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V. PASTUKHOV

Ivan Franko National University of Lviv, Department for Theoretical Physics
(12, Drahomanov Str., Lviv, Ukraine; e-mail: volodyapastukhov@gmail.com)

BOSE GAS IN CLASSICAL ENVIRONMENT AT LOW TEMPERATURES

The properties of a dilute Bose gas with the non-Gaussian quenched disorder are analyzed. Being more specific, we have considered a system of bosons immersed in the classical bath consisting of the non-interacting particles with infinite mass. Making use of perturbation theory up to the second order, we have studied the impact of environment on the ground-state thermodynamic and superfluid characteristics of the Bose component.

Keywords: dilute Bose gas, weak non-Gaussian disorder, superfluid properties.

1. Introduction

The properties of a Bose gas with quenched disorder was studied extensively during last two decades. This rise of an interest in such a system was stimulated by the possibility to observe Bose glass state transition [1], where the superfluidity disappears [2] even at very low temperatures. The first attempts for microscopic description within the approximate second-quantization method adopted for Bose systems with disorder at low temperatures were undertaken in [3]. The further developments [4], particularly the non-perturbative extensions to the case of an arbitrary two-body coupling strength [5] and a strong external potential [6, 7] generally confirm these findings. Diffusive Monte Carlo simulations [8] also agree with the Bogoliubov-like result in the dilute limit, but the increase of the disorder potential makes the differences more visible. The finite-temperature phase diagram of the system was clarified extensively in [9–11]. A shift of the Bose–Einstein condensation transition temperature was determined in [12, 13], and the critical parameters of a Bose gas in the disordered medium were calculated [14] using quantum Monte Carlo methods. No less interesting is the structure of the quasiparticle spectrum and damping for a Bose systeam in the weak random external potential [15–18]. In particular, it was shown that the presence of a disorder broadens the phonon peak of the dynamic structure factor. For a case of liquid ^{4}He with the randomly distributed static impurities, this factor was computed [19] by means of the Path Integral Monte Carlo method and experimentally measured in Ref. [20]. Another consequence of a disorder is the necessity to correct the exact universal identities which are characteristic of many-boson systems like the Hugenholtz–Pines theorem [21] and Josephson's relation [22].

In recent experiments, the disorder is usualy produced by the employment of an optical speckle potential, whose characteristics are precisely controllable [23]. The behavior of bosons in a random potential created by laser speckles was also investigated theoretically [24–26]. The realization of a disorder simple for the understanding can be achieved, however, by the immersion of randomly distributed static impurities in the Bose condensate. The quenched disorder in this system can be then produced by averaging over positions of impurities. When the concentration of impurity particles is small or the interaction with bosons is weak, the function governing the distribution of the random external potential acting on Bose particles can be modeled by the Gaussian. This is exactly the situation considered practically in all available theoretical studies concerning interacting bosons with disorder. But, for weakly-interacting Bose systems, the role of higher-order boson-impurity scattering processes increases that requires to go beyond the standard model of weak disorder and to include the distribution non-Gaussianity. The latter forms the main goal of present study.

2. Formulation of Problem

We consider the system of N interacting bosons immersed in the bath formed by $\mathcal{N}$ non-interacting classical (infinite mass) particles. This model is described by the following Euclidean action after the elimination of "rapidly" varying fields (see, [27] for more details):

$$S = \int dx\, \psi^*(x) \left\{ \frac{\partial}{\partial\tau} + \frac{\hbar^2\nabla^2}{2m} + \mu - \tilde{g}\rho(\mathbf{r}) \right\} \psi(x) - \\ - \frac{g}{2} \int dx |\psi(x)|^4, \quad (1)$$

where the complex field $\psi(x)$ describes bosonic degrees of freedom, μ is the chemical potential that fixes the Bose gas density. The integration over $x = (\tau, \mathbf{r})$ is carried out in a $(3+1)$-domain of the volume βV (β is the inverse temperature) with periodic boundary conditions. The quantity $\rho(\mathbf{r}) = \sum_{1\le j\le \mathcal{N}} \delta(\mathbf{r} - \mathbf{r}_j)$ represents the density of homogeneously distributed classical particles. Both boson-boson and boson-impurity two-body interactions are assumed to be short-ranged that are characterized by the coupling constants g and $\tilde{g}$, respectively. The latter should be related to the appropriate s-wave scattering lengths at the end of calculations. We have

