doi: 10.15407/ujpe61.05.0440

E.A. PONEZHA

Bogolyubov Institute for Theoretical Physics, Nat. Acad. of Sci. of Ukraine
(14b, Metrologichna Str., Kyiv 03680, Ukraine)

PACS 05.40.-a, 05.10.Gg, 73.40.Gk

DECAY OF INTENSITY CORRELATION FUNCTION NEAR INSTABILITY POINT FOR THE MODEL OF RESONANT TUNNELING

The decay of the correlation function $C(t)$ of an electron flow intensity near an instability point for the process of resonant tunneling through a double barrier structure is considered. It is supposed that the intensity of the incoming flow may fluctuate under the influence of an external noise, both white and colored. The correlation function $C(t)$ behavior is analyzed, by using methods leading to the single-exponential approximation such as the method of projection operator and the method of mean relaxation time. Moreover, the method based on a combination of high- and low-frequency expansions of the Laplace transform of $C(t)$, which has allowed the correlation function to be approximated by two decaying exponentials, is applied. The numerical simulation has shown that the latter approach unlike the others gives correct results near the instability point.

Keywords: resonant tunneling, intensity correlation function decay, instability point.

1. Introduction

One of the fundamental problems in the study of nonequilibrium systems is their behavior under the influence of noises near an instability point. To analyze the behavior of such systems, the correlation function in the nonequilibrium steady state and the associated relaxation time are often used.

For the calculation of the correlation functions, the method of projection operator leading to the expansion in continued fractions has been proposed in [1, 2]. It gives good results, when the not very different time scales are involved. In particular, the method accurately describes the short-time behavior of the correlation function.

Near instabilities, the processes evolve slower (the critical slowing down). In this case, the approximation called the approximation of *mean relaxation time* is used [3, 4] to describe the long-time behavior of correlation functions. Both of these methods describe the systems with essentially one time scale that allows one to get the single-exponential approach for a correlation function.

The methods mentioned above are approximations of a lower order, as compared to the more general approach, namely, the method of *generalized moment expansion* (or the method of *double expansion* (DE)), which is based on the simultaneous expansion of the correlation function in the regions of high and low frequencies [5, 6, 7]. In this case, the correlation function is represented as a superposition of two decaying exponentials. The description of the method will be given briefly in Section 3.

In the non-Markovian case, the dependence of the correlation function on noise parameters may have some specific features, as compared to the Markovian limit [2, 6, 7, 8]. The generalization of the DE method to the case of a Gaussian colored noise in the first approximation with respect to the noise correlation time τ has been done in [2, 9]. In particular, the connection between the non-Markovian correlation function $C(t)$ and the effective Markovian correlation function has been established. The behavior of $C(t)$ in the case of a colored noise has some characteristic features. For instance, there appears the initial plateau in the short-time regime. It is known from the numerical simulation [10] that the colored noise leads to a retardation of the process in the sense that the relaxation time is monotonically increased as a function of the noise correlation time τ.

In the present paper, we analyze the influence of the external noise on the intensity correlation function decay in a model system that describes the resonant tunneling of electrons through a double barrier nanostructure. It is known that such systems can enhance dramatically a transmittance coefficient under the resonance condition that allows one to fabricate diodes and transistors on their base with great prospects of using them in electronic devices [11]. That is why the investigation of noise effects in such systems in the instability region is very important.

In our model of tunneling process, the instability takes place at the point of transition from the state with low efficiency of tunneling to a state with high efficiency. In [12, 13], we considered the influence of a white noise on the dynamics of this system. In [14], the influence of a colored noise on the mean first passage time near the instability point was investigated. The problem of the mutual influence of amplitude and phase noises in the incoming flow on the intensity correlation function of the outgoing flow was studied in [15]. In the present paper, we consider the decay of the intensity correlation function $C(t)$ of the outgoing electron flow, by using the DE method. The results of this method are compared with those obtained with the help of methods leading to a single-exponential expansion on short- and long-time scales and with the help of a numerical simulation of the equations of motion.

The work is organized as follows. The model of the stochastic tunneling process is presented in Section 2. The methods of calculation of correlation functions known from the literature and used in our work are described in Section 3. In Section 4, the behavior of the intensity correlation function near the instability point is analyzed, by using the methods mentioned in Section 3, and a comparison with the numerical simulations is given. The conclusions are done in Section 5.

