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INSTABILITY OF DIRECTOR ORIENTATION IN A PLANAR NEMATIC CELL UNDER TUNABLE BOUNDARY CONDITIONS IN THE ELECTRIC FIELD

The planar-planar director reorientation in a nematic liquid crystal (NLC) cell by an external static electric field has been studied. The gliding of the NLC director easy axis over the polymer substrate due to the interaction between the easy axis and the electric field is taken into account. The contribution of this interaction to the surface free energy of the system is assumed to be linear in the electric field, if the elastic fragments of substrate polymer molecules possess their own dipole moments. If the dipole moments of the elastic fragments of polymer molecules are induced by the electric field, the corresponding contribution is taken to be quadratic in the electric field. Depending on the character of the interaction between the easy axis and the electric field, the orientation instability of the director is shown to either have a threshold or not. In both cases, the dynamics of the director and the easy axis has been analyzed, by starting from the field switching-on time moment, during the transition of the system to a stationary state, and until the field is switched-off and the system returns to the initial homogeneous state.

Keywords: nematic liquid crystal, orientational instability, easy axis gliding, director evolution, switching-on/off time, Fréedericksz transition threshold.

1. Introduction

Within the last decades, liquid crystals (LCs) have found a wide application as an element base for a series of information displays owing to their unique magneto- and electro-optical properties. The latter are closely connected with the orientational ordering in the mesophase.

The orientational ordering of LC in the cell volume depends on the conditions at the cell substrates, namely, the anchoring energy, direction of the axis of easy director orientation, and so forth. Polymer films are widely applied in liquid crystal cells as substrates, in which the orientational anisotropy (easy orientation axis) can be induced by illuminating them with polarized light (so-called photo-alignment) [1, 2]. It was found that the easy director axis induced in this way at the polymer surface can change its orientation (gliding) in an external light or dc electric/magnetic field. This phenomenon affects the equilibrium configuration of the director field in the LC cell and opens possibilities for the creation of such conditions for the director at the orienting surface that can be controlled with the use of external fields.

The easy axis gliding in the surface plane of a photo-aligned polymer was observed experimentally in a nematic liquid crystal (NLC) cell in the presence of an electric [3–5] or magnetic [5–7] field. In the opinion of the cited authors [3, 6], the easy axis gliding is possible due to the interaction between NLC molecules and elastic fragments of polymer molecules. At low anchoring energies between the NLC and the substrate, the rotation of the near-surface nematic layer under the action of an external field gives rise to the easy axis reorientation. In works [4, 5], the explanation of the easy axis reorientation was based on the adsorption of NLC molecules on the surface of a polymer substrate. The mechanical moment produced by the external field is transferred from the NLC volume to the nematic molecules adsorbed on the surface and, thus, stimulates the easy axis to shift. Fields, in which the easy axis gliding was observed, were found to be by 1 to 2 orders of magnitude stronger, if NLC molecules were absorbed on the substrate surface than if not. The both mentioned mechanisms governing the rotation of the easy orientation axis of the NLC director at the substrate surface were considered in work [7].

The processes of NLC director and easy axis reorientations in the polymer substrate plane in a strong ac electric field and the relaxation of a system after the field switching-off were studied experimentally and theoretically in work [8]. The simultaneous action of the ac electric field and polarized light on the NLC director dynamics near the polymer surface preliminarily illuminated with ultraviolet was studied in works [9, 10]. In works [8–10], the easy axis reorientation was explained in the framework of a scenario that is based on the adsorption of NLC molecules on the substrate surface. The phenomenological model of the phenomenon, which was proposed in those works, involves the influence of the nematic volume on the easy axis motion, decelerating action of polymer, and viscosity of the easy axis motion. However, in work [11], using the NLC 5CB, it was experimentally revealed that a dc electric field can also induce an orientational anisotropy at the electrosensitive azopolymer surface. The origin of this phenomenon was assumed to be the reorientation of elastic hydrocarbon chains of azopolymer molecules, which is associated with the interaction between their own or induced dipole moments and the external electric field [12].

In this work, the orientational instability of the director in an NLC cell with planar alignment and embedded into a dc electric field will be studied. The latter is oriented along the cell surface and perpendicularly to the initial uniform director orientation. A possibility for the axis of easy director orientation to glide in the plane of either of the cell polymer substrates is assumed. The motion of the easy axis is supposed to be governed by its interaction with the electric field. The contribution of this interaction to the surface free-energy density of the NLC cell is considered to be linear or quadratic for the electric field strength depending on whether the dipole moments of the elastic segments of polymer substrate molecules are their own or are induced by the electric field.

It is found that the orientational instability of NLC has no threshold in the former case and has a threshold in the latter one. The time dependences of the director and moving easy axis deviation angles are calculated and analyzed from the time moment, when the electric field is switched-on, then when the system transits into a stationary state, and until the field is switched-off and the system returns into the initial homogeneous state.

2. NLC Free Energy and Equation for the Director

Let a plane-parallel NCL cell be confined by the planes $z = 0$ and $z = L$ and have a uniform initial orientation of the director along the axis Ox. The cell is embedded into an external uniform dc electric field with the strength vector directed along the axis Oy, $\mathbf{E} = (0, E, 0)$. We assume that the axis of easy director orientation $\mathbf{e}$ interacts with the electric field $\mathbf{E}$ at the upper ($z = L$) polymer substrate of the cell. As a result of this interaction, the easy axis $\mathbf{e}$ can glide in the substrate plane. The NLC anchoring with the lower ($z = 0$) substrate is uniform and infinitely strong.

