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FINITE LARMOR RADIUS EFFECTS ON A TEST-PARTICLE DIFFUSION

Particle diffusion in a static random electric field across a uniform magnetic field is considered. Earlier, we have proposed the closure for the statistical equation that describes particle diffusion in the drift approximation with account for the effect of particle trapping. Here, a generalization of our approach for a finite Larmor radius is given. It is shown that the statistical characteristics of a particle ensemble found as solutions of the analytical model are consistent with the results of direct numerical simulations within a wide range of Larmor radii.

Keywords: finite Larmor radius, diffusion of particles, random field, numerical simulation.

1. Introduction

Nowadays, for understanding a plasma behavior in laboratory devices and space, numerical experiments are widely used. Along with this to interpret experimental observations and results of numerical simulation, as well to describe basic mechanisms underlying complex processes, analytical models might be helpful.

An important role in the evolution of plasma systems is played by transport processes. In nonequilibrium plasma with developed collective motion, a particle transport is anomalous in the sense that it is caused not by pair particle collisions, but their interaction with intense waves. Thus, one of the objectives of an analytical description of transport processes is to find the statistical characteristics of a particle ensemble from the known statistical characteristics of random electric fields that can be measured in experiments.

Most plasma systems are penetrated by a magnetic field, which can be taken as constant and uniform on the temporal and spatial scales of consideration. Whether the correlation time of random electric fields is not small in comparison with the time of a particle guiding center drift at the distances of characteristic spatial inhomogeneities of the electric field, a significant role in the particle transport across a magnetic field will be played by the effect of particle trapping by the electric field. This effect appears most strongly in a frozen electric field with an infinite correlation time. Accounting for the trapping effect presents difficulties for the statistical description. Thus, a random field constant in time is a good test for the statistical theory.

Consequently, a model of particle diffusion in steady fields might serve as a basis for the transition to more general situations, i.e., to fields varying in time. Moreover, a method of calculation of the particle transport in given random fields, if is found, might be incorporated in a self-consistent description.

The evolution of particle ensembles in external fields is described by the nonlinear integro-differential equation for distribution function [1] derived from the first principles. Unfortunately, there is no regular methods of its solution. For this reason, as in most statistical theories, a certain assumption about a statistical equation closure should be used. The example of closure in a kinetic theory of plasma is an expression of correlation functions of higher order through lower ones. A closure in hydrodynamics gives higher moments of a distribution function in terms of lower ones, for example, a pressure through a density and a temperature.

The widely known closure that is used for the calculation of a Lagrangian correlation function from the Eulerian one was proposed by Corrsin [2,3]. However, it can be applied only to fields with a small correlation time and is not valid otherwise [4] and, even more, for the particle trapping. To account for the trapping effect the percolation approach [5], method of continuous time random walks (CTRW) [6], diffusion equations with fractal derivatives [7], and decorrelation trajectory method [8, 9] were used. The percolation approach gives asymptotic diffusion coefficients, but does not consider the temporal evolution of the statistical characteristics of a particle ensemble. CTRW and the diffusion equation with fractal derivatives have a phenomenological component associated with free parameters such as the probability of particle jump and fractional derivative exponents. The approach nearest to our one is the decorrelation trajectory method, where a particular closure along with the concept of subensembles were used.

Earlier, we have proposed the analytical model of 2d particle diffusion in frozen random electric and uniform magnetic fields in the drift approximation [14, 15]. Its solutions were shown to be consistent with the results of numerical simulation. The comparison of our approach and the decorrelation trajectory method [9] showed [16] that the former is technically simpler and has no unphysical dependence on a number of subensembles.

The diffusion coefficient calculated in the drift approximation tends asymptotically to zero. A small but finite radius of gyration changes a qualitative behavior of the diffusion coefficient so that it remains finite all the time. However, the effect of a finite Larmor radius is more pronounced for fast particles, when it becomes of an order of the size of spatial inhomogeneities of a random field.

In this paper, our approach is generalized to account for the effects of a finite Larmor radius. The main condition is a small displacement of the guiding center in comparison with the size of a spatial field inhomogeneity during a particle gyration period. We do not assume that the Larmor radius is small on the same spatial scale. Obviously, this requires to consider the effect in all orders in the Larmor radius. The method of calculation was formulated with a help of the analysis [18] of various approaches used in the decorrelation trajectory method to account for the effects of a finite Larmor radius [10, 11] and [12, 13].

