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SEMIDISCRETE INTEGRABLE NONLINEAR SCHRÖDINGER SYSTEM WITH BACKGROUND-CONTROLLED INTERSITE RESONANT COUPLING. SHORT SUMMARY OF KEY PROPERTIES [1]

The most featured items characterizing the semidiscrete nonlinear Schrödinger system with background-controlled intersite resonant coupling are summarized. The system is shown to be integrable in the Lax sense that makes it possible to obtain its soliton solutions in the framework of a properly parametrized dressing procedure based on the Darboux transformation accompanied by the implicit form of Bäcklund transformation. In addition, the system integrability inspires an infinite hierarchy of local conservation laws, some of which were found explicitly in the framework of the generalized recursive approach. The system consists of two basic dynamic subsystems and one concomitant subsystem, and its dynamics is embedded into the Hamiltonian formulation accompanied by the highly nonstandard Poisson structure. The nonzero background level of concomitant fields mediates the appearance of an additional type of the intersite resonant coupling. As a consequence, it establishes the triangular-lattice-ribbon spatial arrangement of location sites for the basic field excitations. At tuning the main background parameter, we are able to switch system's dynamics between two essentially different regimes separated by the critical point. The physical implications of system's criticality become evident after a rather sophisticated procedure of canonization of basic field variables. There are two variants to standardize the system equal in their rights. Each variant is realizable in the form of two nonequivalent canonical subsystems. The broken symmetry between canonical subsystems gives rise to the crossover effect in the nature of excited states. Thus, in the under-critical region, the system supports the bright excitations in both subsystems; while, in the over-critical region, one of the subsystems converts into the subsystem of dark excitations.

K e y w o r d s: nonlinear lattice, integrable system, soliton, conservation laws, symmetry breaking, canonical field variables.

1. Introduction

The semidiscrete integrable nonlinear Schrödinger systems on one-dimensional or quasi-one-dimensional lattices [1–5, 13, 26, 29, 30, 43] play a significant role in modeling a wide variety of phenomena from various branches of physics, inasmuch as they might give us a clue what type of nonlinear excitations could be expected, when considering real physical situations. It is sufficient to mention that the concept of nonlinear excitations related to one or another model of nonlinear Schrödinger type is applicable to the investigation of nonlinear effects in discrete electric transmission lines [19], to the modeling of soliton-mediated energy and charge transport in macromolecules [6, 9, 10, 23], as well as to the theoretical treatment of experimentally observed light patterns in the cross-sections of coupled optical fibers [7, 12].

In this respect, the semidiscrete integrable nonlinear Schrödinger system with background-controlled intersite resonant coupling [32, 35, 36] is able to find considerable physical applications as a multi-component system with two types of free coupling parameters giving rise to rather unusual properties

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[37, 38] and as a system, whose underlying lattice structure is closely related to that of (1,1) boron nanotube [18] belonging to the new class of low-dimensional synthetic materials known as nanoribbons [14, 16, 20]. The most representative results concerning this system have recently been published in our review articles [41, 42]. Hence, the present short communication should be considered as the concise guidance to the above reviews [41, 42], where the interested reader can find all mathematical details and nontrivial extrications associated with the system under study, as well as the numerous dynamical implications caused by the system criticality against the main background parameter.

2. Dynamic Equations of the System

Having been written in terms of two pairs of basic field amplitudes $q_+(n)$, $r_+(n)$ and $q_-(n)$, $r_-(n)$ accompanied by one pair of concomitant field amplitudes $\mu(n)$, $\nu(n)$, the dynamics of the integrable nonlinear Schrödinger system on a triangular-lattice ribbon is governed by the following set of equations [32, 35–42]:

$$\begin{aligned}&+\mathrm{i}\dot{q}_+(n)+\beta q_-(n-1)[1+q_+(n)r_+(n)]+\\&+\alpha q_+(n+1)[q_+(n)r_-(n)-\nu(n)]+\\&+\alpha\,[q_-(n)+q_+(n)\mu(n)]=0,\end{aligned}\tag{2.1}$$

$$\begin{aligned}&-\mathrm{i}\dot{r}_+(n)+\alpha r_-(n-1)[1+r_+(n)q_+(n)]+\\&+\beta r_+(n+1)[r_+(n)q_-(n)-\mu(n)]+\\&+\beta\,[r_-(n)+r_+(n)\nu(n)]=0,\end{aligned}\tag{2.2}$$