The free energy of the NLC cell can be written in the form

$$F = F_{\text{el}} + F_E + F_S + F_{SE}, \tag{1}$$

where

$$F_{\text{el}} = \frac{1}{2}\int_V \Big\{K_1(\operatorname{div}\mathbf{n})^2 + K_2(\mathbf{n}\cdot\operatorname{rot}\mathbf{n})^2 + K_3[\mathbf{n}\times\operatorname{rot}\mathbf{n}]^2\Big\}\, dV,$$

$$F_E = -\frac{1}{8\pi}\int_V \mathbf{ED}\, dV,$$

$$F_S = -\frac{W}{2}\int_S (\mathbf{en})^2\, dS,$$

$$F_{SE} = -\frac{\alpha}{m}\int_S \big(\mathbf{eE}\big)^m\, dS.$$

Here, F_{el} is the NLC elastic energy; F_E the anisotropic contribution of the electric field to the free energy; F_S the surface free energy written in the form of Rapini potential; F_{SE} the contribution to the surface free energy from the interaction between the moving axis of easy director orientation and the electric field at the upper ($z = L$) polymer substrate; K_1, K_2, and K_3 are the NLC elastic constants; $\mathbf{n}$ is the director; $\mathbf{D} = \hat{\epsilon}\mathbf{E}$ is the electric induction vector; $\hat{\epsilon} = \epsilon_\perp \hat{\mathbf{1}} + \epsilon_a \mathbf{n}\otimes\mathbf{n}$ and $\epsilon_a = \epsilon_\parallel - \epsilon_\perp > 0$ are the tensor and the anisotropy, respectively, of the static dielectric permittivity of NLC; W is the azimuthal energy of NLC anchoring at the upper ($z = L$) substrate of the cell, which is associated with the director deviations in the plane xOy. The parameter m is put equal to 1 or 2, depending on whether the molecules of a polymer orientant have their own or induced dipole moments, respectively.

Owing to the homogeneous character of the system in the plane xOy, the director $\mathbf{n}$ in the cell bulk and the easy axis $\mathbf{e}$ at the upper substrate of the cell can be described by the relations

$$\begin{aligned} \mathbf{n} &= \mathbf{i}\cdot\cos\varphi(z,t) + \mathbf{j}\cdot\sin\varphi(z,t), \\ \mathbf{e} &= \mathbf{i}\cdot\cos\psi(t) + \mathbf{j}\cdot\sin\psi(t), \end{aligned} \tag{2}$$

where $\mathbf{i}$ and $\mathbf{j}$ are the orts of the Cartesian coordinate system.

The free energy (1) per unit area of the cell surface reads

$$F = \frac{K_2}{2}\int\limits_0^L \left[\left(\frac{\partial\varphi}{\partial z}\right)^2 - \frac{E^2}{4\pi K_2}\left(\epsilon_\perp + \epsilon_a\sin^2\varphi\right)\right]dz - \\ - \frac{W}{2}\cos^2(\varphi_L - \psi) - \frac{\alpha}{m}\left(E\sin\psi\right)^m, \tag{3}$$

where $\varphi_L = \varphi(z = L)$ is the director rotation angle at the upper substrate of the cell. Expression (3) involves the solution of electrostatic equations for the electric field $\mathbf{E}$ in the NLC bulk.

By minimizing the free energy (3) with respect to the angles φ and ψ, we obtain the equation

$$\varphi''_{zz} + \frac{\varepsilon_a E^2}{4\pi K_2}\sin\varphi\cos\varphi = \eta_1\varphi'_t, \tag{4}$$

as well as the corresponding boundary conditions

$$\varphi\big|_{z=0} = 0, \tag{5}$$

$$K_2\,\varphi'_z\big|_{z=L} + W\sin(\varphi_L - \psi)\cos(\varphi_L - \psi) = 0, \tag{6}$$

$$\big[W\sin(\varphi - \psi)\cos(\varphi - \psi) + \\ + \alpha E^m\cos\psi\sin^{m-1}\psi\big]_{z=L} = \eta_2\psi'_t. \tag{7}$$

Here, the primed functions φ and ψ mean the derivatives with respect to the corresponding arguments. The right-hand side of Eq. (4) involves, as was done in works [1, 13], dissipative processes in the NLC bulk, when the director rotates, in the approximation where the director coupling with hydrodynamic motions of the nematic is neglected; η_1 is the coefficient of NLC bulk viscosity. The easy axis gliding in the plane $z = L$ of the upper substrate of the cell is described, as was done in works [8–10], by the right-hand side of condition (7), where η_2 is the coefficient of viscosity for the easy axis. In the general case, the solution of Eq. (4) that satisfies the boundary conditions (5)–(7) can be determined only numerically.