2. The Model of the Stochastic Tunneling Process

We consider the process of resonant tunneling, whose model in the deterministic limit was given in [16], and its stochastic model was developed in [13, 14]. In the deterministic limit, the tunneling process can be described by the following dimensionless equation for the intensity I of an electron flow outgoing from the tunneling system:

$$\frac{dI}{dt} = -I + \frac{I_0}{1+(z-I)^2} = F(I). \quad (1)$$

Here, I_0 is the dimensionless intensity of an incoming electron flow, z is a parameter proportional to a shift of the working frequency from the resonant value. When its value satisfies the inequality $z > \sqrt{3}$, the system exhibits the bistability as a function of the parameter I_0.

Solving Eq. (1) with the value of parameter z that guarantees the presence in the bistability region and slowly changing the intensity I_0 in the forward and reverse directions, we get a hysteresis in the plot of I vs I_0. At the end point of the hysteresis loop I_{0K} (the so-called marginal point), the transition from the lower branch to the higher one takes place. The values of outgoing electron flow intensity at the moment of the transition from the lower state to a higher one, I_K, and from the higher state to a lower one, I_k, are determined by the expression [16],

$$I_{K,k} = \frac{1}{3}(2z \pm \sqrt{z^2-3}). \quad (2)$$

The critical value of incoming intensity I_{0K}, at which the transition occurs, can be defined from the relation [16]

$$I_{0K} = I_K[1+(z-I_K)^2]. \quad (3)$$

At $z = 3.5$, the value of I_{0K} equals 7.59. This value for z will be used in all the following calculations.

Let us consider the incoming electron flow intensity I_0 as a stochastic quantity, $I_0 = \langle I_0\rangle + q(t)$, where $\langle I_0\rangle$ is the mean value of the incoming intensity. Let q model intensity fluctuations and be taken to be a Gaussian noise with zero mean and correlation $\langle q^*(t)q(t)\rangle = \frac{D}{\tau}\exp\left(-\frac{|t-t'|}{\tau}\right)$, where D is the noise intensity, and τ is its correlation time. Then we get the following Langevin equation:

$$\frac{dI}{dt} = F(I) + g(I)q(t), \quad (4)$$

where the function $F(I)$ determines the dynamic behavior of the system in the deterministic limit (see Eq. (1)), and $g(I) = \frac{1}{1+(I-z)^2}$ is the term multiplying the noise.

We are interested in the behavior of the correlation function in the neighborhood of the marginal point I_K. To get analytical expressions for the correlation functions, the probability density of the process should be calculated. To ensure that the probability maximum is in a proximity of the marginal point I_K, we expand the function $F(I)$ in a series near this point:

$$F(I) = F(I_K) + F'(I - I_K) + + \frac{F''(I_K)}{2!}(I - I_K)^2 + O(I - I_K)^3. \quad (5)$$

We also introduce the change of variables: $\beta = I_0 - - I_{0K}$ and $x = I - I_K$. At the marginal point I_K, the first derivative of $F(I)$ with respect to I equals zero. Then we get the following approximate equation:

$$f(x, \beta) = -\alpha(x, \beta)x^2 - p(x)\beta, \quad (6)$$

where $\alpha(x, \beta)$ and $p(x)$ are coefficients, whose values can be obtained from Eqs. (2) and (3). They are, respectively,

$$\alpha(x, \beta) = \frac{0.2[1 - 3(x + I_K - z)^2](x + I_K)^3}{(I_{0K} + \beta)^2},$$

$$p(x) = (x + I_K)/I_{0K}.$$

With account of noise $q(t)$, the dynamics of the considered tunneling system near the marginal point I_K is defined by the equation:

$$\dot{x} = f(x, \beta) + k(x)q(t). \quad (7)$$

In the case of intensity fluctuations with a finite small correlation time τ, the Fokker–Planck equation for the probability density $P(x, t)$ connected with the Langevin equation (7) will be given by the following expression [8, 10]:

$$\frac{\partial}{\partial t}P(x,t) = -\frac{\partial}{\partial x}f(x)P(x,t) + \frac{\partial}{\partial x}k(x)\frac{\partial}{\partial x}H(x)P(x,t), \quad (8)$$

where

$$H(x) = k(x) - \tau[f(x, \beta)k'(x) - f'(x, \beta)k(x)]. \quad (9)$$

We assume, for simplicity, that the coefficient multiplying the noise term equals unity, $k(x) = 1$. This means that the noise is considered to be of additive character. Such an approximation has a sense at the transition points, where the process of transition is determined mostly by an additive noise (see [18]).

In the case of a small correlation time τ of the noise, the next expression for the steady probability density was obtained [10]:

$$P_{\rm st}(x, \beta, \tau) = \frac{N}{D1(x, \tau)} \exp\left[\int^x \frac{f(x', \beta)}{D1(x', \tau)}dx'\right], \quad (10)$$

where the diffusion coefficient $D1(x, \tau)$ has been defined by the formula $D1(x, \tau) = \left(\frac{D}{1+f'(x,\beta)\tau}\right)^{1/2}$, and N is the normalization coefficient, $N^{-1} = = \int_0^x P_{\rm st}(x, \beta, \tau)dx$.