The paper is organized as follows. The equations of particle motion are given in Sec. 2, and the statistical approach is presented in Sec. 3. The results of the analytical model and direct numerical simulations are compared in Sec. 4 with conclusions in Sec. 5.

2. Equations of Particle Motion

We consider the 2d motion of test particles in a frozen random electrostatic field $\mathbf{E}(\mathbf{r})$ perpendicular to a constant magnetic field $\mathbf{B}$ oriented along $0z$. The particle coordinates $\mathbf{r} = \{x, y\}$ and velocities $\mathbf{v}$ are governed by the equations

$$\frac{d\mathbf{r}}{dt} = \mathbf{v}, \tag{1}$$

$$\frac{d\mathbf{v}}{dt} = \frac{e}{m}\left(\mathbf{E}(\mathbf{r}) + \frac{1}{c}[\mathbf{v} \times \mathbf{B}]\right). \tag{2}$$

The random electric field is taken as a superposition of N harmonics distributed according to the Gaussian with a width Δk and the amplitude of the potential ϕ_0:

$$\mathbf{E}(\mathbf{r}) = -\frac{\partial}{\partial \mathbf{r}}\phi(\mathbf{r}) = A\phi_0 \times$$
$$\times \sum_{s=1}^{N} \mathbf{k}_s \exp\left(-\frac{1}{2}\left(\frac{\mathbf{k}_s}{\Delta k}\right)^2\right) \sin(\mathbf{k}_s\mathbf{r} - \alpha_s). \tag{3}$$

The set of $N = N_\kappa \times N_\theta$ wave vectors is given by N_κ absolute values in the interval $(0, k_{\max})$ and N_θ directions

$$\mathbf{k}_s = k_l \mathbf{e}_m,\ k_l = l\ \delta k = l\ \frac{k_{\max}}{N_\kappa},\ l = 1, ..., N_\kappa;$$

$$\mathbf{e}_m = (\cos(\theta_m), \sin(\theta_m)),\ \theta_m = m\ \frac{2\pi}{N_\theta},\ m = 1, ..., N_\theta.$$

A particular realization of the random electric field $\mathbf{E}(\mathbf{r})$ is determined by a set of random phases $\{\alpha_i\}$, $i = 1, ..., N$. The normalization coefficient

$$A = \sqrt{\frac{4k_{\max}}{\pi^{1/2}\Delta k\ N_\kappa N_\theta}}$$

is chosen to meet the condition $\langle \mathbf{E}(\mathbf{r})\mathbf{E}(\mathbf{r})\rangle = 1$.

The equations of motion (1), (2) can be rewritten in terms of the Larmor radius $\mathbf{r}_\mathrm{L} = -[\mathbf{v} \times \mathbf{e}_B]/\Omega_B$, and the guiding center coordinate $\mathbf{r}_d = \mathbf{r} - \mathbf{r}_\mathrm{L}$:

$$\frac{d\mathbf{r}_d}{dt} = \frac{1}{\Omega_B}\frac{e}{m}\left[\mathbf{E}(\mathbf{r}_d + \mathbf{r}_\mathrm{L}) \times \mathbf{e}_B\right], \tag{4}$$

$$\frac{d\mathbf{r}_\mathrm{L}}{dt} = -\frac{1}{\Omega_B}\frac{e}{m}\left[\mathbf{E}(\mathbf{r}_d + \mathbf{r}_\mathrm{L}) \times \mathbf{e}_B\right] + \Omega_B\left[\mathbf{r}_\mathrm{L} \times \mathbf{e}_B\right], \tag{5}$$

where $\mathbf{e}_B = \mathbf{B}/B$ is a unit vector along the magnetic field, and $\Omega_B = eB/mc$ is the Larmor frequency. For convenience, we introduce the dimensionless spatial $\{\boldsymbol{\chi}, \boldsymbol{\chi}_d, \boldsymbol{\rho}\} = \{\mathbf{r}, \mathbf{r}_d, \mathbf{r}_\mathrm{L}\} \times \Delta k/(2\pi)$ and time $\tau = t\sigma_0\Omega_B/(2\pi)$ variables. The normalization of time with $\sigma_0 = e\phi_0\Delta k^2/(m\Omega_B^2)$ makes particle orbits in the drift approximation independent of a field amplitude.