3. Linear Interaction between the Easy Axis and the Electric Field

3.1. Dynamics of the director and the moving easy axis

Let the contribution of the interaction between the moving axis of easy director orientation $\mathbf{e}$ and the electric field $\mathbf{E}$ at the upper ($z = L$) substrate of the cell to the NLC surface free energy be linear in the field strength (the parameter $m = 1$ in expression (1) for F_{SE}). We assume the electric field strength $\mathbf{E}$ to be small in comparison with the threshold $E^\infty_{\rm th} = \frac{\pi}{L}\sqrt{\frac{4\pi K_2}{\epsilon_a}}$ for the Fréedericksz transition at the infinitely strong NLC anchoring at the cell substrates and in the absence of easy axis gliding. Then, assuming the angles φ and ψ to be small, let us confine the consideration to Eq. (4) linearized in φ and ψ,

$$\varphi''_{\xi\xi} + e^2\varphi = \varphi'_\tau \tag{8}$$

and to the boundary conditions (5)–(7) also linearized in φ and ψ,

$$\varphi\big|_{\xi=0} = 0, \tag{9}$$

$$\varphi'_\xi\big|_{\xi=1} + \varepsilon(\varphi_L - \psi) = 0, \tag{10}$$

$$\varepsilon(\varphi_L - \psi) + \tilde{\alpha}e = \gamma\psi'_\tau. \tag{11}$$

Here, the following dimensionless parameters are used: the coordinate $\xi = z/L$, time $\tau = tK_2/(\eta_1 L^2)$, viscosity coefficient $\gamma = \eta_2/(\eta_1 L)$, anchoring energy $\varepsilon = WL/K_2$, electric field strength $e = \pi E/E^\infty_{\rm th}$, and interaction parameter $\tilde{\alpha} = \alpha\sqrt{4\pi/(\epsilon_a K_2)}$.

In view of Eq. (11), the boundary condition (10) reads $\varphi'_\xi\big|_{\xi=1} = \tilde{\alpha}e - \gamma\psi'_\tau$. Then let us construct the function

$$u(\xi,\tau) = \varphi(\xi,\tau) - \xi(\tilde{\alpha}e - \gamma\psi'_\tau). \tag{12}$$

As follows from Eqs. (8)–(10), it satisfies the equation

$$u''_{\xi\xi} + e^2 u = u'_\tau - \xi\left(\gamma\psi''_{\tau\tau} - \gamma e^2\psi'_\tau + \tilde{\alpha}e^3\right) \tag{13}$$

and the homogeneous boundary conditions

$$u\big|_{\xi=0} = u'_\xi\big|_{\xi=1} = 0. \tag{14}$$

The solution of Eq. (13) that satisfies the boundary conditions (14) is sought as a series

$$u(\xi,\tau) = \sum_{n=0}^{\infty} u_n(\tau)\sin\big[\pi(n + 1/2)\xi\big], \tag{15}$$

where u_n are the unknown expansion coefficients. Substituting expression (15) into Eq. (13) and taking advantage of the linear independence of the functions $\sin[\pi(n+1/2)\xi]$ within the interval $[0,1]$, we obtain the following equation for the coefficients u_n:

$$u_n' + \left(\pi^2(n+1/2)^2 - e^2\right)u_n = \\ = \frac{2(-1)^n}{\pi^2(n+1/2)^2}\left(\gamma\psi''_{\tau\tau} - \gamma e^2\psi'_\tau + \tilde{\alpha}e^3\right), \quad (16)$$

where $n = 0, 1, 2, \ldots$.

If the electric field is low in comparison with the critical one $E_{\rm th}^{\infty}$, the higher harmonics in expansion (15) can be regarded as small and neglected. As a solution of Eq. (13), we take only the first term in expansion (15),

$$u(\xi,\tau) = u_0(\tau)\sin(\pi\xi/2). \quad (17)$$

The substitution of expression (17) into the boundary condition (11) written for the function $u(\xi,\tau)$ brings about

$$u_0 = \psi + (1 + 1/\varepsilon)(\gamma\psi'_\tau - \tilde{\alpha}e). \quad (18)$$

With regard for expression (18) for u_0 in Eq. (16), we arrive at the equation for the function $\psi(\tau)$,

$$a\psi'' + b\psi' + c\psi = d, \quad (19)$$

where

$$a = \gamma\left[1 + \left(1 - 8/\pi^2\right)\varepsilon\right],$$
$$b = \gamma\left[8\varepsilon e^2/\pi^2 + \varepsilon/\gamma + (\varepsilon+1)\left(\pi^2/4 - e^2\right)\right],$$
$$c = \varepsilon\left(\pi^2/4 - e^2\right),$$
$$d = \tilde{\alpha}e\left[8\varepsilon e^2/\pi^2 + (\varepsilon+1)\left(\pi^2/4 - e^2\right)\right].$$

The solution of Eq. (19) gives us the time dependence of the rotation angle of the moving easy axis at the cell surface $z = L$ in the form

$$\psi(\tau) = \frac{k_2 d/c - \tilde{\alpha}e/\gamma}{k_1 - k_2}e^{-k_1\tau} - \frac{k_1 d/c - \tilde{\alpha}e/\gamma}{k_1 - k_2}e^{-k_2\tau} + \frac{d}{c}, \quad (20)$$

where $k_{1,2} = (b \pm \sqrt{b^2 - 4ac})/(2a) > 0$. The coefficients of $e^{-k_{1,2}\tau}$ in Eq. (20) are obtained from the initial conditions $\psi(\tau = 0) = 0$ and $\psi'_\tau(\tau = 0) = \tilde{\alpha}e/\gamma$. The latter follows from Eq. (11) as a consequence of $\varphi(\xi, \tau = 0) = 0$. The substitution of $\psi(\tau)$-dependence (20) into Eq. (18) gives the value of $u_0(\tau)$ and, accordingly, the function $u(\xi,\tau)$ (17). Taking the latter and the $\psi(\tau)$-value (20) into account, relation (12) yields the dependence $\varphi(\xi,\tau)$ for the director rotation angle in the NLC bulk. The $\varphi(\xi,\tau)$-expression is cumbersome and is not presented here.