$$\frac{1}{g} = \frac{1}{t} - \frac{1}{V}\sum_{\mathbf{k}} \frac{1}{2\varepsilon_k}, \quad \frac{1}{\tilde{g}} = \frac{1}{\tilde{t}} - \frac{1}{V}\sum_{\mathbf{k}} \frac{1}{\varepsilon_k}, \quad (2)$$

where $t = 4\pi\hbar^2 a/m$, $\tilde{t} = 2\pi\hbar^2\tilde{a}/m$, and $\varepsilon_k = \hbar^2k^2/2m$ is the free-particle dispersion. Introducing phase-density representation for bosonic fields $\psi^*(x) = \sqrt{n(x)}e^{-i\varphi(x)}$, $\psi(x) = \sqrt{n(x)}e^{i\varphi(x)}$ and making use of the Fourier transformation for $n(x)$ and $\varphi(x)$, we get

$$n(x) = n + \frac{1}{\sqrt{\beta V}} \sum_K e^{iKx} n_K, \\ \varphi(x) = \frac{1}{\sqrt{\beta V}} \sum_K e^{iKx} \varphi_K, \quad (3)$$

where $K = (\omega_k, \mathbf{k})$ stands for the bosonic Matsubara frequency ω_k and three-dimensional wave-vector $\mathbf{k}$, as well as for the classical component density

$$\rho(\mathbf{r}) = \rho + \frac{1}{\sqrt{V}} \sum_{\mathbf{k}\neq 0} \mathrm{e}^{i\mathbf{kr}} \rho_{\mathbf{k}}, \quad (4)$$

where $\rho = \mathcal{N}/V$ is the average density of bath particles and $\rho_{\mathbf{k}} = \frac{1}{\sqrt{V}} \sum_{1\le j\le \mathcal{N}} \mathrm{e}^{-i\mathbf{kr}_j}$. So, we rewrite action (1) in the following way:

$$S = S_B + S_d. \quad (5)$$

The first term describes the Bose gas itself [28]

$$S_B = \beta V \mu n - \frac{1}{2}\beta V g n^2 - \\ - \frac{1}{2}\sum_K \Big\{ \omega_k \varphi_K n_{-K} - \omega_k \varphi_{-K} n_K + \\ + 2n\varepsilon_k \varphi_K \varphi_{-K} + \left[\frac{\varepsilon_k}{2n} + g\right] n_K n_{-K} \Big\} + \\ + \frac{1}{3!\sqrt{\beta V}} \sum_{K+Q+P=0} \frac{1}{4n^2} (\varepsilon_k + \varepsilon_q + \varepsilon_p) n_K n_Q n_P + \\ + \frac{1}{2\sqrt{\beta V}} \sum_{K,Q} \frac{\hbar^2}{m} \mathbf{kq} \varphi_K \varphi_Q n_{-K-Q} + ..., \quad (6)$$

while S_d accounts for the presence of the environment:

$$S_d = -\beta V \tilde{g} n\rho - \sqrt{\beta}\tilde{g} \sum_K \delta_{\omega_k,0} \rho_{-\mathbf{k}} n_K. \quad (7)$$

Dots in Eq. (6) stand for terms with products of four, five, *etc.* density fluctuation fields n_K and describe the higher-order quasiparticle scattering processes, which are less probable for weakly interacting Bose systems and, therefore, negligible. The thermodynamic relation $-\partial\Omega/\partial\mu = N$ for the grand potential together with explicit form of Eq. (6) fix $n = N/V$ [29,30] to be the density of the Bose system. This observation allows us to proceed in the canonical ensemble. In order to obtain a physically meaningful result, the averaging over the positions of classical particles should be performed for the free energy of our system. This particularly means that, first, we have to calculate the free energy F of the Bose gas in the presence of a local external potential $\tilde{g}\rho(\mathbf{r})$ and then to identify the Helmholtz potential of the system "Bose gas + classical bath" with

$$\bar{F} = \frac{1}{V^{\mathcal{N}}} \int_V d\mathbf{r}_1 ... \int_V d\mathbf{r}_{\mathcal{N}} F. \quad (8)$$

