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# POSITIVE PARITY LEVELS OF $^{21,23}$Na ISOTOPES BY USING THE NUCLEAR SHELL MODEL

*The energy levels and transition probabilities* $B(E2;\downarrow)$ *and* $B(M1;\downarrow)$ *have been investigated for* $^{21,23}$*Na isotopes by using the (USDA and USDB) interactions in the (sd-shell) model space. In the calculations of the shell model, it has been assumed that all possible many-nucleon configurations are specified by the* $(0d_{5/2}$, $1s_{1/2}$, *and* $0d_{3/2})$ *states above* $^{16}$*O doubly magic nucleus. The available empirical data are in a good agreement with predictions of theoretical energy levels. Spins and parities are affirmed for new levels, transition probabilities* $B(E2;\downarrow)$ *and* $B(M1;\downarrow)$ *are predicted as well.*

*Keywords*: shell model, energy levels, OXBASH code, $B(E2;\downarrow)$ and $B(M1;\downarrow)$.

## 1. Introduction

In view of the absence of a comprehensive and interrelated theory of the nuclear structure, several attempts have been made to link nuclear data to a number of different nuclear models, where important physical foundations had been used as a basis for the accurate structure. There are a number of research that try to describe the atomic nuclei and to dismantle these nuclei to different components. A lot of researchers in nuclear physics focus their efforts on some nuclear models, which are the first step to understand the data observation, to measure the links between them, and to draw some conclusions [1]. These models are aimed at the description of data and the interpretation of nuclear properties. One of the most important nuclear models proposed to describe the interaction between nucleons is the shell model. The nuclear shell model is an essential step in understanding the nature of nuclei as well. It is very successful in the microscopic description of the nuclear structure, which represents many features of nuclear energy levels. The importance of the nuclear shell model lies in its ability to give a true accurate approximation of the energy levels of nucleons with different values of angular momentum [2, 3]. The shell model had been modified significantly to involve large neutron systems, which is very necessary in the field of astronomy and has many applications in the study of the great explosions called supernova and other fields [4]. The nuclear shell model solves the many-body Schŕodinger equation in a truncated model space. Modern nuclear shell models include the configuration mixing and the residue interactions. The observed binding energies, levels, electromagnetic properties, $\beta$ decays, and many other properties can be well described through the shell model approach in the light and medium mass regions. Moreover, the bounded systems of interacting particles within their motion in a central potential show some shell structure [5]. This explains that the nuclear shell is one of the most important models that describe the structure of nuclei. In order to be successful, any shell model should focus on two main points:

***First:*** An attempt to understand the characteristics of the force that have already been measured and identified.

***Second:*** It must predict additional properties that can be measured through new experiments [6, 7].

The study of low-lying excited states of nuclei around double magic shells provides information about the specific nuclear orbital of a nucleus, because a few nuclear orbits dominate the contribution to its wave function. This is well proved by the attention focused on these nuclei in various recent works. The previous studies of low-lying states and transition probabilities for isotopes in the *sd*-shell region were reviewed in [7–10].

## 2. Theory

The shell model within a restricted model space is one of the models, which succeeded in describing the static properties of nuclei, when effective charges are used. With the light nuclei, the good approximation for the nuclear potential in a nucleus is the three-dimensional oscillator potential which has the easily manipulating advantage of single-particle wave functions. From the mean-field approximation, each nucleon of mass $(m)$ moves independently in a potential that represents the average interaction with the other nucleons in a nucleus. Suppose that the potential depth at the center of the nucleus is $U_0$. Then each particle moves in a spherically symmetric harmonic oscillator potential with angular frequency $(\omega)$, and the Hamiltonian becomes [11]

$$\left.\begin{aligned} H_0 &= \frac{-\hbar^2}{2m}\nabla^2 + \frac{m\omega^2 r^2}{2} - U_0, \\ U_0 &= \frac{m\omega^2 R_0^2}{2}, \quad R_0 = 1.07\,\mathrm{A}^{1/3}\,\mathrm{fm} \end{aligned}\right\}. \tag{1}$$