According to Eq. (3), the dimensionless potential (a stream function) $\sigma(\boldsymbol{\chi})$ takes the form

$$\sigma(\boldsymbol{\chi}) = \sqrt{\frac{\kappa_{\max}}{\pi^{3/2} N_\kappa N_\theta}} \times$$
$$\times \sum_{i=1}^{N} \exp\left(-\frac{1}{2}\kappa_i^2\right) \cos\left(2\pi \boldsymbol{\kappa}_i \boldsymbol{\chi} - \alpha_i\right), \qquad (6)$$

where $\boldsymbol{\kappa} = \boldsymbol{k}/\Delta k$ are normalized wave vectors.

Then the dimensionless coordinate of the particle guiding center $\boldsymbol{\chi}_d$ and the Larmor radius $\boldsymbol{\rho}$ are governed by the equations

$$\frac{d\chi_{di}}{d\tau} = -\epsilon_{ik} \frac{\partial}{\partial \chi_{dk}} \sigma(\boldsymbol{\chi}_d + \boldsymbol{\rho}), \qquad (7)$$

$$\frac{d\rho_i}{d\tau} = \epsilon_{ik} \left(\frac{\partial}{\partial \boldsymbol{\chi}_{dk}} \sigma(\boldsymbol{\chi}_d + \boldsymbol{\rho}) + \frac{2\pi}{\sigma_0}\rho_k\right), \qquad (8)$$

where ϵ_{ik} denotes the antisymmetric second-order tensor $\epsilon_{xy} = -\epsilon_{yx} = 1$.

The numerical simulation is performed by the integration of Eqs. (7), (8) with Eq. (6) and gives us exact particle trajectories $\boldsymbol{\chi}(\tau) = \boldsymbol{\chi}_d(\tau) + \boldsymbol{\rho}(\tau)$. Statistical characteristics of an ensemble of particles is obtained from these data by averaging over N_r realizations of random fields. Thus, the particle mean displacement is obtained from the simulation as follows:

$$\langle \chi_i \rangle^{(\mathrm{NS})}(\tau) = \frac{1}{N} \sum_{r=0}^{N_r} \chi_{i,r}(\tau), \quad i = x, y.$$

The dispersion measure is a variance

$$\langle \Delta\chi_i^2 \rangle^{(\mathrm{NS})}(\tau) = \frac{1}{N} \sum_{r=0}^{N_r} \left(\chi_{i,r}(\tau) - \langle \chi_i \rangle(\tau)\right)^2.$$

For an isotropic random field, the mean displacement is zero, and the dispersion is measured by the mean-square displacement $\langle \Delta\chi_i^2 \rangle^{(\mathrm{NS})}(\tau) = \langle \chi_i^2 \rangle^{(\mathrm{NS})}(\tau)$.

The mean displacement of guiding centers and their dispersion are calculated similarly. Thus, the dispersion of guiding center coordinates for the isotropic random field $\Delta_i^{(\mathrm{NS})}$ is found as

$$\Delta_i^{(\mathrm{NS})}(\tau) = \frac{1}{N} \sum_{r=0}^{N_r} \left(\chi_{d\,i,r}(\tau)\right)^2. \qquad (9)$$

The Lagrangian correlation functions can be found in a direct numerical simulation as well. For example, the Lagrangian correlation function of drift velocities is calculated as follows:

$$C^{\mathrm{L}}_{\upsilon_i \upsilon_j}{}^{(\mathrm{NS})}(\tau) = \frac{1}{N} \sum_{r=0}^{N_r} \left(\upsilon_{i,r}(\boldsymbol{\chi}(\tau))\ \upsilon_{j,r}(0)\right), \qquad (10)$$

where the drift velocity $\boldsymbol{\upsilon}$ in an exact particle position, according to Eq. (7), is given as

$$\upsilon_i = \epsilon_{ik} \frac{\partial}{\partial \chi_k} \sigma(\boldsymbol{\chi}). \qquad (11)$$

Dispersion (9) and the Lagrangian correlation function (10) calculated from the simulation data independently satisfy the equations

$$\frac{1}{2}\Delta_i^{(\mathrm{NS})}(\tau) = \int_0^\tau d\tau' D_i^{(\mathrm{NS})}(\tau') =$$
$$= \int_0^\tau \tau' \int_0^{\tau'} \tau'' C^{\mathrm{L}}_{\upsilon_i \upsilon_i}{}^{(\mathrm{NS})}(\tau''), \qquad (12)$$

where $D_i^{(\mathrm{NS})}(\tau)$ is a time-dependent diffusion coefficient. Statistical characteristics found in the direct numerical simulation will be compared with calculations made on a basis of the analytical model considered in the next section.