In our work, we are interested how the decay of the correlation function $C(t)$ defined by the expression [2]

$$C(t) = \frac{\langle\delta x(t + t')\delta x(t')\rangle_{\rm st}}{\langle(\delta x)^2\rangle_{\rm st}} \quad (11)$$

with $\delta x(t) = x(t) - \langle x(t)\rangle$ is dependent on the shift β from the bifurcation point, the noise strength D, and the noise correlation time τ.

3. Methods of Calculation of the Correlation Function

In this section, we consider the main available methods of calculation of the correlation function. The calculation of the steady correlation function (11) begins usually with the Laplace transformation of this function [2, 3, 4],

$$\tilde{C}(w) = \int_0^\infty \exp(-wt)C(t)dt. \quad (12)$$

In [2, 3, 4] with the use of a projection-operation method that leads to the expansion in continued fractions, the following expression for $\tilde{C}(w)$ has been obtained in the first order:

$$\tilde{C}(w) = C_0[w + \gamma_s - K(w)]^{-1},$$

where $C_0 = 1$, which follows from the normalization condition. Here,

$$\gamma_s = -(d/dt)|_{t=0}C(t)/C(0)$$

is the relaxation rate in the short-time regime. It is defined by the formula [2]

$$\gamma_s = \frac{\langle D1(x, \tau\rangle_{\rm st}}{\langle(\delta x)^2\rangle_{\rm st}}. \quad (13)$$

The term $K(w)$ is responsible for the memory effects. Neglecting the memory effects leads to a single-exponential approach to $C(t)$ [2]:

$$C(t) \approx C_0 \exp(-\gamma_s t). \tag{14}$$

As was shown in [2], such an approximation gives satisfactory results in many cases. However in some cases, in particular, near instabilities, the corrections due to the memory term $K(w)$ are essential and must be included in order to achieve the correct behavior of the correlation functions at large times.

In [3], it was shown that, in the long-time regime, the single-exponential approximation can be used with the relaxation rate γ_l (the long-time relaxation) defined by the expression

$$\gamma_l = T_0^{-1}, \tag{15}$$

$$T_0 = \frac{-1}{\langle x^2\rangle - \langle x\rangle^2} \int\limits_0^\infty \frac{G_0^2(x)dx}{D1(x,\tau)^2 H(x) P_{\mathrm{st}}(x,\beta,\tau)}. \tag{16}$$

Here, $G_0(x) = -\int_0^x (x' - \langle x'\rangle) P_{\mathrm{st}}(x,\beta,\tau)dx$, $H(x)$ is defined by Eq. (9), and $P_{\mathrm{st}}(x,\beta,\tau)$ is the steady probability density of the process.

Thus, γ_s defines fast processes, and γ_l does slow ones. When the relaxation processes evolve on the same time scales, γ_s and γ_l will have similar values, and the single-exponential approach with any of these relaxation rates gives a satisfactory description of the correlation function behavior. However, if the relaxation process evolves on different time scales with equal weights, as is often takes place near instabilities, then γ_s gives the correct behavior at small times and incorrect at large times, while γ_l correctly describes the behavior at large times.

The method of double expansion (DE) [3–6] has allowed one to describe satisfactorily the behavior of the correlation function near instabilities for the complete time regime. The method is based on the combination of the low- and high-frequency expansion of the Laplace transform of $C(t)$. Coefficients of these expansions are derivatives and the so-called relaxation moments, respectively.

The expansion of the Laplace transform $\tilde{C}(w)$ for high frequencies takes the form

$$\tilde{C}(w) = \sum_{k=0}^{\infty} \mu_k (1/w)^k, \tag{17}$$

and, for low frequencies,

$$\tilde{C}(w) = \sum_{k=0}^{\infty} \mu_{-k-1}(-w)^k, \tag{18}$$

where the coefficients of the expansion, μ_k, are the derivatives at $t = 0$,

$$\mu_k = \frac{d^k C(t)}{dt^k}|_{t=0}$$

and $\mu_{-k-1} \equiv T_k$ are the relaxation moments,

$$T_k = \int\limits_0^\infty t^k C(t)dt.$$

The relaxation moment of zero order, T_0, is defined by expression (16) and is the usual relaxation time that contains information on the long-time scale and relates to the area under the curve of $C(t)$. Equation (17) describes the behavior of $C(t)$ on the short-time scale. This expansion was also used in the method of expansion in continued fractions.