Let us briefly discuss the above averaging procedure and how it can be understood from the point of view of an experimental realization. Suppose that the prepared mixture of bosons and heavy particles is large enough to be divided (at least imaginably) into many independent macroscopic "regions". In every such "region," the positions of classical particles distinguish from the neighboring ones. Therefore, by probing the properties of the whole system, we actually observe the averaged impact of all "regions". The described situation is the simplest realization of the quenched disorder in many-body systems. Note that it sufficiently differs from the case of the so-called annealed disorder, which can be also realized on the basis of our simple two-component model at finite temperatures. In that case, the positions of heavy impurities vary slowly, but the relaxation time of the "light" component (bosons in the present study) is assumed to be small enough to keep the system at the thermodynamic equilibrium.

In the following, we will assume that bosons are weakly coupled to one another and will study the impact of the classical component on the properties of a Bose gas in the zero-temperature limit. It is well known that the presence of a disorder or an interaction with other quantum systems depletes the superfluid density of the Bose gas even at absolute zero [31–34]. Of course, this phenomena is observed in our case too. In order to calculate the normal density of a superfluid, we have to assume that the Bose subsystem moves as a whole with velocity $\mathbf{v}$. Simple analysis [35] shows that the account for this motion only leads to a shift $\omega_k \to \omega_k - i\hbar\mathbf{v}\mathbf{k}$ of the Matsubara frequency in the action S_B. Then the general consideration in the spirit of perturbation theory in terms of $\tilde{g}$ leads to

$$\bar{E}_{\mathbf{v}} = E_B + N\frac{m\mathbf{v}^2}{2} + V\tilde{g}n\rho - \\ - \frac{1}{2}\sum_{\mathbf{k}} \tilde{g}^2\overline{\rho_{\mathbf{k}}\rho_{-\mathbf{k}}}\langle n_K n_{-K}\rangle|_{\omega_k=0} + \\ + \frac{1}{3!}\sum_{\mathbf{k}+\mathbf{q}+\mathbf{p}=0} \tilde{g}^3\overline{\rho_{\mathbf{k}}\rho_{\mathbf{q}}\rho_{\mathbf{p}}}\sqrt{\beta}\langle n_K n_Q n_P\rangle|_{\omega_k,\omega_q,\omega_p=0} \quad (9)$$

for the ground-state energy of the system, where E_B is the contribution of the Bose gas in rest alone, and $\langle n_K n_{-K}\rangle$, $\langle n_K n_Q n_P\rangle, ...$ denote the irreducible density correlation functions of pure bosons moving with velocity $\mathbf{v}$. The structure factors of classical non-interacting particles are fully determined by their density and can be easily evaluated by using the procedure described above: $\overline{\rho_{\mathbf{k}}\rho_{-\mathbf{k}}} = \rho$, $\overline{\rho_{\mathbf{k}}\rho_{\mathbf{q}}\rho_{\mathbf{p}}} = \rho\delta_{\mathbf{k}+\mathbf{q}+\mathbf{p},0}/\sqrt{V}$, *etc.* Expanding r.h.s. of Eq. (9) in powers of velocity

$$\bar{E}_{\mathbf{v}} = \bar{E}_{\mathbf{v}=0} + V\frac{m\mathbf{v}^2}{2}n_s + o(\mathbf{v}^2) \quad (10)$$