3. Analytical Approach

The aim of our analytical approach is to find a temporal evolution of the statistical characteristics of an ensemble of particles in a random field. The complete information is given by the particle distribution function (or a transition probability), which is a solution of the nonlocal nonlinear equation. As far as there are no regular methods to find an exact solution, when the effect of particle trapping is not negligible, we limit ourselves to the finding of the particle dispersion that is the second moment of the distribution function.

The particle dispersion $\Delta_i(\tau)$ is given in terms of the diffusion coefficient $D_i(\tilde{\tau})$ as follows:

$$\Delta_i(\tau) = 2 \int_0^\tau d\tilde{\tau} D_i(\tilde{\tau}), \quad i = x, y, \qquad (13)$$

and the Taylor relation gives the diffusion coefficient in terms of the Lagrangian correlation function of velocity components

$$D_i(\tau) = \int_0^\tau d\tilde{\tau} C^{\mathrm{L}}_{\upsilon_i \upsilon_i}(\tilde{\tau}). \qquad (14)$$

These two equations are equivalent to Eqs. (12) that were verified in numerical simulations.

The Lagrangian velocity correlation function

$$C^{\mathrm{L}}_{\upsilon_i \upsilon_i}(\tau) = \langle \upsilon_i(\boldsymbol{\chi}(\tau))\ \upsilon_i(\boldsymbol{\chi}(0))\rangle \tag{15}$$

is calculated along partial trajectories by averaging over an ensemble of realizations of the random field denoted by angular brackets $\langle ... \rangle$. It is unknown, and the key problem is to deduce it from the Eulerian correlation function. The Eulerian velocity correlation function is defined in a laboratory frame as

$$C^{\mathrm{E}}_{\upsilon_i \upsilon_i}(\boldsymbol{\chi}) = \langle \upsilon_i(\boldsymbol{\chi})\ \upsilon_i(\mathbf{0})\rangle \tag{16}$$

and is supposed to be given. It is measured either in experiments or can be calculated for given fields, i.e. ones defined by Eq. (6). The Eulerian potential correlation function

$$C^{\mathrm{E}}_{\sigma\sigma}(\boldsymbol{\chi}) = \langle \sigma(\boldsymbol{\chi})\ \sigma(\mathbf{0})\rangle \tag{17}$$

is related to the Eulerian velocity correlation function as follows:

$$C^{\mathrm{E}}_{\upsilon_i \upsilon_i}(\boldsymbol{\chi}) = -\frac{\partial^2}{\partial \chi_i^2}\ C^{\mathrm{E}}_{\sigma\sigma}(\boldsymbol{\chi}). \tag{18}$$

For statistically isotropic random fields, the Eulerian correlation function of potentials depends on a distance between two points

$$C^{\mathrm{E}}_{\sigma\sigma}(\chi) = \langle \sigma(\boldsymbol{\chi})\ \sigma(\mathbf{0})\rangle. \tag{19}$$

If we introduce $C^{\mathrm{E}}_{\upsilon\upsilon}$ as the sum of components

$$C^{\mathrm{E}}_{\upsilon\upsilon}(\chi) = C^{\mathrm{E}}_{\upsilon_x \upsilon_x}(\boldsymbol{\chi}) + C^{\mathrm{E}}_{\upsilon_y \upsilon_y}(\boldsymbol{\chi}), \tag{20}$$

it will be dependent on the absolute value of $|\boldsymbol{\chi}|$ as well.

The exact relation between Eulerian and Lagrangian velocity correlation functions could be established with a help of the transition probability. However, as was noted, it is unknown. So, we should make assumption about such relation, which is equivalent to a particular closure of statistical equations. This is the main assumption in the formulation of our analytical model. Further, we shall avoid to use free parameters as well.