$$\begin{aligned}&+\mathrm{i}\dot{q}_-(n)+\alpha q_+(n+1)[1+q_-(n)r_-(n)]+\\&+\beta q_-(n-1)[q_-(n)r_+(n)-\mu(n)]+\\&+\beta\,[q_+(n)+q_-(n)\nu(n)]=0,\end{aligned}\tag{2.3}$$

$$\begin{aligned}&-\mathrm{i}\dot{r}_-(n)+\beta r_+(n+1)[1+r_-(n)q_-(n)]+\\&+\alpha r_-(n-1)[r_-(n)q_+(n)-\nu(n)]+\\&+\alpha\,[r_+(n)+r_-(n)\mu(n)]=0,\end{aligned}\tag{2.4}$$

$$\begin{aligned}&+\mathrm{i}\dot{\mu}(n)+\alpha q_+(n+1)[r_+(n)+r_-(n)\mu(n)]+\\&+\beta\,[q_+(n)r_+(n)-q_-(n)r_-(n)]-\\&-\alpha r_-(n-1)[q_-(n)+q_+(n)\mu(n)]=0,\end{aligned}\tag{2.5}$$

$$\begin{aligned}&-\mathrm{i}\dot{\nu}(n)+\beta r_+(n+1)[q_+(n)+q_-(n)\nu(n)]+\\&+\alpha\,[r_+(n)q_+(n)-r_-(n)q_-(n)]-\\&-\beta q_-(n-1)[r_-(n)+r_+(n)\nu(n)]=0,\end{aligned}\tag{2.6}$$

where the amplitudes within each pair are related by the symmetry of complex conjugation $r_+(n)=q_+^*(n)$, $r_-(n)=q_-^*(n)$, $\nu(n)=\mu^*(n)$, and the overdot denotes the differentiation with respect to the time variable τ. The coupling parameters α and β can be taken as arbitrary complex-valued functions of the time restricted by the only property of complex conjugation $\beta^*=\alpha$. The chosen symmetries of field amplitudes and coupling parameters ensure the attractive type of nonlinearities of the system. As for the boundary conditions, we assume the basic field amplitudes to be rapidly vanishing at both spatial infinities $|n|\to\infty$. We adopt the concomitant field amplitudes to be supported by an arbitrarily fixed spatially uniform background $\nu=\lim_{|n|\to\infty}\nu(n)$ and $\mu=\lim_{|n|\to\infty}\mu(n)$. In the general case (*viz* for nonzero values of the limiting quantities μ and ν) the last two conditions $\nu=\lim_{|n|\to\infty}\nu(n)$ and $\mu=\lim_{|n|\to\infty}\mu(n)$ are suitable for treating the suggested semidiscrete nonlinear system (2.1)–(2.6) as a system given on the ribbon of a triangular lattice. In so doing, the quantities μ and ν acquire the meaning of additional (background-controlled) coupling parameters.

It can be shown [35, 36, 41, 42] that the local densities

$$\rho_-(n)=\ln[\mu(n)-q_-(n)r_+(n)],\tag{2.7}$$

$$\rho_+(n)=\ln[\nu(n)-q_+(n)r_-(n)],\tag{2.8}$$

$$\rho_0(n)=\ln[1+\mu(n)\nu(n)+q_+(n)r_+(n)+q_-(n)r_-(n)],\tag{2.9}$$

entering the three lowest local conservation laws of the system under study (2.1)–(2.6), are mutually dependent due to the property

$$\dot{\rho}_-(n)=\dot{\rho}_0(n)=\dot{\rho}_+(n).\tag{2.10}$$

On the one hand, the chain of equalities (2.10) forces the limiting values μ and ν of concomitant fields $\mu(n)$ and $\nu(n)$ to be time-independent. On the other hand, it should be treated as the differential version of two natural constraints

$$\frac{\mu(n)-q_-(n)r_+(n)}{1+\mu(n)\nu(n)+q_+(n)r_+(n)+q_-(n)r_-(n)}=\frac{\mu}{1+\mu\nu},\tag{2.11}$$

$$\frac{\nu(n)-q_+(n)r_-(n)}{1+\mu(n)\nu(n)+q_+(n)r_+(n)+q_-(n)r_-(n)}=\frac{\nu}{1+\mu\nu},\tag{2.12}$$

where the main background parameter $\mu\nu$ can acquire only the nonnegative values by virtue of its definition. The natural constraints (2.11) and (2.12) imply that the concomitant fields $\mu(n)$ and $\nu(n)$ are actually dependent on the basic fields $q_+(n)$, $r_+(n)$ and $q_-(n)$, $r_-(n)$. Namely, this observation prescribes us to call the fields $\mu(n)$, $\nu(n)$ as the concomitant ones.