(actually in powers of the dimensionless parameter v/c, where c is the sound velocity) to the quadratic order, one obtains the density n_s of the superfluid component. It should be noted that Eq. (10) represents the energy of a Bose system with disorder in the laboratory frame, and it assumes that only the superfluid component is moving with velocity $\mathbf{v}$. Equation (10) also suggests that, even at absolute zero, the superfluid component of the Bose gas is depleted due to the disorder-induced loss of coherence. The considered situation is somewhat similar to the Bose system with mobile impurities [36] (for instance, small amount of ^{3}He atoms immersed in liquid ^{4}He), where the effective mass of impurity particles can be straightforwardly related to the depletion of the superfluid density (see, for example, [37]). Furthermore, the increase of the disorder strength may lead to the total destruction of the superfluidity in Bose systems, but this is not a case of weak disorder addressed in this study. Another consequence following from Eq. (10) is that, even at very low temperatures, the presence of a quenched disorder modifies the equations of the two-fluid hydrodynamics providing that the velocity of the first sound should decrease (because the total density of bosons in front of the derivative in the equation $c^2 = \frac{n}{m}\frac{\partial\mu}{\partial n}$ has to be replaced with n_s). The latter fact can be used for the experimental measurements of the superfluid density depletion in disordered Bose systems.

In addition to a depletion of the superfluid density, the interaction with the bath also decreases the number of Bose particles with zero momentum. To calculate the condensate density of a Bose gas, we use the following prescription: first, within the variational differentiation

$$N_k = \left(\frac{\delta\bar{E}}{\delta\varepsilon_k}\right)_n, \quad (11)$$

we determine the distribution function of particles with non-zero momentum and then obtain the condensation fraction

$$\frac{n_0}{n} = 1 - \frac{1}{N}\sum_{\mathbf{k}\neq 0} N_k. \quad (12)$$

To this stage, our consideration is formally exact, and the problem is actually reduced to the calculation of irreducible density correlators of a pure Bose gas. But even in the Bogoliubov approximation, these calculations are very cumbersome. Therefore, we restrict ourselves below to the case of weak interaction of interspecies, where the perturbation theory in terms of $\tilde{g}$ can be used.

3. Perturbation Theory

By treating the Bose subsystem on the basis of Bogoliubov's theory, we greatly simplify the analysis below. But, at that time, we restrict it to consideration of dilute gases. This is exactly the situation realized in experiments with cold alkali atoms. From the point of view of further calculations in the dilute limit, we are free to drop the beyond Bogoliubov corrections to the various density correlation functions. Additionally, working with the same accuracy, one should treat the ground-state energy of pure bosons $E_B = E_{\rm LHY}$ on level of the Lee–Huang–Yang [38] formula.

Following the above-mentioned approximation scheme in the first order of perturbation theory, we have to neglect the last term in Eq. (9) and substitute the pair density correlation function

$$\langle n_K n_{-K}\rangle = \frac{2n\varepsilon_k}{E_k^2+(\omega_k - i\hbar\mathbf{kv})^2}, \tag{13}$$

with $E_k = \sqrt{\varepsilon_k^2 + 2ng\varepsilon_k}$ being Bogoliubov's spectrum. The resulting formula has to be used for obtaining the particle distribution N_k and condensate density

$$\frac{n_0}{n} = \frac{n_0^B}{n} - \frac{1}{V}\sum_{\mathbf{k}\neq 0}\rho\tilde{t}^2\frac{\varepsilon_k^2}{E_k^4}, \tag{14}$$

(n_0^B/n is the Bogoliubov result for pure bosons) in the adopted approximation and after the renormalization of the coupling constant (2) for the explicit evaluation of the energy correction

$$\frac{\bar{E}^{(1)}_{\mathbf{v}=0}}{N} = \rho\tilde{t} - \frac{1}{V}\sum_{\mathbf{k}\neq 0}\rho\tilde{t}^2\left[\frac{\varepsilon_k}{E_k^2}-\frac{1}{\varepsilon_k}\right], \tag{15}$$

and a depletion of the superfluid component

$$\frac{n_s}{n} = 1 - \frac{4}{3V}\sum_{\mathbf{k}\neq 0}\rho\tilde{t}^2\frac{\varepsilon_k^2}{E_k^4}. \tag{16}$$

From general principles as well as from the above formulas, it is clear that, in the first-order approximation, the results derived for our system are identical to those obtained for a dilute Bose gas with weak disorder [8]. The differences appear in the next orders of the formulated perturbation theory.