For zero Larmor radius, when particle motion is considered in the drift approximation, the following closure was proposed [14]:

$$C^{\mathrm{L}}_{\upsilon\upsilon}(\tau) = C^{\mathrm{E}}_{\upsilon\upsilon}(\Delta^{1/2}(\tau)), \tag{21}$$

where $\Delta = \Delta_x + \Delta_y$. The development of this model with account for subensembles was given in [15], and the better agreement with a numerical simulation was achieved. In work [16], our approach was compared with the decorrelation trajectory method [9], and some advantages of the former were shown.

Here, a generalization of our analytical model to account for the finite Larmor radius effects is proposed. We assume that the displacement of the particle guiding center during the cyclotron period in comparison with the size of spatial field inhomogeneities is small and average the random potential (6) over the particle gyration angles φ_c:

$$\sigma(\boldsymbol{\chi}_d, \rho) = \frac{1}{2\pi}\int\limits_0^{2\pi} d\varphi_c\ \sigma(\boldsymbol{\chi}_d + \boldsymbol{\rho}(\varphi_c)) = \\ = \int d\boldsymbol{\kappa}\ \sigma(\boldsymbol{\kappa}) \exp(i\boldsymbol{\kappa}\boldsymbol{\chi}_d) J_0\left(\kappa\rho\right). \tag{22}$$

The corresponding gyroaveraged Eulerian potential correlation function is of the form

$$C^{\mathrm{E}}_{\sigma\sigma}(\chi_d, \rho) = \langle \sigma(\boldsymbol{\chi}_d + \boldsymbol{\chi}_{d1}, \rho)\ \sigma(\boldsymbol{\chi}_{d1}, \rho)\rangle = \\ = \int d\boldsymbol{\kappa}\ C^{\mathrm{E}}_{\sigma\sigma}(\boldsymbol{\kappa}) \exp(i\boldsymbol{\kappa}\boldsymbol{\chi}_d) J_0^2(\kappa\rho). \tag{23}$$

Such gyroaveraging of the correlation function in the application to the method of decorrelated trajectories was considered in [12]. The averagings of fields over the ensemble and over the angles of gyration do not commute. A similar expression, but with the Bessel function in the first degree $J_0(\kappa\rho)$, is obtained when the averaging over an ensemble is performed before the gyroaveraging. This method was used earlier in the paper [11] for the decorrelation trajectory method. Both orders of averaging in the application to our closure were compared in the work [16], and it was found that the former gives a better agreement with numerical simulations. Our choice is consistent with conclusions in [12]. Thus we will use Eq. (23) for the gyroaveraged Eulerian correlation function.

The generalization of closure (21) for a finite Larmor radius is the following:

$$C^{\mathrm{L}}_{\upsilon\upsilon}(\tau) = C^{\mathrm{E}}_{\upsilon\upsilon}\left(\Delta^{1/2}(\tau), \rho\right). \tag{24}$$

Then Eqs. (13), (14), and (24) yield the final equation for a mean square displacement:

$$\frac{d^2}{d\tau^2}\Delta(\tau) = C^{\mathrm{E}}_{\upsilon\upsilon}\left(\Delta^{1/2}(\tau), \rho\right), \tag{25}$$

where $C^{\mathrm{E}}_{\upsilon\upsilon}(\chi)$ is a known function.

The solution of this equation gives the temporal evolution of the mean-square displacement, the running diffusion coefficient, and the Lagrangian correlation function for a given finite Larmor radius. The solution of the equation averaged over the angles of gyration will be compared in the next section with the results of numerical simulation performed on a basis of the exact nonaveraged equations of motion (7), (8).

4. Comparison with Results of Numerical Simulation

Equations (7) and (8) have been solved numerically using the Runge–Kutta method of the fifth order. The random potential (6) was taken as a superposition of $N = 1440$ harmonics ($N_\kappa = 20$, $N_\theta = 72$) with a unique set of random phases for each realization. The maximal absolute value of the dimensionless wave vector in the numerical simulation was taken as $\kappa_{\max} = 2$ because of the decay of partial intensities distributed according to Gaussian (6). These parameters were chosen to give a smooth curve for the Eulerian correlation function of the potential (stream function) $C^{\rm E}_{\sigma\sigma}$. The number of realizations was taken as $N_r = 10^4$.