The HO length parameter $b$ which is related to the mass of the nucleon $(m)$ and the frequency $(\omega)$ associated with the HO potential

$$b = \sqrt{\frac{\hbar}{m\omega}}. \tag{2}$$

The mean field potential with the HO potential is augmented by the spin-orbit $(s \cdot l)$ term and the orbit-orbit $(l \cdot l)$ force which lead to a successful reproduction of the pattern of magic numbers, if the strong spin-orbit interaction term $(V_{ls})$ is added to the single-particle Hamiltonian [12]:

$$V_{ls} = f(r)\,\mathbf{l}\cdot\mathbf{s}. \tag{3}$$

## 3. Results and Discussion

The nuclear shell-model codes, such as OXBASH, are used to investigate the structure of nuclei. The essential inputs to the most shell-model configuration mixing codes are sets of the single-particle matrix elements (SPEs) and two-body matrix elements (TBMEs). These sets are termed “effective interactions” or “model-space Hamiltonians”. The *sd* model space includes the $0d_{5/2}$, $0d_{3/2}$, and $1s_{1/2}$ valence orbits. For this model space, there are three SPEs and 63 TBMEs which in the mass region of $A = 16$–$40$ can determine the energies and wave functions for about 106 levels. The calculations were performed by using the OXBASH code for Windows [13, 14]. The code uses the M-scheme Slater determinant basis, a projection technique, and wave functions with good angular momentum $(J)$ and isospin $(T)$. The universal sd (USD) of the Wildenthal interaction Hamiltonian has provided realistic *sd*-shell wave functions to use in the study of the nuclear structure, nuclear spectroscopy, and nuclear astrophysics. Brown and Richter had derived and developed a new Hamiltonian for the *sd*-shell (USDA and USDB), and it leads to a new state of precision for realistic shell model eigenvectors [15, 16]. The first formulation includes *sd*-model spaces consisting of ($0d_{5/2}$, $1s_{1/2}$, and $0d_{3/2}$) shells above the ($^{16}$O) nucleus with the USDA and USDB Hamiltonians defining the effective interaction with single-particle energies of $\{(0d_{5/2} = -3.944,\ 1s_{1/2} = -3.061$ and $d_{3/2} = 1.980),\ (0d_{5/2} = -3.926,\ 1s_{1/2} = -3.208$ and $0d_{3/2} = 2.112)\}$ MeV for $\{0d_{3/2},\ 1s_{1/2}$, and $0d_{5/2}\}$, respectively. The transition probability is considered as one of the most sensitive parameters in determining the effective interactions. In order to elucidate this sensitivity, the reduced electric quadruple transition probability $B(E2)$ and the reduced magnetic dipole transition probability $B(M1)$ are calculated. The results show that the default values of effective charge and $g$-factors are used in the OXBASH program according to the values of effective charges $\{e_p = 1.35e$ and $e_n = 0.35\}$, and the free nucleon $g$ factors are $\{g_s(p) = 5.586,\ g_s(n) = -3.826,\ g_l(p) = 1$, and $g_l(n) = 0\}$ which are determined from fits to a large number of magnetic and quadrupole moments. The values of the effective $g$ factors are chosen according to the minimum (*rms*) values for USDA in the fits.

The purpose of these calculations is to describe an optimal set in USDA and SDBA to calculate $B(E2)$ and $B(M1)$ transitions. The experimental data that were adopted in this work are the latest results for the corresponding nucleus.

### *3.1. ($^{21}$Na) nucleus*

According to the shell model, the ground state of $^{21}$Na is a closed $^{16}$O core with five nucleons distributed as three protons and two neutrons in the (*sd*) space, which is similar to other (Na) isotopes ($11 \leqslant N \leqslant 10$) for the closed core and distribution of protons. Other excited states are formed by the configurations of these nucleons in the (*sd*) shell model space. Predictions by using USDA and USDB interactions for the first sequence $\{5/2_1^+,\ 7/2_1^+,\ 9/2_1^+\}$ are presented in Table 1, which shows a good agreement with the experimental data [17]. $(7/2_2^+, 3/2_2^+, 3/2_3^+, 7/2_3^+)$ spins were confirmed for levels which were practically uncertain. The spins and parities of levels ($11/2_1^+$ and $7/2_7^+$) are affirmed experimentally. The states: ($1/2_3^+$, $3/2_5^+$, $5/2_6^+$, $3/2_6^+$, $5/2_7^+$, $3/2_4^+$, $3/2_7^+$, $1/2_4^+$ and $3/2_8^+$) were predicted at energies close to the experimental values. The state ($5/2_9^+$) was determined for an experimental energy of 10.050 MeV that undetermined in the spin and parity. Theoretically, the levels $\{1/2_2^+$, $9/2_2^+$, $13/2_1^+$, $5/2_4^+$, $7/2_4^+$, $7/2_5^+$, $9/2_3^+$, $9/2_4^+$, $11/2_2^+$, $9/2_5^+ 13/2_2^+$, $5/2_8^+$, $11/2_3^+$, $15/2_1^+$, $17/2_1^+$, $9/2_6^+$, and ($11/2_4^+$ to $17/2_{10}^+)\}$ with the angular momentum, parity, and energy values were expected in our calculations and were not predicted previously.