The aim of the DE method is to get such an expression for $C(t)$ that contains information of both the short- and long-time regimes simultaneously. For the Markovian case, this problem was solved in [5, 6], where the expression for $C(t)$ was given in the form of a superposition of N exponentials:

$$C(t) = \sum_n^N a_n \exp(-\gamma_n t).$$

In the approximation where $N = 2$, the expression for $C(t)$ is written as

$$C(t) = a_1 \exp(-\gamma_1 t) + a_2 \exp(-\gamma_2 t). \tag{19}$$

The expression for the calculation of the correlation function of the non-Markovian process was obtained in [8, 9]. In the short-time regime, it has the form

$$C(t) = e^{-\gamma_s t} + \tau\gamma_s(1 - e^{-t/\tau})e^{-\gamma_s t}. \tag{20}$$

The non-Markovian correlation function in the DE approximation is given by the expression [8, 9]

$$C(t) = a_1 \exp(\gamma_1 t) + a_2 \exp(\gamma_2 t) + \tau\gamma_s(1 - e^{-t/\tau})e^{-\gamma_s t}. \tag{21}$$

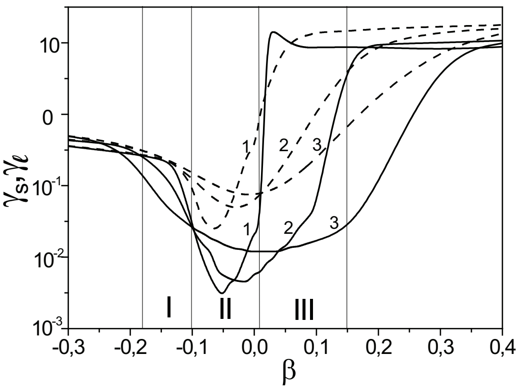


Fig. 1. Relaxation rates γ_s (dashed curves) and γ_l (solid curves) as functions of the deviation β for various noise intensities ($D = 0.1$ – curves with number *1*, $D = 0.2$ – curves with number *2*, $D = 0.3$ – curves with number *3*)

In these formulas, the relaxation rates are determined by expressions

$$\gamma_{1,2} = \frac{T_0-\mu_1T_1 \pm \sqrt{(T_0-\mu_1T_1)^2 - 4(T_0^2 - T_1)(1-\mu_1T_1)}}{2(T_0^2-T_1)}, \tag{22}$$

and the coefficients of the expansion are

$$a_1 = \frac{\mu_1 - \gamma_2}{\gamma_1 - \gamma_2}, \quad a_2 = 1 - a_1.$$

The quantities in (22) have the following meaning:

• $\mu_1 = \gamma_s$ defines the relaxation rate at short times and is described by formula (13).

• $T_0 \equiv \mu_{-1}$ defines the mean relaxation time, which is given by formula (16).

• $T_1 \equiv \mu_{-2}$ is defined by the expression

$$T_1 = \frac{-1}{\langle x^2\rangle - \langle x\rangle^2} \int\limits_0^x \frac{G_0^2(x)G_1(x)dx}{Dg(x)H(x)P_{\rm st}(x,\beta,\tau)}.$$

Here, $G_1(x)$ can be calculated by the formula

$$G_1(t) = \int\limits_0^x P_{\rm st}(x,\beta,\tau)\left[\int\limits_0^{x'} \frac{G_0(x'')}{D(x'')P_{\rm st}(x'',\beta,\tau)}dx'' - \left\langle \int\limits_0^x \frac{G_0(x')}{D(x')P_{\rm st}(x',\beta,\tau)}dx' \right\rangle\right]dx'.$$

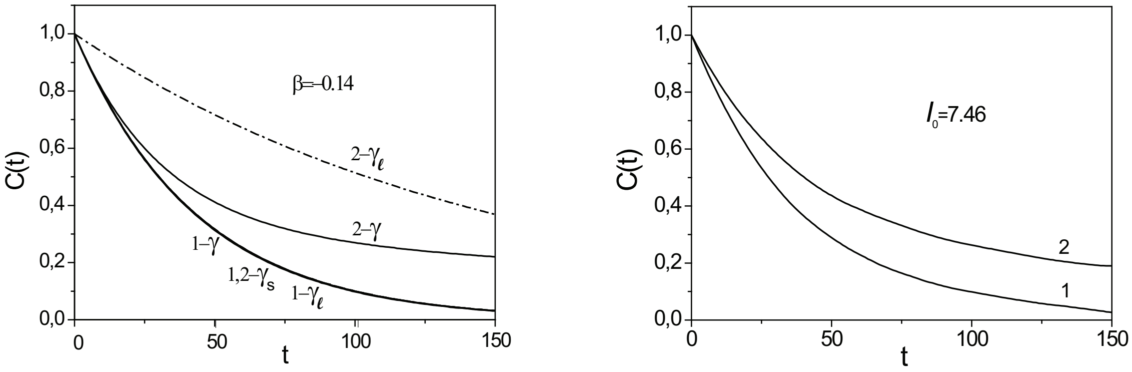
4. The Decay of the Intensity Correlation Function for the Model of Resonant Tunneling

At first, we analyze the behavior of the intensity correlation function of an electron flow passing through a tunneling system in the Markovian limit. The results of calculation of the correlation function within the DE method with the use of formula (19) will be compared with those based on the projection operator procedure (formula (14)) and on the mean relaxation time method (formula (15)).

