoms can be confined in a trap formed by the bichromatic field of counter-propagating waves and simultaneously cooled down to approximately the Doppler temperature limit. The fact that the confinement and the cooling of atoms simultaneously in the trap is observed at such a value of parameter  $s$ , at which the cooling is impossible in the case of weak fields (see Fig. 3), has a simple explanation: if the intensity of even one standing wave exceeds the value, at which the absorption saturates, the derivative of the light pressure force with respect to the velocity calculated at the zero velocity value can be both negative and positive [24].

Figure 9 illustrates the dependence of the temperature of an ensemble consisting of 100 atoms on the detuning  $s$  calculated by the Monte Carlo method. One can see that, at a detuning of 15–20 MHz, a temperature minimum of about 170 K is reached, which is a little lower than  $T_D$  described by Eq. (52).

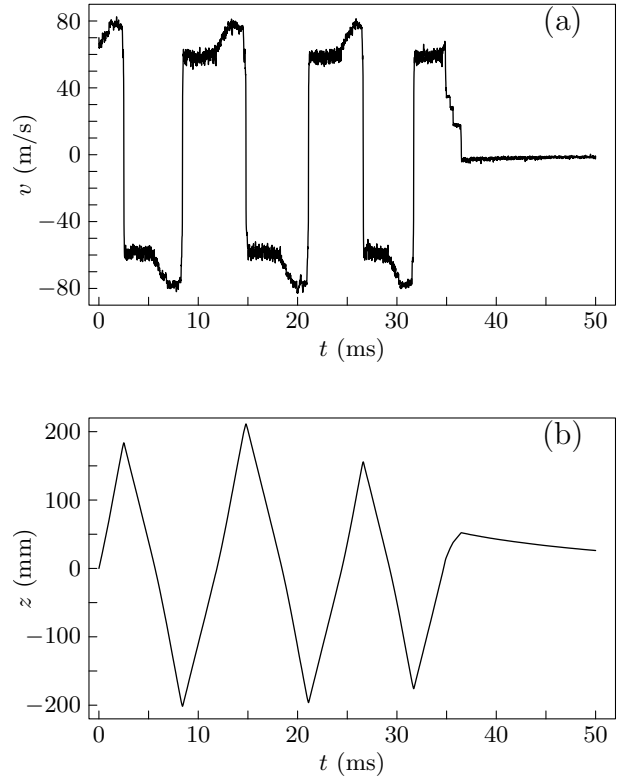
The interval of atomic velocities, at which the atom is captured and cooled by the field, depends on the trap size. For the parameters of Fig. 9 and the detuning  $s = 2\pi \times 20$  MHz, which provides a temperature of atoms in the trap that is close to the minimum, the trap size should not be smaller than 0.2 mm for



**Fig. 9.** Dependence of the temperature of  $^{23}\text{Na}$  atoms on the detuning  $s$  of the atomic transition frequency from the average carrier frequency of counter-propagating bichromatic waves. The Rabi frequencies of the waves are identical and equal  $\Omega_0 = 2\pi \times 100$  MHz, the frequency difference of standing waves  $\Omega = 2\pi \times 200$  MHz,  $N_a = 100$ . Circles denote the results of calculations, vertical bars mark the temperature limits evaluated from the fluctuations of the mean square of atomic velocities

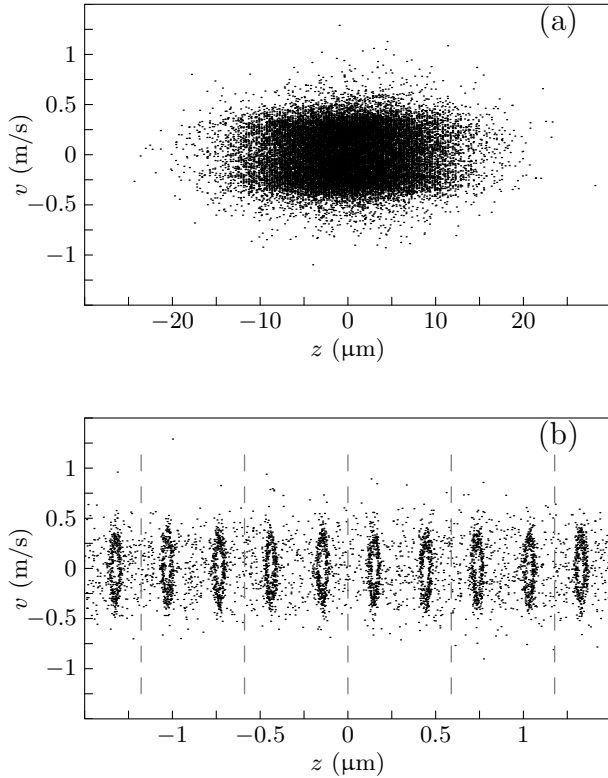
the initial velocity  $v_0 = 5$  m/s, 1 mm for  $v_0 = 9$  m/s, and tens of centimeters for  $v_0 > 50$  m/s. The reason for such drastic growth of the necessary trap dimensions lies in the change of the atomic braking scenario, when the velocity exceeds a certain value (about 10 m/s for the indicated parameters). In particular, at low initial velocities, the velocity of an atom almost monotonically decreases in time. On the other hand, if the initial velocity is high, the velocity of an atom changes in a wide interval between positive and negative values until it almost vanishes, and the atom is located at this moment near the trap center; then the atom is decelerated (see Fig. 10). The time needed for all that scenario to be realized is different for different realizations of the atomic motion in accordance with the fact that the atomic motion trajectory considerably depends on the time distribution of the moments of spontaneous photon emission. The velocity of the atom changes approximately within the limits of  $\pm\Omega/(2k_B)$ , which corresponds to a variation of the force sign in the case where the atom interacts with the bichromatic field at  $s = 0$  [10]. The step-like temporal dependence of the velocity in the interval from 35 to 37 ms may probably corresponds to doppleron resonances [32].