However, if we confine the consideration to the quantities of the first order of smallness with respect to $1/\gamma$, the time dependences of the deviation angles for the director and the easy axis in the approximation $\gamma \gg 1$ ($\eta_2 \gg \eta_1 L$) [8] read

$$\varphi(\xi,\tau) \approx \tilde{\alpha}e\left(\frac{32e^2}{\pi^4}\sin\frac{\pi\xi}{2} + \xi\right)\times \\ \times\left[1 - \exp\left(-\frac{\varepsilon\tau}{\gamma(1+\varepsilon)}\right)\right], \quad (21)$$
$$\psi(\tau) \approx \tilde{\alpha}e\frac{1+\varepsilon}{\varepsilon}\left[1 - \exp\left(-\frac{\varepsilon\tau}{\gamma(1+\varepsilon)}\right)\right].$$

As one can see from Eq. (21), the director field is deformed, and the moving easy axis deviates from the initial direction even in an infinitesimally weak electric field. Therefore, the orientational instability of NLC in the electric field has no threshold.

The characteristic switching-on time is equal to

$$t_{\rm on} = \frac{\eta_2 L(1+\varepsilon)}{\varepsilon K_2}. \quad (22)$$

As the anchoring energy ε grows, the time $t_{\rm on}$ decreases and approaches the value $\eta_2 L/K_2$ in the case of infinitely strong NLC anchoring with the upper ($z = L$) substrate of the cell. The time $t_{\rm on}$ does not depend on the interaction parameter α and the coefficient of NLC bulk viscosity η_1.

In general, the dynamics of the director $\mathbf{n}$ and the moving easy axis $\mathbf{e}$ after the electric field is switched-on, but before the system reaches a stationary state, is governed by the field strength E and the NLC cell parameters. In Fig. 1, the time dependences of the deviation angles of the director, φ_L, and the easy axis, ψ, at the cell surface $z = L$ are plotted for various values of the electric field strength E. The dependences $\varphi_L(t)$ and $\psi(t)$ were calculated, by numerically solving Eq. (4) with the boundary conditions (5)–(7) for nematic 5CB of the thickness $L = 10$ μm. The coefficients of NLC bulk viscosity and easy axis viscosity were put equal to $\eta_1 = 0.5$ P [14] and $\eta_2 = 0.01$ P $\times$ cm [11, 12], respectively.

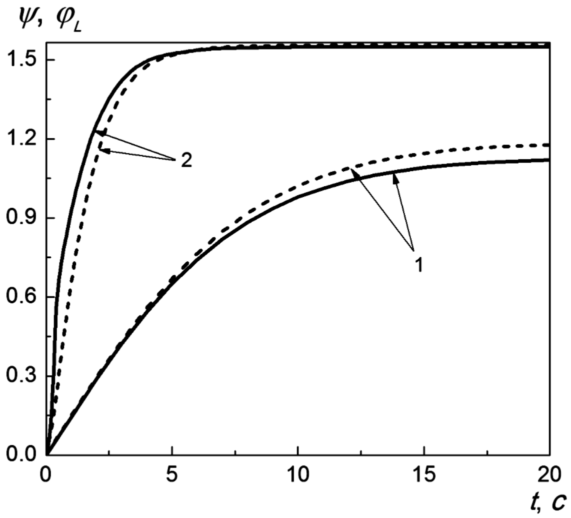

Fig. 1. Time dependences of the director (φ_L, solid curves) and moving easy axis (ψ, dashed curves) deviation angles at the cell surface in the case of linear interaction between the easy axis and the electric field. $\varepsilon = 10$, $\tilde{\alpha} = 1$. $E = 0.5E_{\rm th}^{\infty}$ (*1*) and $1.5E_{\rm th}^{\infty}$ (*2*)

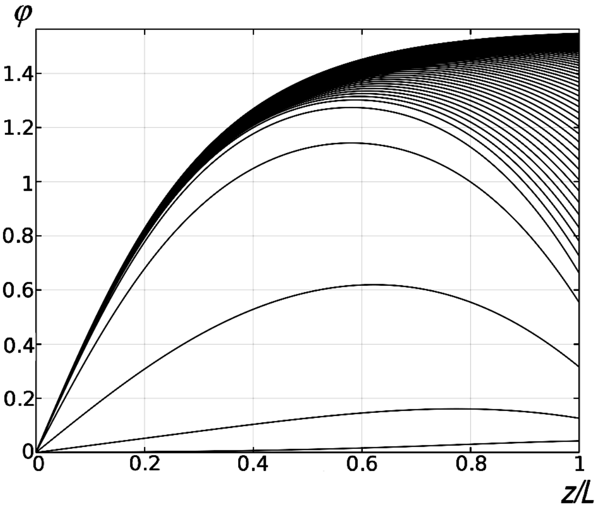

Fig. 2. Dynamics of the director deviation angle across the cell thickness from the time moment of electric field switching-on ($t = 0$ s) to the transition into the stationary state ($t = 10$ s) with a step of 0.1 s in the case of linear interaction between the easy axis and the electric field. $\varepsilon = 10$, $\tilde{\alpha} = 1$, and $E = 1.5E_{\rm th}^{\infty}$