The spatial arrangement of lattice sites and intersite resonant bonds related to the system of our interest (2.1)–(2.6) represents the two-leg ladder lattice that, according to modern nanoribbon terminology [11], can be referred to as the simplest triangular-lattice ribbon with linear edges. In order to justify the triangular-lattice-ribbon configuration of the underlying space lattice, it is sufficient to consider the linear part of our nonlinear system (2.1)–(2.6) and to observe that the quantities α, β and $-\alpha\nu$, $-\beta\mu$ should be understood, respectively, as the parameters of intersite linear and composite intersite linear couplings between the basic fields. These parameters responsible for the coherent (nondissipative) interaction between the basic fields are analogous to the parameters of the intersite resonant coupling typical of the theory of molecular (small-radius) excitons [8].

The quasi-one-dimensionality of a lattice relevant to the suggested system (2.1)–(2.6) appears to be a favorable property required in physical applications, inasmuch as already the quasi-one-dimensionality (in contrast to the pure one-dimensionality) of a real macromolecular lattice structure (in general, any lattice structure considered on a spatially microscopic scale) is known to be an indispensable factor for the structure thermodynamic stability [44]. On the other hand, namely due to the ladder-like geometry of its underlying lattice structure, system (2.1)–(2.6) (when dealing with the electrically charged excitations) acquires the property to experience the action of an external uniform magnetic field in terms of magnetic fluxes threading the elementary (triangular) plackets of a lattice ribbon and modeled by the phases of complex-valued coupling parameters treated as the Peierls phases [22, 30, 33].

Moreover, the coupling parameters are capable to incorporate the impact of an external linear potential on the dynamics of the primary system (2.1)–(2.6) via the appropriate modification of their time dependences [37].

The semidiscrete nonlinear system under study (2.1)–(2.6) permits the zero-curvature representation

$$\dot{L}(n|z) = A(n+1|z)L(n|z) - L(n|z)A(n|z) \qquad (2.13)$$

in terms of either 4×4 [32] or 2×2 [35, 36, 41, 42] square auxiliary matrices $L(n|z)$ and $A(n|z)$ referred to as the spectral and evolution matrices, respectively. Such a representability is known to define system's integrability in the Lax sense [21, 24].

3. Summary of Main Results

In the referred review articles [41, 42], we considered the most important properties of the semidiscrete integrable nonlinear Schrödinger system with background-controlled intersite resonant coupling in view of a significant role that the semidiscrete integrable models of the nonlinear Schrödinger-type play in the description of different phenomena from various branches of physics. The core of reviewed results [41,42] lies in eight original articles [32,34–40], though the impact of works [17, 27, 28] dealing with the standardization of the famous Ablowitz–Ladik semidiscrete nonlinear system seems to be indispensable.

As the matter of fact, author's activity in the standardization of the Ablowitz–Ladik system [27, 28] has been inspired by the rather critical attitude of Professor A.S. Davydov toward the nonstandard field amplitudes as those lacking the direct physical sense [33]. The similar problem of standardization concerns also the semidiscrete integrable nonlinear Schrödinger system with background-controlled intersite resonant coupling, however, on a more sophisticated level [38–42] as compared with the standardization problems appearing in simple semidiscrete integrable nonlinear systems [25, 27, 28, 31] characterized by the splittable structure (symplectic) matrices.

On the one hand, the splittability of a structure matrix assumes that each of the two diagonal blocks of a structure matrix is a zero matrix, while each of the two off-diagonal blocks of a structure matrix is a diagonal matrix. On the other hand, the splittability requires that the each element of the structure matrix to be given by the field variables belonging to one separate subsystem. There is no universal recipe how to overcome both of the above conditions simultaneously.

As for the system characterized by the splittable structure matrix, the problem of its canonization turns out to be more or less a trivial (though very cumbersome, sometimes) task. Thus, the main problem in the canonization of a semidiscrete integrable nonlinear Schrödinger system with background-controlled intersite resonant coupling was to find out such a nonlinear transformation to new field variables that the corresponding structure matrix will be splittable. To proceed with this program, we were obliged to make a number of logical steps.

First of all, we have obtained several lowest local densities from the infinite hierarchy and established the Poisson and Hamiltonian structures of the system in terms of the primary field variables. Then, relying upon the so-called natural constraints, we have revealed system's criticality against the background parameter. With regard for system's criticality, we have managed to introduce the set of intermediate field variables and then the two variants of primary-intermediate field variables. Each variant of the primary-intermediate field variables is characterized by the splittable structure matrix. Hence, the main obstacle for the canonization of the system have been surmounted. The last step in the standardization procedure has been made by the proper choices of nonlinear point transformations from each of the above-mentioned variants of primary-intermediate field variables to each of two relevant variants of canonical field variables.