*Table 1.* **The calculated energy levels with their spin and parity for $^{21}$Na nucleus with the USDA and USDB interactions versus the experimental data [17]**

| $J^\pi$ | Theoretical values for $E$ (MeV) | | Experimental values | |
|---|---|---|---|---|
| | USDA results | USDB results | $E$ (MeV) | $J^\pi$ |
| 1 | 2 | 3 | 4 | 5 |
| $3/2_1^+$ | 0.000 | 0.000 | 0.000 | $3/2^+$ |
| $5/2_1^+$ | 0.299 | 0.266 | 0.331 | $5/2^+$ |
| $7/2_1^+$ | 1.776 | 1.757 | 1.716 | $7/2^+$ |
| $1/2_1^+$ | 2.890 | 2.859 | 2.423 | $1/2^+$ |
| $9/2_1^+$ | 2.801 | 2.831 | 2.829 | $9/2^+$ |
| $5/2_2^+$ | 3.770 | 3.718 | 5.544 | $5/2^+$ |
| $5/2_3^+$ | 4.701 | 4.627 | 4.294 | $5/2^+$ |
| $11/2_1^+$ | 4.374 | 4.378 | 4.419 | $(11/2^+)$ |
| $3/2_2^+$ | 5.059 | 4.913 | 5.020 | $(3/2, 5/2, 7/2)^+$ |
| $7/2_2^+$ | 5.515 | 5.393 | 5.380 | $(3/2, 5/2, 7/2)^+$ |
| $3/2_3^+$ | 5.695 | 5.528 | 5.770 | $(3/2, 5/2, 7/2)^+$ |
| $1/2_2^+$ | 5.941 | 5.962 | – | – |
| $9/2_2^+$ | 6.139 | 6.116 | – | – |
| $13/2_1^+$ | 6.230 | 6.245 | – | – |
| $7/2_3^+$ | 6.306 | 6.168 | 6.341 | $(3/2, 5/2, 7/2)^+$ |
| $3/2_4^+$ | 6.682 | 6.739 | 6.468 | $3/2^+$ |
| $1/2_3^+$ | 7.237 | 7.070 | 7.253 | $1/2^+$ |
| $5/2_4^+$ | 7.238 | 7.285 | – | – |
| $7/2_4^+$ | 7.312 | 7.076 | – | – |
| $7/2_5^+$ | 7.439 | 7.324 | – | – |
| $9/2_3^+$ | 6.368 | 6.255 | – | – |
| $3/2_5^+$ | 7.676 | 7.845 | 7.609 | $3/2^+$ |
| $9/2_4^+$ | 7.853 | 7.795 | – | – |
| $11/2_2^+$ | 8.073 | 7.935 | – | – |
| $5/2_5^+$ | 7.899 | 7.909 | 7.930 | $5/2^-$ |
| $5/2_6^+$ | 8.433 | 8.153 | 8.595 | $5/2^+$ |
| $3/2_6^+$ | 8.810 | 8.746 | 8.715 | $3/2^+$ |
| $5/2_7^+$ | 8.906 | 8.790 | 8.976 | $5/2^+$ |
| $7/2_6^+$ | 9.021 | 9.110 | 9.051 | $7/2^+$ |
| $3/2_7^+$ | 9.080 | 8.942 | 9.155 | $3/2^+$ |
| $9/2_5^+$ | 9.178 | 8.973 | – | – |
| $1/4_4^+$ | 9.291 | 9.145 | 9.217 | $1/2^+$ |
| $13/2_2^+$ | 9.294 | 9.234 | – | – |
| $7/2_7^+$ | 9.301 | 9.139 | 9.280 | (3/2, 5/2, 7/2) |
| $5/2_8^+$ | 9.490 | 9.320 | – | – |
| $3/2_8^+$ | 9.605 | 9.657 | 9.725 | $3/2^+$ |