Let the electric field strength E be lower than 0.5 $E_{\rm th}^{\infty}$ ($E_{\rm th}^{\infty}$ is the Fréedericksz transition threshold) in the absence of NLC anchoring at the upper ($z = L$) substrate of the cell, i.e. $\varepsilon = 0$. In this case, after the electric field has been switched-on and until the system reaches the stationary state, the moving easy axis deviates faster owing to its interaction with the electric field. The director in the NLC bulk rotates following the easy axis, so that $\varphi_L \leqslant \psi$ (see Fig. 1). The deformations of the director field are the largest at the surface $z = L$ with the moving easy axis, $\varphi(z,t) \leqslant \varphi_L(t)$, unlike the case of fixed easy axis positions at both cell substrates where the largest deviation of the director is reached only in the NLC bulk.

When the system transits into a stationary state in the electric field $E > 0.5E_{\rm th}^{\infty}$, the director, as a whole, deviates and draws the moving easy axis after itself, so that $\varphi_L \geqslant \psi$ (see Fig. 1). Now, the director deviation reaches a maximum in the NLC bulk. As the system approaches the stationary state, the maximum deformations of the director field become shifted from the NLC bulk toward the surface with the moving easy axis. The distributions of the director deviation angle over the cell thickness calculated for various time moments before the system reaches the stationary state are depicted in Fig. 2. In the stationary state, $\psi \geqslant \varphi_L$ at any time, irrespective of the electric field strength. We emphasize that if the NLC bulk viscosity and the easy axis viscosity increase, the time dependences $\varphi(z,t)$ and $\psi(t)$ saturate more slowly.

In the stationary state, $\varphi'_t = \psi'_t = 0$, and, in accordance with the boundary conditions (6) and (7), we have $\varphi'_z\big|_{z=L} > 0$ at the upper surface of a cell. Evidently, $\varphi'_z > 0$ in the NLC bulk as well. Therefore, the maximum deviation of the director is reached at the surface $z = L$. By twice integrating Eqs. (4) over z and taking the boundary conditions (5)–(7) into account, we obtain the equation

$$\frac{K_2}{E}\int\limits_0^{\varphi}\frac{d\varphi}{\left[\dfrac{\varepsilon_a K_2}{4\pi}(\sin^2\varphi_L - \sin^2\varphi) + \alpha^2\cos^2\psi\right]^{1/2}} = z. \tag{23}$$

From this equation taken at $z = L$ and making allowance for condition (7), we find the deviation angles for the director, φ_L, and the easy axis, ψ, at the cell surface. Substituting the obtained φ_L- and ψ-values into Eq. (23), we obtain the distribution of the director rotation angle φ over the cell thickness for the given electric field strength E.

The dependences $\varphi(z)$ calculated for various values of field strength E and interaction parameter α are depicted in Fig. 3. As the anchoring energy ε and the parameter α increase, the deformations of the NLC director field grow and reach the maximum at the surface $z = L$. If the field strength E grows, the deviation angles of the director, φ_L, and the easy axis, ψ, monotonically increase in such a way that $\varphi_L \leqslant \psi$ and reach the maximum possible value $(\pi/2)$ already at $E \gtrsim 2E^{\infty}_{\rm th}$.

Let us consider the relaxation of the system from the stationary state into the initial homogeneous state, when the electric field is switched-off, $E = 0$. By linearizing φ and ψ, we obtain the equation

$$\varphi''_{\xi\xi} = \varphi'_\tau \tag{24}$$

and the corresponding boundary conditions

$$\varphi\big|_{\xi=0} = 0, \tag{25}$$

$$\varphi'_\xi\big|_{\xi=1} + \varepsilon(\varphi_L - \psi) = 0, \tag{26}$$

$$\varepsilon(\varphi_L - \psi) = \gamma\psi'_\tau. \tag{27}$$

By solving Eqs. (24)–(27) and considering the finite character of the solution as $\tau \to +\infty$, we find the following expressions for the deviation angles of the director and the moving easy axis, respectively:

$$\varphi(\xi,\tau) = \sum_{n=1}^{\infty} A_n e^{-\varkappa_n^2\tau}\sin(\varkappa_n\xi), \tag{28}$$

$$\psi(\tau) = \sum_{n=1}^{\infty} \frac{A_n}{\gamma\varkappa_n} e^{-\varkappa_n^2\tau}\cos\varkappa_n, \tag{29}$$

where A_n are constants, and $\varkappa_n$ are positive roots of the equation

$$\operatorname{tg}\varkappa = \frac{1}{\gamma\varkappa} - \frac{\varkappa}{\varepsilon}. \tag{30}$$