The time dependences of the root-mean-square deviations from the average values of velocity and coordinate, which are depicted in Fig. 8, allow one to



**Fig. 10.** Simulated time dependences of the velocity (a) and the coordinate (b) of  $^{23}\text{Na}$  atom at its interaction with a strong field of bichromatic counter-propagating waves. The initial velocity of the atom  $v_0 = 70$  m/s, its initial coordinate  $z_0 = 0$ ,  $s = 20$  MHz ( $\delta_1 = \delta_2 = 120$  MHz and  $\delta_3 = \delta_4 = -80$  MHz), the Rabi frequencies of the waves are identical and equal  $\Omega_0 = 2\pi \times 100$  MHz, the frequency difference of standing waves  $\Omega = 2\pi \times 200$  MHz

estimate the size and the temperature of the atomic cloud. A better understanding of the atomic distribution over coordinates and velocities is given by its presentation in the coordinate-velocity phase plane, as is shown in Fig. 11. While simulating the motion of atoms, we accepted all of them to be in the excited state at the initial moment, and all initial phases were put equal to zero. Calculations were carried out for  $s = 2\pi \times 20$  MHz, which provides the cooling of the atomic ensemble to a temperature that is close to the minimum at the  $\Omega_0$ - and  $\Omega$ -values indicated in Fig. 11. The initial velocity of atoms was chosen to equal zero in order to obtain a symmetric distribution of atoms with respect to the trap center from the very beginning. The initial state of the atom, owing



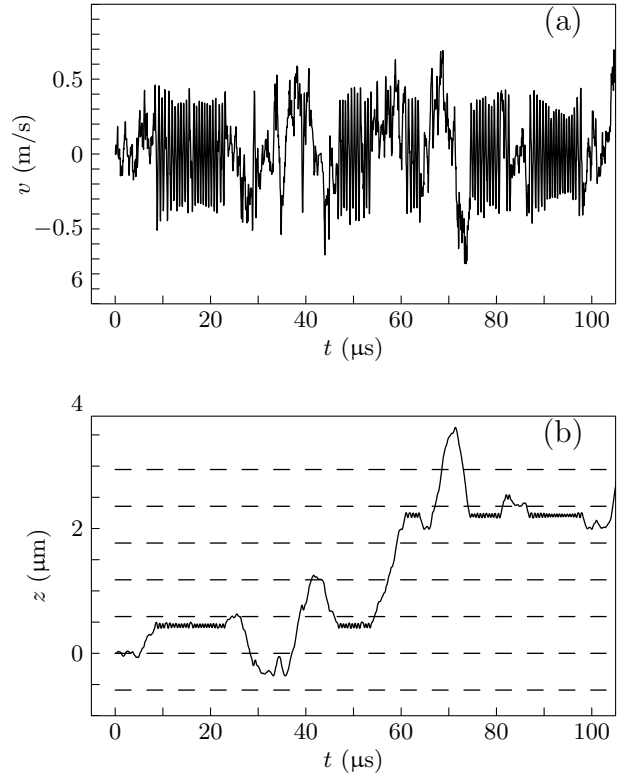
**Fig. 11.** Distribution of 25000 atoms in the phase plane in 500  $\mu\text{s}$  after they started to move (a) and its scaled-up region near the coordinate origin (b). The initial velocity of the atom  $v_0 = 0$  m/s, its initial coordinate  $z_0 = 0$ ,  $s = 20$  MHz ( $\delta_1 = \delta_2 = 120$  MHz and  $\delta_3 = \delta_4 = -80$  MHz), the Rabi frequencies of the waves are identical and equal  $\Omega_0 = 2\pi \times 100$  MHz, the frequency difference of standing waves  $\Omega = 2\pi \times 200$  MHz. At  $t = 0$ , all atoms are in the excited state, and  $\phi_1 = \phi_2 = \phi_3 = \phi_4 = 0$ . Dashed lines mark distances multiple to  $\lambda$  and reckoned from the trap center

to multiple spontaneous radiation emissions, does not affect the atomic distribution in the phase plane.