The results of the analytical calculations will be compared with those obtained by a numerical simulation of the Langevin equation (4) for the considered tunneling process, which was carried out with the use of the procedure described in [10]. The correlation function was obtained by averaging of 1000 trajectories along 15000 steps of integration in each trajectory. To be sure in reaching the stationary state, the first 2500 steps were excluded. The step of integration was $\Delta = 0.005$.

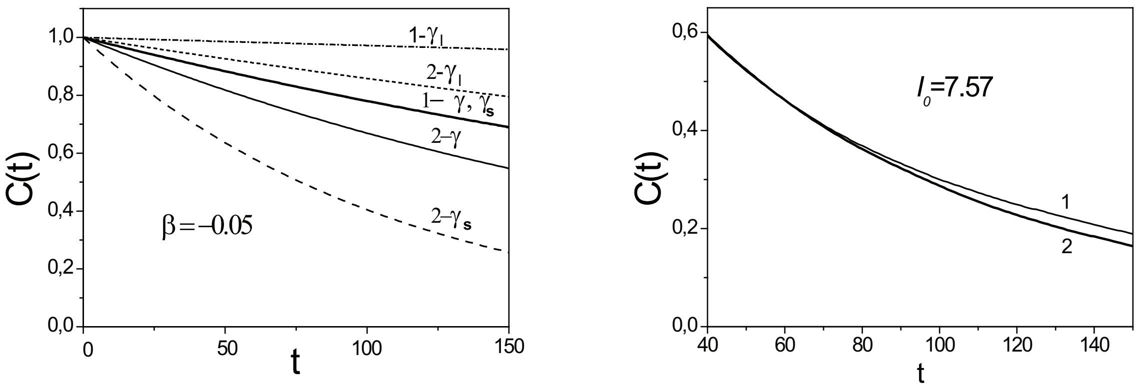
At first, the relaxation rates in the short- and long-time regimes in dependence on the deviation β and the noise strength D were calculated and compared. Figure 1 shows the dependences of the short-time relaxation rates γ_s (dashed curves) and the long-time relaxation rates γ_l (solid curves) as functions of the deviation β of the mean incoming intensity $\langle I_0\rangle$ from the value I_{0K}, at which the transition from one state to the other occurs in the deterministic limit. The numbers on the curves determine various noise intensities D (*1* – $D = 0.1$; *2* – $D = 0.2$; *3* – $D = 0.3$). It can be seen in the figure that the coincidence of the relaxation rates takes place at the considerable removal from the instability point ($\beta = 0$), either toward higher or lower values. What is more, in these regions, the dependence on the noise intensity is not observed either.

For our problem, we are interested in the behavior of these rates in the close neighborhood of the instability point, where their values differ significantly. The decrease of the relaxation rates near the instability point indicates a critical slowing down. Moreover, as can be expected, this decrease is more significant for γ_l, i.e. in the long-time regime. We note that an increase in the noise intensity leads to a shift of the minimum values of relaxation rates to the side of increasing the control parameter I_0. This is the ev-



a b

Fig. 2. *a* – Correlation functions $C(t)$ for $\beta = -0.14$, obtained in the one-exponential approximation with relaxation rates γ_s and γ_l, and by DE method (curves with symbol γ) with various values of D ($D = 0.1$ – curves with number *1*), $D = 0.2$ – curves with number *2*). *b* – Correlation functions $C(t)$ obtained by a numerical simulation with $I_0 = 7.46$ and $D = 0.1$ (curve *1*), $D = 0.2$ (curve *2*)