### *3.2. ($^{23}$Na) nucleus*

In the nucleus of $^{23}$Na, the seven valence fermions are three protons and four neutrons in the *sd*-space above $^{16}$O core with, the excited states formed by the configuration of these nucleons in the *sd*-shell model space. In Table 2, the predictions with the USDA and USDB interactions for the first sequence $\{5/2_1^+,\ 7/2_1,\ 9/2_1^+,\ 1/2_2^+,\ 5/2_3^+,\ 11/2_1^+,\ 1/2_3^+,\ 3/2_3,\ 3/2_4^+,\ 5/2_9^+,\ 1/2_5^+,\ 1/2_6^+,\ 3/2_{10},\ 1/2_7,\ 1/2_8\}$ are in a good agreement with the experimental data [18]. Experimentally, the energies {(6.114, 6.618, 6.947, and 7.487 MeV}, are with confirmed spins ($3/2_7^+$, $7/2_4^+$, $11/2_2^+$, $1/2_{10}^+$, $1/2_9^+$, $1/2_4^+$), but without parity. The energy levels {(10.906 and

*Continuation of Table 1*

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| $11/2^+_3$ | 9.619 | 9.334 | – | – |
| $15/2^+_1$ | 9.712 | 9.894 | – | – |
| $17/2^+_1$ | 9.715 | 9.595 | – | – |
| $9/2^+_6$ | 9.781 | 9.603 | – | – |
| $1/2^+_5$ | 9.751 | 9.834 | 9.775 | $1/2^-$ |
| $7/2^+_8$ | 9.959 | 9.871 | 9.779 | $7/2^-$ |
| $5/2^+_9$ | 10.065 | 9.888 | 10.050 | – |
| $11/2^+_4$ | 10.143 | 10.023 | – | – |
| $7/2^+_9$ | 10.261 | 10.176 | – | – |
| $13/2^+_3$ | 10.283 | 10.178 | – | – |
| $9/2^+_7$ | 10.389 | 10345 | – | – |
| $11/2^+_5$ | 10.446 | 10.247 | – | – |
| $3/2^+_9$ | 10.624 | 10.366 | – | – |
| $11/2^+_6$ | 10.661 | 10.470 | – | – |
| $5/2^+_{10}$ | 10.702 | 10.293 | – | – |
| $7/2^+_{10}$ | 10.704 | 10.648 | – | – |
| $1/2^+_6$ | 10.783 | 10.420 | – | – |
| $13/2^+_4$ | 10.868 | 10.911 | – | – |
| $9/2^+_8$ | 10.873 | 10.857 | – | – |
| $1/2^+_7$ | 11.074 | 10.589 | – | – |
| $3/2^+_{10}$ | 11.258 | 11.095 | – | – |
| $9/2^+_9$ | 11.598 | 11.562 | – | – |
| $15/2^+_2$ | 11.651 | 11.405 | – | – |
| $11/2^+_7$ | 11.903 | 11.805 | – | – |
| $9/2^+_{10}$ | 11.944 | 11.757 | – | – |
| $15/2^+_3$ | 12.291 | 12.019 | – | – |
| $11/2^+_8$ | 12.352 | 12.174 | – | – |
| $11/2^+_9$ | 12.652 | 12.355 | – | – |
| $13/2^+_5$ | 12.685 | 12.565 | – | – |
| $11/2^+_{10}$ | 12.839 | 12.950 | – | – |
| $13/2^+_6$ | 13.100 | 12.967 | – | – |
| $17/2^+_2$ | 13.111 | 12.887 | – | – |
| $1/2^+_8$ | 13.285 | 13.167 | – | – |
| $1/2^+_9$ | 13.558 | 13.948 | – | – |
| $15/2^+_4$ | 13.738 | 13.507 | – | – |
| $13/2^+_7$ | 13.821 | 13.856 | – | – |
| $15/2^+_5$ | 13.822 | 13.814 | – | – |
| $17/2^+_3$ | 14.350 | 14.666 | – | – |
| $19/2^+_1$ | 14.480 | 14.352 | – | – |
| $1/2^+_{10}$ | 14.504 | 14.268 | – | – |
| $13/28^+$ | 14.669 | 14.791 | – | – |
| $13/2^+_9$ | 15.025 | 14.435 | – | – |
| $13/2^+_{10}$ | 15.372 | 15.199 | – | – |
| $15/2^+_6$ | 16.503 | 16.021 | – | – |