Note that expression (29) for the angle $\psi(\tau)$ was obtained from the condition $\varphi'_\xi\big|_{\xi=1} + \gamma\psi'_\tau = 0$, which holds, when both boundary conditions (26) and (27) are taken into account. As follows from Eqs. (28) and (29), the characteristic time of relaxation is determined by the slowest damped mode–in particular, with $n = 1$–and is equal to

$$t_{\rm off} = \frac{\eta_1 L^2}{\varkappa_1^2 K_2}. \tag{31}$$

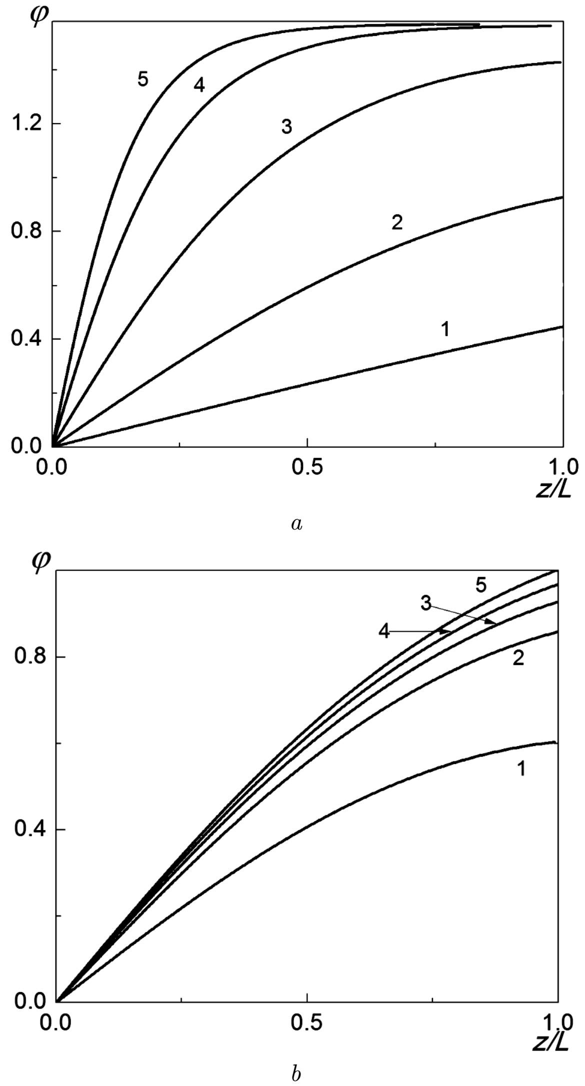

Fig. 3. Dependences of the director deviation angle φ on the coordinate z at $\varepsilon = 1$. (*a*) $\tilde\alpha = 1$; $E/E^{\infty}_{\rm th} = 0.2\,(1)$, $0.5\,(2)$, $1\,(3)$, $2\,(4)$, and $3\,(5)$. (*b*) $E/E^{\infty}_{\rm th} = 0.5$; $\tilde\alpha = 0.1\,(1)$, $0.5\,(2)$, $1\,(3)$, $2\,(4)$, and $10\,(5)$

The possibility for the easy axis to glide over the cell substrate $z = L$ results in the growth of the relaxation time $t_{\rm off}$ in comparison with the case where the gliding is absent. In particular, if the NLC anchoring with both cell substrates is infinitely strong, we obtain $t_{\rm off} = 4t^{\infty}_{\rm off}$, where $t^{\infty}_{\rm off} = \frac{\eta_1 L^2}{\pi^2 K_2}$ is the relaxation time of the system in the absence of easy axis gliding at $W = \infty$. However, the time $t_{\rm off}$ does not depend on the interaction parameter α. The growth of the anchoring energy ε leads to a reduction of the

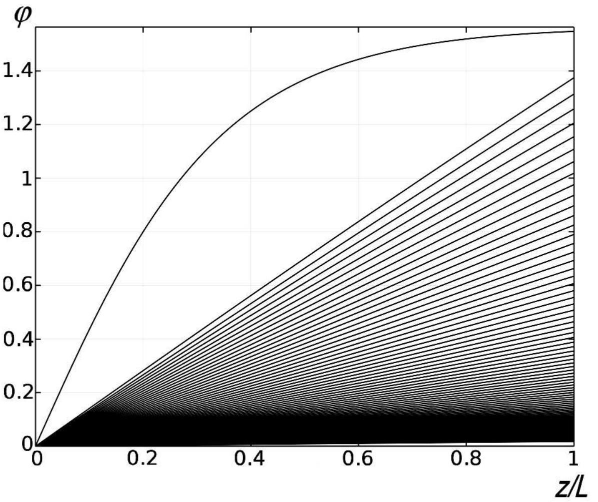

Fig. 4. Dynamics of the director deviation angle across the cell thickness from the electric field switching-off ($t = 0$ s) to the return into the initial homogeneous state ($t = 50$ s) with a step of 0.5 s. $\varepsilon = 10$ and $\tilde{\alpha} = 1$

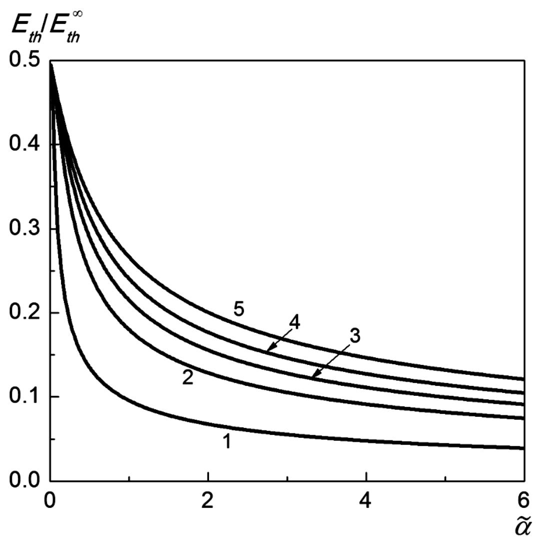

Fig. 5. Dependences of the threshold magnitude $E_{\rm th}$ on the interaction parameter α; $\varepsilon = 0.1$ (*1*), 0.5 (*2*), 1 (*3*), 2 (*4*), and 10 (*5*)

time $t_{\rm off}$, which is explained by a stronger interaction between the director and the easy axis.