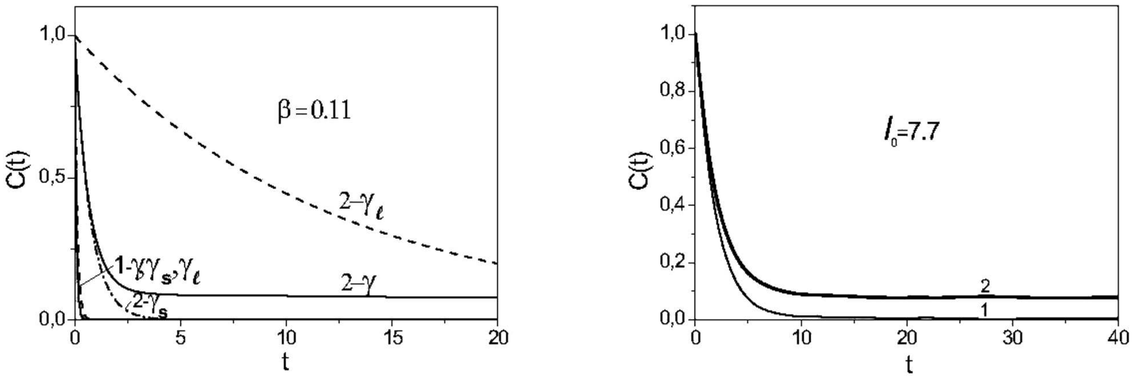
a b

Fig. 3. *a* – Correlation functions $C(t)$ for $\beta = -0.05$ obtained by the one-exponential approach with relaxation rates γ_s and γ_l, and by the DE method (curves with symbol γ) with different values of D ($D = 0.1$ – curves with number *1*), $D = 0.2$ – curves with number *2*). *b* – Correlation functions $C(t)$ obtained by the numerical simulation with $I_0 = 7.57$ and $D = 0.05$ (the curve *1*), $D = 0.1$ (curve *2*)

idence that the transition will begin later than in the deterministic case, i.e., the stability domain increases.

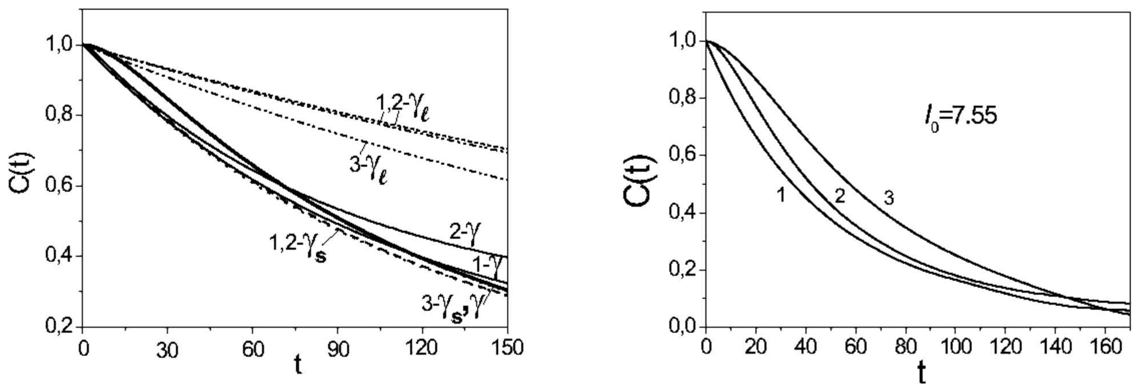
To analyze the dependence of the correlation function decay on the noise intensity D, it is convenient to divide the region near the instability point into three intervals according to the values of β: interval I ($\beta = -0.17 \div -0.1$), II ($\beta = -0.1 \div 0.01$), and III ($\beta = 0.01 \div 0.15$) (see Fig. 1).

In interval I, the values of rates at small times γ_s (dashed curves) almost do not depend both on the value of β and on the noise intensity D. At the same time, the rates at large times γ_l (solid curves) decrease with increasing D. This means that the behavior of the correlation function in the dependence on D is dominated in this interval by the rate γ_l. The results of calculations of the correlation functions in this interval obtained with the use of different methods are shown in Fig. 2, *a* for $\beta = -0.14$ and various noise intensities D. The curves in Fig. 2, *a* are marked by two symbols. The first symbol is number 1 or 2 that defines the noise strength (*1* – $D = 0.1$; *2* – $D = 0.2$). The second symbol determines the method, with which the function was obtained. The correlation function $C(t)$ obtained with the use of the single-exponentional approximation is marked by the symbol γ_s for the relaxation rate γ_s and by the symbol γ_l for the relaxation rate γ_l. The correlation function calculated by means of the DE method is marked as



a *b*

Fig. 4. *a* – Correlation functions $C(t)$ for $\beta = 0.115$ obtained by one-exponential approach with relaxation rates γ_s and γ_l, and by the DE method (curves with symbol γ) with different values of D ($D = 0.1$ –curves with number *1*), $D = 0.2$ – curves with number *2*). *b* – Correlation functions $C(t)$ obtained by numerical simulation with $I_0 = 7.7$ and $D = 0.1$ (curve *1*), $D = 0.25$ (curve *2*)



a *b*

Fig. 5. *a* – Correlation functions $C(t)$ for $\beta = -0.1$ and $D = 0.1$ obtained by the one-exponential approach with relaxation rates γ_s and γ_l, and by the DE method (curves with symbol γ) with various values of τ ($\tau = 0.01$ – the curves with number *1*, $\tau = 0.1$ – with number *2*, $\tau = 0.8$ – with number *3*). *b* – Correlation functions $C(t)$ obtained by a numerical simulation with $I_0 = 7.55$, $D = 0.1$, and $\tau = 0.01$ (curve *1*), $\tau = 0.5$ (curve *2*), $\tau = 1$ (curve *3*)