*End of Table 1*

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| $15/2^+_7$ | 16.690 | 16.509 | – | – |
| $15/2^+_8$ | 16.855 | 17.004 | – | – |
| $17/2^+_4$ | 17.168 | 17.301 | – | – |
| $15/2^+_9$ | 17.412 | 17.262 | – | – |
| $17/2^+_5$ | 18.150 | 17.712 | – | – |
| $15/2^+_{10}$ | 18.913 | 18.744 | – | – |
| $17/2^+_6$ | 19.081 | 18.266 | – | – |
| $19/2^+_2$ | 20.547 | 20.037 | – | – |
| $17/2^+_7$ | 21.439 | 21.694 | – | – |
| $17/2^+_8$ | 22.932 | 23.105 | – | – |
| $17/2^+_9$ | 23.868 | 23.628 | – | – |
| $17/2^+_{10}$ | 26.349 | 25.165 | – | – |

*Table 2.* **The calculated energy levels with their spin and parity for $^{23}$Na nucleus using USDA and USDB interactions versus the experimental data [18]**

| $J^\pi$ | Theoretical values for $E$ (MeV) | | Experimental values | |
|---|---|---|---|---|
| | USDA results | USDB results | $E$ (MeV) | $J^\pi$ |
| 1 | 2 | 3 | 4 | 5 |
| $3/2^+_1$ | 0.000 | 0.000 | 0.000 | $3/2^+$ |
| $5/2^+_1$ | 0.446 | 0.399 | 0.439 | $5/2^+$ |
| $7/2^+_1$ | 2.185 | 2.168 | 2.076 | $7/2^+$ |
| $1/2^+_1$ | 2.232 | 2.173 | 2.390 | $1/2^+$ |
| $9/2^+_1$ | 2.828 | 2.722 | 2.703 | $9/2^+$ |
| $3/2^+_2$ | 2.772 | 2.761 | 2.982 | $3/2^+$ |
| $5/2^+_2$ | 3.763 | 3.749 | 3.914 | $5/2^+$ |
| $1/2^+_2$ | 4.555 | 4.463 | 4.429 | $1/2^+$ |
| $7/2^+_2$ | 4.634 | 4.661 | 4.774 | $7/2^+$ |
| $5/2^+_3$ | 5.513 | 5.422 | 5.378 | $5/2^+$ |
| $7/2^+_3$ | 5.363 | 5.245 | – | – |
| $11/2^+_1$ | 5.527 | 5.790 | 5.534 | $11/2^+$ |
| $3/2^+_3$ | 6.044 | 5.827 | 5.766 | $3/2^+$ |
| $5/2^+_4$ | 5.809 | 5.793 | 5.776 | – |
| $9/2^+_2$ | 6.098 | 6.032 | – | – |
| $11/2^+_2$ | 6.112 | 6.023 | 6.114 | $(7/2, 9/2, 11/2)^+$ |
| $9/2^+_3$ | 6.198 | 6.090 | – | – |
| $1/2^+_3$ | 6.317 | 6.329 | 6.307 | $1/2^+$ |
| $13/2^+_1$ | 6.320 | 6.223 | – | – |
| $5/2^+_5$ | 6.573 | 6.697 | – | – |