In the course of the standardization, we have discovered that each particular incarnation of the standardized system consists of the weak and strong subsystems. The symmetry between the weak subsystem and the strong subsystem is essentially broken and can be restored only at the zero value of main background parameter. In the under-critical region of the main background parameter, both canonical subsystems are the subsystems of bright nonlinear excitations; while, in the over-critical region, the weak subsystem converts into the subsystem of dark nonlinear excitations. Here, the terms "bright nonlinear excitations" and "dark nonlinear excitations" should be understood by analogy with the terms "bright solitons" and "dark solitons" typical of the nonlinear optics [15]. At the very critical point, the weak subsystem turns out to be completely unexcitable. The crossover in the types of nonlinear excitations has been confirmed by the standardized minus-asymmetric and plus-asymmetric multicomponent one-soliton solutions both analytically and graphically with the formulas for the primary (unstandardized) soliton solution being taken into account.

The primary soliton solution itself has been obtained in the framework of the rather nontrivial Darboux dressing approach developed specially for this purpose. This approach is based on restoring the Darboux matrix relying upon its spectral properties and on involving the implicit form of the multicomponent Bäcklund transformation. The successive application of the Darboux dressing procedure to the generation of multisoliton solutions was outlined. However, the explicit realization of this program seems to be cumbersome due to the multicomponent character of system-governing equations.

One more important property of the semidiscrete integrable nonlinear Schrödinger system with background-controlled intersite resonant coupling is linked with the *a priori* arbitrary time dependences of the transverse coupling parameters capable to incorporate the effect of an external linear potential. As a consequence, the primary integrable nonlinear system with appropriately adjusted parametrical driving becomes isomorphic to the system modeling the Bloch oscillations of charged nonlinear carriers in the electrically biased ribbon of a triangular lattice. The justification of this statement can be found in our recent paper [37].

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НАПІВДИСКРЕТНА НЕЛІНІЙНА ШРЬОДІНҐЕРОВА СИСТЕМА З ФОНОВО-КОНТРОЛЬОВАНИМИ РЕЗОНАНСНИМИ ЗВ'ЯЗКАМИ. СТИСЛИЙ ПЕРЕЛІК КЛЮЧОВИХ ВЛАСТИВОСТЕЙ

Р е з ю м е

Ми підсумовуємо найхарактерніші властивості напівдискретної нелінійної Шрьодінґерової системи з параметрами міжвузлового резонансного зв'язку керованими фоновими значеннями допоміжних полів. Показано, що система є інтеґровною в сенсі Лакса і, як наслідок, уможливлює побудову своїх солітонних розв'язків в рамках належно параметризованої процедури одягання на основі перетворення Дарбу. З іншого боку, інтеґровність системи породжує нескінченну ієрархію локальних законів збереження, декотрі з яких знайдено явно із застосуванням узагальненого рекурсивного підходу. Система складається з двох основних динамічних підсистем та однієї супутньої (допоміжної) підсистеми і допускає Гамільтонове формулювання, супроводжуване доволі нестандартною Пуассоновою структурою. Ненульовий фоновий рівень супутніх полів опосередковує появу додаткового типу міжвузлового резонансного зв'язку, внаслідок чого просторове впорядкування вузлів розміщення основних польових збуджень уособлює найпростішу драбинчасту стьожку трикутної ґратки. Підлаштовуючи керівного фонового параметра, ми маємо змогу переключати динаміку системи між двома суттєво відмінними режимами, розділеними критичною точкою. Критичність динаміки системи відносно фонового параметра проявляється як опосередковано в рамках допоміжної лінійної спектральної задачі, так і безпосередньо в поведінці самих нелінійних динамічних рівнянь. Фізичний підтекст критичности динаміки системи стає ясним після досить витонченої процедури канонізації основних польових змінних. Наразі існує два рівноправні варіанти стандартизації польових змінних досліджуваної нелінійної динамічної системи. Кожен з варіантів є реалізовним у формі двох нееквівалентних канонічних підсистем. Порушена симетрія між канонічними підсистемами є запорукою ефекту зміни природи збуджених станів при переході через критичну точку. Отже, в докритичній області система обумовлює світлі збудження в обох підсистемах, тоді як в надкритичній області одна із підсистем перетворюється на підсистему з темними збудженнями.