For the considered random potential (6), the Eulerian correlation function in the continuous limit takes the form

$$C^{\rm E}_{\sigma\sigma}(\boldsymbol{\chi}) = \frac{1}{4\pi^2}\exp\left(-\frac{\pi^2\chi^2}{2}\right) I_0\left(\frac{\pi^2\chi^2}{2}\right). \qquad (26)$$

The Fourier transform of this correlation function is

$$C^{\rm E}_{\sigma\sigma}(\boldsymbol{\kappa}) = \frac{1}{4\pi^{7/2}\kappa}\exp\left(-\frac{\kappa^2}{8\pi^2}\right). \qquad (27)$$

For the correlation function of drift velocity components, we obtain

$$C^{\rm E}_{\upsilon_i\upsilon_i}(\boldsymbol{\kappa}) = \frac{\kappa_i^2}{4\pi^{7/2}\kappa}\exp\left(-\frac{\kappa^2}{8\pi^2}\right). \qquad (28)$$

The substitution of the Fourier transform (28) into Eq. (25) gives the equation for the mean-square displacement with account for a finite Larmor radius in the gyroaveraged random potential (22)

$$\frac{d^2}{d\tau^2}\Delta(\tau) = \int d\kappa\ \frac{\kappa^2}{4\pi^{7/2}}\exp\left(-\frac{\kappa^2}{4\pi^2}\right)\times$$
$$\times J_0(\kappa\Delta^{1/2})J_0^2(\kappa\rho). \qquad (29)$$

The integral on the right-hand side of Eq. (29) was calculated numerically and is valid in a wide range of Larmor radii. The numerical solutions of this equation gives the temporal evolution of the mean-square displacement of particle guiding centers for fixed values of the Larmor radii that correspond to the initial values in the numerical simulation.

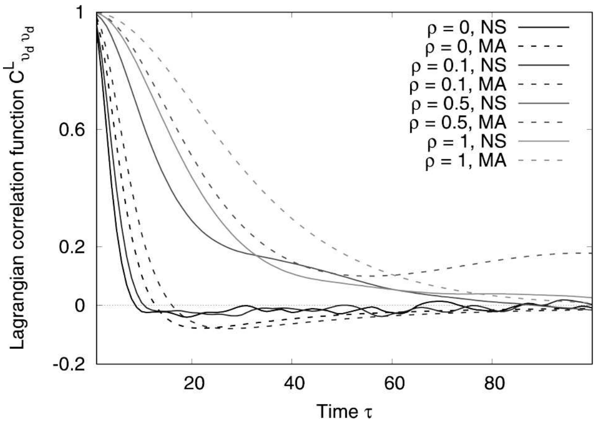

Fig. 1. Lagrangian correlation function for $\rho = 0, 0.1, 0.5, 1.$ obtained in the numerical simulation (NS, $N_r = 10^4$) and by the moment approximation (MA)

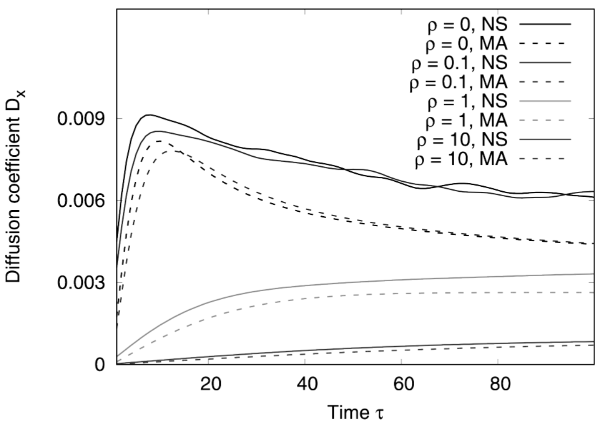

Fig. 2. Diffusion coefficient for Larmor radii $\rho = 0, 0.1, 1, 10$ obtained in the numerical simulation (NS) and as a solution of the analytical model (MA)