The second-order calculations require the account for the last term in Eq. (9). The appropriate three-point density correlator reads

$$\begin{aligned}\langle n_K n_Q n_P\rangle = {} & \frac{\delta_{K+Q+P,0}}{\sqrt{\beta V}}\times\\ & \times\left[\frac{\hbar^2}{m}\mathbf{kq}\langle n_K\varphi_{-K}\rangle\langle n_Q\varphi_{-Q}\rangle\langle n_P n_{-P}\rangle + \right.\\ & \left. + \frac{\varepsilon_k}{4n^2}\langle n_K n_{-K}\rangle\langle n_Q n_{-Q}\rangle\langle n_P n_{-P}\rangle + \text{perm.}\right],\end{aligned} \tag{17}$$

in the dilute limit, where we have used shorthand notation for the phase-density correlator

$$\langle \varphi_K n_{-K}\rangle = \frac{\omega_k - i\hbar\mathbf{kv}}{E_k^2+(\omega_k-i\hbar\mathbf{kv})^2}. \tag{18}$$

The further strategy is the same as previously used. By calculating the next correction to the particle distribution N_k, we are in position to obtain the condensate density up to the second-order of perturbation theory. Then the application of the coupling constant renormalization procedure (2) yields

$$\begin{aligned}&\frac{6}{V^2}\sum_{\mathbf{k},\mathbf{q}\neq 0}\rho\tilde{t}^3 nt\frac{\varepsilon_k}{E_k^2}\frac{\varepsilon_q}{E_q^2}\frac{\varepsilon^2_{|\mathbf{k}+\mathbf{q}|}}{E^4_{|\mathbf{k}+\mathbf{q}|}} - \\ &-\frac{2}{V}\sum_{\mathbf{k}\neq 0}\rho\tilde{t}^2\frac{\varepsilon_k^2}{E_k^4}\frac{1}{V}\sum_{\mathbf{q}\neq 0}\tilde{t}\left[\frac{\varepsilon_q}{E_q^2}-\frac{1}{\varepsilon_q}\right]\end{aligned} \tag{19}$$

for the fraction of non-condensed particles. In the same fashion, we obtain the energy correction

$$\begin{aligned}\frac{\bar{E}^{(2)}_{\mathbf{v}=0}}{N} = {} & \frac{1}{V^2}\sum_{\mathbf{k},\mathbf{q}}\rho\tilde{t}^3\left\{\frac{\varepsilon_k}{E_k^2}\frac{\varepsilon_q}{E_q^2}\left[\frac{\varepsilon^2_{|\mathbf{k}+\mathbf{q}|}}{E^2_{|\mathbf{k}+\mathbf{q}|}}-1\right] + \right.\\ & \left. + \left[\frac{\varepsilon_k}{E_k^2}-\frac{1}{\varepsilon_k}\right]\left[\frac{\varepsilon_q}{E_q^2}-\frac{1}{\varepsilon_q}\right]\right\}\end{aligned} \tag{20}$$

and the second-order normal density fraction

$$\begin{aligned}&\frac{2}{3V^2}\sum_{\mathbf{k},\mathbf{q}}\rho\tilde{t}^3\left(\frac{\hbar^2\mathbf{kq}}{m}\right)^2\frac{\varepsilon_{|\mathbf{k}+\mathbf{q}|}}{E_k^2E_q^2E^2_{|\mathbf{k}+\mathbf{q}|}} - \\ &-\frac{4}{3V^2}\sum_{\mathbf{k},\mathbf{q}}\rho\tilde{t}^3\frac{\varepsilon^3_{|\mathbf{k}+\mathbf{q}|}}{E^4_{|\mathbf{k}+\mathbf{q}|}}\frac{\varepsilon_q}{E_q^2}\frac{\varepsilon_k}{E_k^2} - \\ &-\frac{8}{3V^2}\sum_{\mathbf{k},\mathbf{q}}\rho\tilde{t}^3\left\{\frac{\varepsilon_k^2}{E_k^4}\left[\frac{\varepsilon_q^2}{E_q^2}-1\right]\frac{\varepsilon_{|\mathbf{k}+\mathbf{q}|}}{E^2_{|\mathbf{k}+\mathbf{q}|}} + \right.\\ &\left. + \frac{\varepsilon_k^2}{E_k^4}\left[\frac{\varepsilon_q}{E_q^2}-\frac{1}{\varepsilon_q}\right]\right\}\end{aligned} \tag{21}$$

associated with the presence of a disorder.