γ. We can see that when the noise intensity is small the use of all methods gives the same decay of the correlation functions (the curves with number *1*). With the growth of the noise intensity up to $D = 0.2$, the relaxation rate γ_s is practically unchanged, and the correlation function marked $2 - \gamma_s$ coincides with the correlation function $1 - \gamma_s$ with $D = 0.1$. At the same time, the value of γ_l is decreased, which leads to the slowing of the decay of the correlation function (curve $2 - \gamma_l$). The calculation with the DE method gives curve $2 - \gamma$.

Fig. 2, *b* shows the correlation functions obtained by the numerical simulation of Eq. (4) in the parameter region that corresponds to interval I ($I_0 = 7.46$ and $D = 0.1$ – curve *1*, $D = 0.2$ – curve *2*). The main objective in the selection of parameters for numerical calculations was to obtain a typical behavior of the correlation functions in this interval. It is seen in this figure that, as D increases, the decay of the correlation function slows down. Such a behavior coincides with the result of analytical calculations with the use of DE method (compare with the curves $1 - \gamma$ and $2 - \gamma$ in Fig. 2, *a*).

In interval II, the values of relaxation rates with increasing D may not decrease but, on the contrary, may increase (see Fig. 1). The behavior of the correlation functions for various noise strengths D ($D = 0.1$ – the curves with number *1*, $D = 0.2$ –

the curves with number *2*) in the pre-threshold region ($\beta = -0.05$) obtained with the use of analytical calculations is shown in Fig. 3, *a*. These results are compared with those of numerical calculations shown in Fig. 3, *b* (with $I_0 = 7.57$ and $D = 0.05$ – curve *1*, $D = 0.1$ – curve *2*). In Fig. 3, *b*, we can see the non-standard behavior of the correlation function in the dependence on the noise strength, namely, the growth of D leads to a faster decay (curve *2*). It can be seen in Fig. 3, *a* that such a behavior is reproduced qualitatively by the DE method, as follows from the comparison of the curves $1-\gamma$ and $2-\gamma$. The agreement is rather rough, but we show here only the possibility of such a behavior and the fact that it can be described by the DE method.

The correlation function behavior for various noise strengths D in the region above the threshold (interval III, $\beta = 0.11$) is shown in Fig. 4, *a* (analytical calculations) and in Fig. 4, *b* (numerical calculations with $I_0 = 7.7$). When the noise intensity is small ($D = 0.1$ – the curves with number *1*), the correlation function falls abruptly, and its time dependences obtained with the use of different approaches are similar. When the noise intensity increases ($D = 0.2$ – curves with number *2*), the relaxation rates decrease, and its decreasing is more significant in the long-time regime, than in the short-time regime (compare curves $2 - \gamma_l$ and $2 - \gamma_s$). The calculation with the DE method gives curve $2 - \gamma$, which tends to the asymptote at large times. The numerical calculation (Fig. 4, *b*) gives a similar behavior (curve *2*).

In the non-Markovian case, we analyze the dependence of the correlation function on the noise correlation time τ. For analytical calculations, formulas (20) and (21) were used. Calculations reported in the literature, both numerical [10, 14] and analytical [1, 2, 10] ones, and carried out for different models that describe stochastic processes at some distance from the instability point show that an increase of the noise correlation time leads to slowing the correlation function decay, i.e. to the growth of the relaxation time. At the same time, it can be expected that, in the pre-threshold region in a near proximity of the transition point, the behavior will be different. In Fig. 5, *a*, we show the time dependence of the correlation functions obtained with $\beta = -0.1$ and $D = 0.1$ for several noise correlation times ($\tau = 0.01$ – the curves with number *1*; $\tau = 0.1$ – with number *2*; $\tau = 0.8$ – with number *3*), by using different methods marked, as earlier, by the symbol γ with corresponding indices. The growth of the noise correlation time from $\tau = 0.01$ to $\tau = 0.1$ leads to a standard slowing of the correlation function decay (compare curves $1-\gamma$ and $2-\gamma$). However, the further increase of the correlation time up to $\tau = 0.8$ leads, on the contrary, to the decay acceleration at large times (see curve $3-\gamma$). Such a behavior qualitatively agrees with the numerical simulations with $I_0 = 7.55$, $D = 0.1$, and $\tau = 0.01$ (curve *1*), $\tau = 0.5$ (curve *2*), $\tau = 1$ (curve *3*) (Fig. 5, *b*).