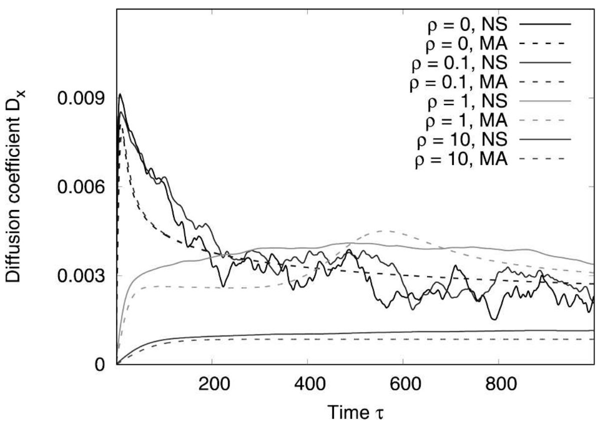

Fig. 3. The same as in Fig. 2, but for longer times

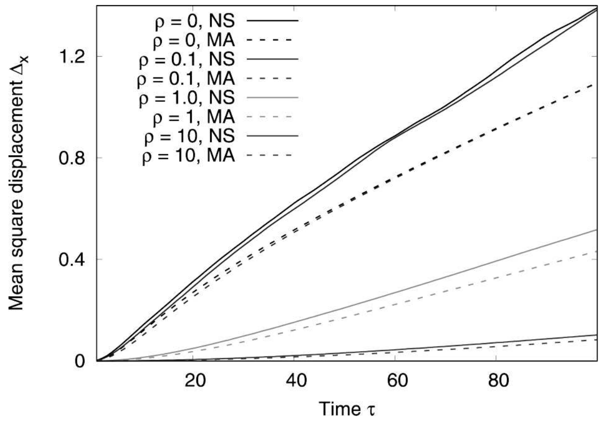

Fig. 4. Mean-square displacement for Larmor radii $\rho = 0$, 0.1, 1, 10 obtained in the numerical simulation (NS) and as a solution of the analytical model (MA)