In Fig. 4, the distributions of the director rotation angle across the cell thickness calculated for various time moments after the electric field in the system that was in the stationary state has been switched-off are exhibited. As one can see, the maximum deformations of the director field are reached at the surface with the moving easy axis. It is remarkable that the dependence $\varphi(z,t)$ becomes almost linear in z rather quickly in comparison with the total time of the complete relaxation of the system into its initial homogeneous state. After the electric field has been switched-off, the director returns back into the initial homogeneous state more rapidly and draws the easy axis after itself, so that $\varphi_L \leqslant \psi$.

4. Quadratic Interaction between the Easy Axis and the Electric Field

Now, let the contribution to the NLC surface free energy from the interaction between the axis of easy director orientation and the electric field at the cell substrate $z = L$ be quadratic in the field strength (the parameter $m = 2$ in expression (1) for F_{SE}). The minimization of the free energy (3) with respect to the angles φ and ψ brings about Eq. (4) and the boundary conditions (5)–(7) with $m = 2$. In the approximation of small φ- and ψ-angles, the dynamics of the system is described by the linearized equation (8) with the boundary conditions (9), (10), and the condition

$$\varepsilon(\varphi_L - \psi) + \tilde{\alpha} e^2 \psi = \gamma \psi'_\tau, \tag{32}$$

where $\tilde{\alpha} = 4\pi\alpha/(\epsilon_a L)$.

In the framework of the linearized problem, the deviation angles for the director and the easy axis read

$$\varphi(\xi,\tau) = \sum_{n=1}^{\infty} B_n e^{\lambda_n^2 \tau} \sin\left(\sqrt{e^2 - \lambda_n^2}\,\xi\right), \tag{33}$$

$$\psi(\tau) = \sum_{n=1}^{\infty} B_n e^{\lambda_n^2 \tau} \left(\frac{1}{\varepsilon}\sqrt{e^2 - \lambda_n^2} \cos\sqrt{e^2 - \lambda_n^2} + \right.$$
$$\left. + \sin\sqrt{e^2 - \lambda_n^2}\right), \tag{34}$$

where B_n are constants, and λ_n are the positive roots of the equation

$$\operatorname{tg}\sqrt{e^2 - \lambda^2} = \sqrt{e^2 - \lambda^2}\left(\frac{1}{\tilde{\alpha} e^2 - \gamma\lambda^2} - \frac{1}{\varepsilon}\right). \tag{35}$$

As one can see from Eqs. (33) and (34), if $\lambda_n \geqslant 0$, the spatial perturbation of the type $\sin\left(\sqrt{e^2 - \lambda_n^2}\,\xi\right)$ for the director field grows exponentially. The first harmonic turns out the strongest. Hence, the orientational instability of NLC in the electric field has a threshold. The magnitude of Fréedericksz transition threshold is given by the least positive root of the equation

$$\left(\frac{1}{\tilde{\alpha}e^2} - \frac{1}{\varepsilon}\right) e = \operatorname{tg} e, \quad (36)$$

which is obtained from Eq. (35) by setting $\lambda = 0$.

In Fig. 5, the dependences of the Fréedericksz transition threshold $E_{\rm th}$ on the interaction parameter α, which were found from Eq. (36) for several values of the anchoring energy ε, are shown. As the parameter α increases and the anchoring energy ε decreases, the threshold magnitude $E_{\rm th}$ diminishes. Note that, irrespective of the anchoring energy value in the limiting case $\alpha \to 0$, the Fréedericksz transition threshold approaches a value of $0.5E_{\rm th}^{\infty}$ obtained in the absence of NLC anchoring at the upper ($z = L$) substrate of the cell.

If the electric field strength E only slightly exceeds the threshold value $E_{\rm th}$, the angles φ and ψ can be considered small. Since the lowest harmonics are excited first, the higher harmonics in expansions (33) and (34) can be neglected. Then the values of angles φ and ψ are given by the terms with $n = 1$ in expressions (33) and (34), respectively. From whence, we obtain the characteristic time of switching-on in the form

$$t_{\rm on} = \frac{\eta_1 L^2}{(1-\sigma)(e^2 - e_{\rm th}^2)K_2}, \quad (37)$$

where

$$\sigma = \frac{2(\gamma - \tilde{\alpha})}{\tilde{\alpha}^2 e_{\rm th}\left(e_{\rm th}\cos^{-2} e_{\rm th} - \operatorname{tg} e_{\rm th}\right) + 2\gamma}, \quad e_{\rm th} = \frac{\pi E_{\rm th}}{E_{\rm th}^{\infty}}.$$

Unlike the case of linear interaction between the easy axis and the electric field, the switching-on time $t_{\rm on}$ decreases with a reduction of the anchoring energy ε, a decrease in the coefficient of nematic bulk viscosity η_1, and the growth of the interaction parameter α. However, as was in the case of linear interaction between the easy axis and the electric field, the time $t_{\rm on}$ decreases together with the viscosity η_2.

The calculations show that the dynamics of the director and the moving easy axis is qualitatively similar to that obtained in the case of linear interaction between the easy axis and the electric field at $E > 0.5E_{\rm th}^{\infty}$.