5. Conclusions

We have analyzed the decay of the intensity correlation function in the neighborhood of the transition from the state with low tunneling efficiency to a state with high efficiency. This transition takes place at the marginal point of the hysteresis cycle. The analytical expressions for the correlation functions in a vicinity of the instability point are obtained with the use of the approximated equation of the process, which ensures that the system is very near this point. The results are compared with the results of numerical simulations of the model tunneling process operating in the neighborhood of the transition point.

The calculations of the correlation functions carried out with the use of different methods allow us to conclude that, in a neighborhood of the instability point, the results achieved with the use of the DE method qualitatively agree with those of a numerical simulation. Outside the region of instability, the single-exponential approximation can be used. Directly in the instability region, the abnormal behavior of the correlation functions is observed both in the dependence on the noise intensity (acceleration of the correlation function decay with increase of D) and on the noise correlation time (acceleration of the correlation function decay with increase of τ).

1. S. Grossman, Phys. Rev. A **17**, 1123 (1978).
2. A. Hernandez-Machado, M. San Miguel, and J.M. Sancho, Phys. Rev. A **29**, 3388 (1984).
3. W. Nadler and K. Schulten, Z. Phys. B: Condensed Matter **59**, 53 (1985).
4. W. Nadler and K. Schulten, Z. Phys. B: Condensed Matter **72**, 535 (1988).
5. W. Nadler and K. Schulten, Phys. Rev. Lett. **51**, 1712 (1983).
6. W. Nadler and K. Schulten, J. Chem. Phys. **82**, 151, (1985).
7. J.M. Noriega, L. Pesquera, and M.A. Rodriguez, Phys. Rev. A **43**, 4008 (1991).

8. J.M. Noriega, L. Pesquera, M.A. Rodriguez, J. Casademunt, and A. Hernandez-Machado, Phys. Rev. A **44**, 2094 (1991).
9. A. Hernandez-Machado, J. Casademunt, M.A. Rodrigues, L. Pesquera, and J.M. Noriega, Phys. Rev. A **43**, 1744 (1991).
10. J.M. Sancho, M. San Miguel, S.L. Katz, and J.D. Gunton, Phys. Rev. A **26**, 1589 (1982).
11. V.V. Mitin, V.A. Kochelap, and M.A. Stroscio, *Introduction to Nanoelectronics. Science, Nanotechnology, Engineering, and Applications* (Cambridge Univ. press, Cambridge, 2008).
12. V.N. Ermakov and E.A. Ponezha, Metallofiz. Nov. Tekhnol. **30**, 585, (2008).
13. E.A. Ponezha, Ukr. Phys. J. **55**, 244 (2010).
14. V.N. Ermakov and E.A. Ponezha, Metallofiz. Nov. Tekhnol. **33**, 45, (2011).
15. E.A. Ponezha, Metallofiz. Nov. Tekhnol. **36**, 713, (2014).
16. A.S. Davydov and V.N. Ermakov, Physica D **28**, 168 (1987).
17. M.R. Young and S. Singh, Phys. Rev. A **38**, 238 (1988).

Received 13.03.16

О.О. Понежа

РЕЛАКСАЦІЯ КОРЕЛЯЦІЙНОЇ ФУНКЦІЇ ІНТЕНСИВНОСТІ ПОБЛИЗУ ТОЧКИ НЕСТАБІЛЬНОСТІ ДЛЯ ПРОЦЕСУ РЕЗОНАНСНОГО ТУНЕЛЮВАННЯ

Р е з ю м е

Досліджувалася релаксація кореляційної функції інтенсивності $C(t)$ потоку електронів у точці нестабільності для процесу резонансного тунелювання електронів крізь двобар'єрну нанометрову структуру. Передбачалося, що інтенсивність падаючого потоку може флуктуювати під дією зовнішнього шуму, як білого, так і кольорового. Поведінка кореляційної функції $C(t)$ аналізувалася як за допомогою методів, що приводять до одноекспоненційного наближення, таких як метод проекційного оператора й метод середнього часу релаксації, так і методу, заснованого на комбінації високо- і низькочастотного розкладання перетворення Лапласа для $C(t)$, при якому кореляційна функція апроксимувалася суперпозицією двох спадаючих експонент. Чисельна симуляція показала, що останній підхід на відміну від інших дає правильні результати в області нестабільності.