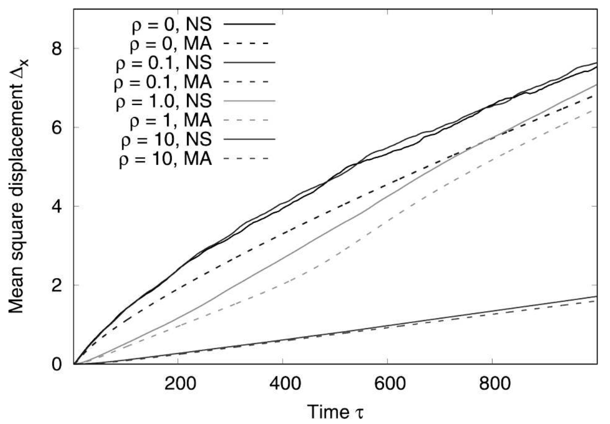

Fig. 5. The same as in Fig. 4, but for longer times

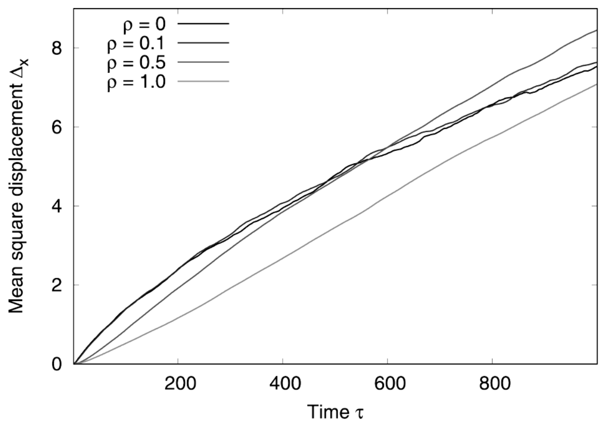

Fig. 6. Intersection of dispersion curves. Numerical simulation

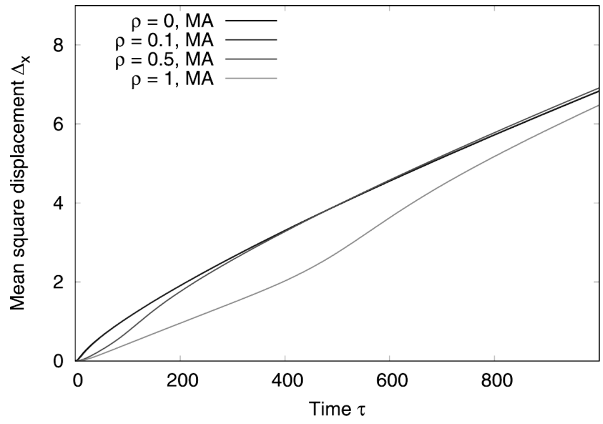

Fig. 7. Intersection of dispersion curves. Analytical model

These solutions are compared with the results of direct numerical simulations of the exact particle motion governed by Eqs. (7) and (8). In simulations, the fields were not gyroaveraged, and the Larmor radius was not fixed. In work [18], we observed that the Larmor radius dispersion is saturated, and this gives a reason to assume it as a constant in the analytical model. Other statistical characteristics of particle ensembles such as the diffusion coefficient $D(\tau)$ and the Lagrangian correlation function $C^{\mathrm{L}}_{\upsilon_i\upsilon_i}(\tau)$ are related to the mean-square displacement by Eqs. (13) and (14) and are compared with the results of simulations as well.

In Fig. 1, the Lagrangian correlation functions found from the analytical model and in simulations are shown. In the drift approximation, $\rho = 0$, as well as for a small Larmor radius $\rho = 0.1$, the particle trapping by a field is apparently reflected by negative values of the correlation function. At larger ρ, it becomes not so strong and evident.

The running diffusion coefficients are shown in Fig. 2 for $\tau = 100$ and in Fig. 3 for a longer temporal interval of simulation, $\tau = 1000$, with higher fluctuations.

In the drift approximation, particles move along counter lines of the stream function. In this sense, their movement for each particular field realization is regular. Herewith, the diffusion coefficient asymptotically tends to zero. This reflects, together with the well-defined maximum at the early stage, a strong particle trapping.

For $\rho \neq 0$, the particles do not move on closed orbits. This causes a weakening of particle trapping and leads to asymptotically finite diffusion coefficients.

The particle mean-square displacement corresponding to these running diffusion coefficients is shown in Figs. 4 and 5. Figures 1–5 show an agreement between solutions of the proposed analytical model and results of direct numerical simulations.

It should be noted that, at the early evolution stage, the particle spread proceeds faster for small Larmor radii, whereas the asymptotic diffusion coefficient grows with the Larmor radius. This leads to the intersection of curves of the mean-square displacement shown for the numerical simulation in Fig. 6 and for a solution of the analytical model in Fig. 7. The analytical model implies that the intersection of curves for the initial gyroradii $\rho(0) = 0$ and $\rho(0) = 0.1$ occurs at the instant $\tau_{i0} \approx 100$; for $\rho(0) = 0$ and $\rho(0) = 0.5$ at $\tau_{i1} \approx 450$; for $\rho(0) = 0$ and $\rho(0) = 1.0$ at $\tau_{i2} \approx 2200$. Earlier, this effect was found in [10] by the decorrelation trajectory method. In random fields with finite correlation times, it may cause a nonmonotonous dependence of the diffusion coefficient on the Larmor radius.

5. Conclusions

The diffusion of a test particle in a random electric field across a constant magnetic field has been considered analytically and numerically. The attention is paid to the particle trapping effect, and, for this reason, a frozen field is considered. Our analytical approach that was earlier developed for the drift motion is generalized to account for finite Larmor radius effects. This effect was calculated in all orders of the Larmor radius, so the model can be used in a wide range of its variation. The consistency of solutions of the analytical model with the results of a direct numerical simulation is shown.

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ВПЛИВ СКІНЧЕНОГО РАДІУСА
ЛАРМОРА НА ДИФУЗІЮ ПРОБНИХ ЧАСТИНОК

Р е з ю м е

Розглянуто дифузію частинок у статичному випадковому електричному полі поперек однорідного магнітного поля. Раніше ми запропонували спосіб замикання для статистичного рівняння, яке описує дифузію частинок у дрейфовому наближенні з урахуванням ефекту захоплення частинок. В цій роботі дано узагальнення нашого підходу з урахуванням скінченого ларморового радіуса. Показано, що статистичні характеристики, знайдені як розв'язки аналітичної моделі, узгоджуються з результатами прямого числового моделювання в широкому діапазоні ларморових радіусів.