Figure 11, b demonstrates the phase diagram for the atoms located in the central part of the trap. One can clearly distinguish ellipse-like structures, which testify to an oscillatory motion of atoms in vicinities of the points

$$z_n = \lambda(2n + 1)/4, \quad (62)$$

where  $n$  runs over the set of integers. The trap center corresponds to the antinode of standing waves. The presence of a small number of atoms between the ellipses indicates a change in the number of atoms oscillating around a certain  $z_n$  due to the transition of



**Fig. 12.** Time dependence of the velocity (a) and the coordinate (b) of an atom from the distribution shown in the phase plane in Fig. 11. Dashed lines mark distances multiple to  $\lambda$  and reckoned from the trap center

some of them to other  $z_n$  with different  $n$ . Note that the similar calculations for  $s$ , which correspond to a higher temperature of the atomic ensemble, e.g., for  $s = 2\pi \times 10$  MHz, also result in ellipses in the phase plane; however, they are less pronounced.

An example of the evolutions of the velocity and the coordinate of one of the atoms in Fig. 11 is shown in Fig. 12. For illustrative reasons and in order to demonstrate the oscillatory motion, only the initial time interval ( $\leq 100 \mu\text{s}$ ) is shown. One can see that the atom oscillates in vicinities of  $z_1$  and  $z_7$ .

## 8. Conclusions

In this work, a possibility to realize an atomic trap proposed in work [12], which is formed by the bichromatic field of counter-propagating waves, is demonstrated. At low intensities of counter-propagating waves, lower than the absorption saturation intensity in the atom, the motion of atoms in the trap

can be interpreted on the basis of perturbation theory for the force of light pressure developed to the second and fourth orders in the field. The force of the second order does not depend on the atomic coordinate and results in the deceleration or acceleration of the atom by the field, provided that the detuning of the frequency of standing waves from the atomic transition frequency is properly chosen, until the atomic velocity reaches a value of  $\pm\Omega/2k$ . The force of the fourth order in the field depends on the coordinate and is approximately proportional to a deviation of the atom from the trap center. The combined action of both force components makes the formation of a trap for atoms possible. Most of the time, the atoms in the trap move at a velocity of  $\pm\Omega/2k$ . Numerical simulations of the atomic motion in a strong field,  $\Omega_0 \gg \gamma$ , demonstrate the realizability of the simultaneous confinement and cooling of sodium atoms to temperatures of the Doppler limit order and, probably, a little lower. The size of the atomic cloud at a Rabi frequency of 100 MHz amounts to approximately 0.1 mm, the temperature is about 200  $\mu\text{K}$ , and the interval of velocities, at which the braking of atoms and their confinement in the trap are possible, is determined by the trap size and ranges from 0 to 9 m/s for a trap 1 mm in size. Atoms in the cloud oscillate around the points  $z_n = \lambda(2n + 1)/4$  corresponding to the nodes of a standing wave and transit from a vicinity of one node to a vicinity of the other one. In such a way, the atoms form a lattice in the space with the period  $\lambda/2$ .

The proposed mechanism of trap formation can also be used to confine and cool molecules with the matrix of Franck–Condon factors close to a diagonal one. In this case, an additional field is required to bring the atoms that, owing to spontaneous radiation, turn out on a level distinct from the ground one back to a cyclic interaction with the cooling and confining field, as was shown while observing light pressure on SrF molecules [31].

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*В.І. Романенко, О.В. Романенко, Л.П. Яценко*

ОПТИЧНА ПАСТКА ДЛЯ АТОМІВ НА ОСНОВІ  
ЗУСТРІЧНИХ БІХРОМАТИЧНИХ СВІТЛОВИХ ХВИЛЬ  
Р е з ю м е

Показано, що полем зустрічних біхроматичних світлових хвиль можна сформувати одновимірну пастку для атомів. Вибираючи належним чином відстроювання монохроматичних компонент хвиль від частоти переходу в атомі, можна досягти одночасного утримання ансамблю атомів у пастці і їх охолодження до температури, близької до доплерівської границі, без застосування додаткових полів. Чисельне моделювання проведено для атомів натрію. Обговорюється можливість застосування подібної пастки для утримання і охолодження молекул з майже діагональною матрицею факторів Франка-Кондона.