4. Results

An interesting feature of the model with a delta-like potential is that all integrals appearing during the calculation of energy corrections and the depletion of the condensate and superfluid densities can be evaluated analytically to the very end. Particularly, for the fraction of condensed particles in the presence of the classical bath, we obtained the following expansion:

$$\frac{n_0}{n} = \frac{n_0^B}{n} - \frac{\sqrt{\pi}}{2}\frac{\rho\tilde{a}^2}{\sqrt{na}} - 6\pi\rho\tilde{a}^3. \tag{22}$$

After the tedious integration, the formula for a superfluid density with a similar structure was also derived in the form:

$$\frac{n_s}{n} = 1 - \frac{2\sqrt{\pi}}{3}\frac{\rho\tilde{a}^2}{\sqrt{na}} - 4\pi\left\{3 - \frac{2}{3}\ln\frac{16}{3}\right\}\rho\tilde{a}^3. \tag{23}$$

The situation with energy is more complicated. There is no problem in the calculation of the first-order correction, but the second one is logarithmically divergent. Let us recall that the same type of problems originally occurs [39] during the computation of the beyond Lee–Huang–Yang ground-state energy of a pure Bose gas and is totally connected with the point-like approximation of the two-body potential. Applying a similar regularization procedure, i.e., cutting off the upper integration limit on a scale of order $1/\tilde{a}$, we finally have (with logarithmic accuracy)

$$\frac{\bar{E}_{\mathbf{v}=0}}{N} = \frac{E_{\mathrm{LHY}}}{N} + \\ + \rho\tilde{t}\left\{1 + 4\sqrt{\pi}\sqrt{na\tilde{a}^2} - 8\pi na\tilde{a}^2\ln\frac{1}{na\tilde{a}^2}\right\}. \tag{24}$$

Of course, this result can be justified within a more sophisticated consideration which particularly intends the explicit momentum dependence of the t-matrix (2). Indeed, in the limit of $k \to \infty$, the leading asymptote is $\tilde{t} \sim 1/(k\tilde{a})^2$ in the limit of vanishing effective interaction range that provides the convergence of integrals and the correctness of the above cut-off procedure.

5. Conclusions

In summary, by means of the hydrodynamic approach, we have studied properties of a dilute Bose gas with the non-Gaussian quenched disorder. The realization of an external random potential is performed by inserting a macroscopic number of non-interacting classical particles with infinite mass into the system. Assuming that the two-body potential describing the interaction between bath particles and bosons is short-ranged and weak, we have perturbatively analyzed the thermodynamic and superfluid characteristics of the system. Particularly, we obtained, in addition to the well-known Bogoliubov-like result, the second-order beyond-mean-field corrections to the energy, condensate fraction, and superfluid density of the Bose gas. It is instructive to note that the presence of the environment generally depletes the superfluid and condensate densities. Furthermore, the second-order terms of these observables do not depend on the number of bosons and are totally determined by the interaction with impurities.

The possible experimental visualization of the calculated next to beyond-mean-field effects can be realized not only on the system of "dirty" bosons. Very promising in this context is a two-component mixture of Fermi particles [40], where the strength of the tunnable interaction can be tuned in a wide range to observe both the weakly non-ideal Fermi gas and the dilute Bose condensate of dimers.

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В. Пастухов

РОЗРІДЖЕНИЙ БОЗЕ-ГАЗ В КЛАСИЧНОМУ СЕРЕДОВИЩІ ПРИ НИЗЬКИХ ТЕМПЕРАТУРАХ

Р е з ю м е

Проаналізовано властивості розрідженого бозе-газу з негаусовим безладом. Більш конкретно, ми розглянули систему бозонів, занурених у класичну ванну, що складається з невзаємодіючих частинок з нескінченною масою. Використовуючи теорію збурень до другого порядку, ми вивчили вплив середовища на термодинамічні та надплинні характеристики основного стану бозе-компоненти.