It is obvious that the process of return of the system from the stationary state into the initial homogeneous one after the electric field switching-off does not depend on the interaction between the easy axis and the electric field. The characteristic relaxation time $t_{\rm off}$ is given by expression (31), as was in the case of linear interaction between the easy axis and the electric field.

5. Conclusions

The reorientation of the director in an NLC cell from one planar state into another one under the influence of a dc electric field has been studied. The electric field is assumed to be directed along the cell surface and perpendicularly to the initial uniform director orientation. The gliding of the axis of easy director orientation in the plane of either of the polymeric substrates of the cell due to the interaction of this axis with the electric field is taken into consideration. The contribution of this interaction to the NLC surface free energy density is considered to be linear or quadratic in the electric field strength, depending on whether the dipole moments of elastic segments of molecules of the polymer substrate material are permanent or induced by the electric field.

It is found that the time behaviors of the director and the moving easy axis are governed by the electric field strength E, parameter α of the interaction between the easy axis and the electric field, energy ε of NLC anchoring at the substrate with the moving easy axis, and coefficients of NLC bulk viscosity and easy axis viscosity. Analytical expressions for the time dependences of the director and easy axis deviation angles after the electric field switching-on/off are obtained by solving the linearized variational equations with corresponding boundary conditions.

If the interaction of the easy axis with the electric field is linear in the field strength E, the orientational instability of NLC has no threshold. In particular, from the time moment, when the electric field with the strength $E \leqslant 0.5E_{\rm th}^{\infty}$ ($E_{\rm th}^{\infty}$ is the Fréedericksz transition threshold provided an infinitely strong NLC anchoring with the cell substrates and the absence of easy axis gliding) is switched-on and until the system transits into the stationary state, the easy axis deviates more rapidly and draws the director after itself. In this case, the maximum deformations of the director field are reached at the surface with the moving easy axis. On the contrary, if the system transits into the stationary state in the electric field with the strength $E > 0.5E_{\rm th}^{\infty}$, the director deviates, on the whole, more rapidly and draws the easy axis after itself. The maximum deformations of the director field are reached in the NLC bulk, being shifted toward the surface with the moving easy axis, as the system approaches a stationary state. The characteristic time of switching-on, $t_{\rm on}$, turns out independent of the NLC bulk viscosity and the interaction parameter α, but it diminishes for larger values of the anchoring energy ε.

If the interaction between the easy axis and the electric field is quadratic in the field strength E, the orientational instability of NLC has a threshold character. The orientational instability threshold becomes lower with increase in the interaction parameter α and decrease in the anchoring energy ε. In the limiting case where the interaction of the easy axis with the electric field is absent, the threshold approaches a value of $0.5E_{\rm th}^{\infty}$ obtained, when the NLC anchoring with the moving easy axis is absent. If the field strength E exceeds the threshold value, the time dependences of the deviation angles of the director and the easy axis are qualitatively similar to the corresponding dependences obtained in the case of linear interaction between the easy axis and the electric field. The switching-on time $t_{\rm on}$ decreases with a reduction of the anchoring energy ε and the growth of the interaction parameter α. Irrespective of the character of the interaction between the easy axis and the electric field, a reduction of the coefficient of easy axis viscosity results in a reduction of the switching-on time $t_{\rm on}$.

In the stationary state, the largest deformations of the director field are concentrated at the surface with the moving easy axis, irrespective of the interaction between this axis and the electric field. The increase in the anchoring energy ε and the interaction parameter α results in the growth of director deformations.

If the system is in a stationary state and the electric field is switched-off, the director returns more quickly to the initial homogeneous state and rotates the easy axis after itself irrespective of the interaction between the easy axis and the electric field. In this case, the largest deviation of the director takes place at the surface with the moving easy axis. Note that, during relaxation, an almost linear profile of the director deviation angle across the cell thickness is established. In both cases of interaction between the easy axis and the electric field, the characteristic time of relaxation, $t_{\rm off}$, is the same. With increase in the anchoring energy and areduction of the coefficients of NLC bulk viscosity and easy axis viscosity, the time $t_{\rm off}$ decreases.

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ОРІЄНТАЦІЙНА НЕСТІЙКІСТЬ ДИРЕКТОРА В ПЛАНАРНІЙ НЕМАТИЧНІЙ КОМІРЦІ З КЕРОВАНИМИ МЕЖОВИМИ УМОВАМИ В ЕЛЕКТРИЧНОМУ ПОЛІ

Р е з ю м е

Досліджується переорієнтація директора із одного планарного стану в інший планарний стан у комірці нематичного рідкого кристала під дією постійного електричного поля. Враховується проковзування осі легкого орієнтування директора в площині полімерної підкладки комірки, зумовлене взаємодією цієї осі з електричним полем. Внесок такої взаємодії в густину поверхневої вільної енергії комірки вважається лінійним по напруженості електричного поля, якщо еластичні частини молекул полімерної підкладки мають власні дипольні моменти. Якщо ж дипольні моменти еластичних частин молекул полімеру наводяться електричним полем, то аналогічний внесок вважається квадратичним по напруженості електричного поля. Показано, що в залежності від характеру взаємодії легкої осі з електричним полем орієнтаційна нестійкість директора може бути як пороговою, так і безпороговою. В обох випадках досліджена часова поведінка директора і легкої осі з моменту ввімкнення електричного поля з наступним виходом системи в стаціонарний стан та закінчуючи поверненням системи в вихідний однорідний стан після вимкнення поля.