*Continuation of Table 2*

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| $7/2^+_4$ | 6.575 | 6.491 | 6.618 | $(3/2, 5/2, 7/2)^+$ |
| $3/2^+_4$ | 6.771 | 6.785 | 6.735 | $3/2^+$ |
| $5/2^+_6$ | 6.931 | 6.764 | 6.867 | $3/2, 5/2^+$ |
| $5/2^+_7$ | 7.183 | 7.048 | 7.122 | – |
| $9/2^+_4$ | 7.194 | 7.037 | 7.186 | – |
| $13/2^+_2$ | 7.247 | 7.219 | 7.267 | – |
| $11/2^+_3$ | 7.248 | 7.168 | – | – |
| $7/2^+_5$ | 7.322 | 7.407 | – | – |
| $9/2^+_5$ | 7.483 | 7.418 | 7.412 | – |
| $11/2^+_4$ | 7.627 | 7.665 | – | – |
| $1/2^+_4$ | 7.639 | 7.629 | 7.487 | $(1/2, 3/2)^-$ |
| $7/2^+_6$ | 7.823 | 7.589 | 7.834 | $5/2^+, 7/2$ |
| $7/2^+_7$ | 7.939 | 7.789 | 7.876 | – |
| $3/2^+_6$ | 7.966 | 7.879 | – | – |
| $5/2^+_8$ | 7.971 | 7.840 | 7.991 | – |
| $7/2^+_8$ | 7.939 | 8.027 | 8.061 | $5/2^+, 7/2, 9/2^+$ |
| $9/2^+_6$ | 8.169 | 8.155 | 8.149 | – |
| $7/2^+_9$ | 8.170 | 8.541 | 8.173 | – |
| $3/2^+_7$ | 8.280 | 8.353 | 8.220 | – |
| $9/2^+_7$ | 8.290 | 8.396 | 8.261 | – |
| $5/2^+_9$ | 8.426 | 8.512 | 8.475 | $3/2^+, 5/2^+$ |
| $13/2^+_3$ | 8.644 | 8.483 | 8.631 | – |
| $1/2^+_5$ | 8.588 | 8.655 | 8.646 | $1/2^+$ |
| $5/2_10^+$ | 8.759 | 8.728 | 8.721 | – |
| $7/2^+_9$ | 8.077 | 8.541 | 8.797 | – |
| $15/2^+_1$ | 8.928 | 8.889 | – | – |
| $9/2^+_8$ | 8.981 | 8.77 | 8.822 | – |
| $11/2^+_5$ | 9.042 | 8.982 | 9.000 | – |
| $3/2^+_8$ | 9.059 | 9.060 | 9.038 | – |
| $7/2^+_{10}$ | 9.092 | 9.160 | 9.072 | – |
| $11/2^+_6$ | 9.215 | 9.163 | 9.171 | – |
| $1/2^+_6$ | 9.230 | 9.403 | 9.252 | $1/2^+$ |
| $9/2^+_9$ | 9.394 | 9.391 | 9.322 | – |
| $3/2^+_9$ | 9.525 | 9.548 | 9.582 | – |
| $9/2^+_{10}$ | 9.617 | 9.485 | 9.627 | – |
| $3/2^+_{10}$ | 9.689 | 9.866 | 9.652 | $3/2^+$ |
| $13/2^+_4$ | 9.689 | 9.664 | – | – |
| $15/2^+_2$ | 9.703 | 9.586 | 9.738 | – |
| $1/2^+_7$ | 9.892 | 10.138 | 9.850 | $1/2^+$ |
| $13/2^+_5$ | 10.112 | 10.087 | 10.183 | – |
| $11/2^+_7$ | 10.380 | 10.740 | 10.221 | – |
| $1/2^+_8$ | 10.592 | 10.378 | 10.507 | $1/2^+$ |
| $1/2^+_9$ | 10.743 | 10.905 | 10.906 | $(1/2^-)$ |
| $11/2^+_8$ | 10.807 | 10.993 | – | – |

*Continuation of Table 2*

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| $17/2^+_1$ | 11.009 | 10.925 | 11.004 | – |
| $11/2^+_9$ | 11.042 | 11.384 | 11.155 | – |
| $13/2^+_6$ | 11.238 | 11.023 | – | – |
| $13/2^+_7$ | 11.273 | 11.244 | – | – |
| $15/2^+_3$ | 11.318 | 11.260 | – | – |
| $11/2^+_{10}$ | 11.396 | 11.384 | – | – |
| $17/2^+_2$ | 11.517 | 11.528 | – | – |
| $1/2^+_{10}$ | 11.558 | 11.640 | – | – |
| $13/2^+_8$ | 12.016 | 11.764 | – | – |
| $15/2^+_4$ | 12.278 | 12.268 | – | – |
| $15/2^+_5$ | 12.299 | 12.415 | – | – |
| $13/2^+_9$ | 12.458 | 12.391 | – | – |
| $17/2^+_3$ | 12.715 | 12.582 | 12.592 | – |
| $13/2^+_{10}$ | 12.928 | 12.816 | 12.920 | – |
| $15/2^+_6$ | 13.464 | 13.421 | 13.210 | – |
| $17/2^+_4$ | 13.913 | 13.883 | 13.720 | – |
| $15/2^+_7$ | 14.146 | 14.127 | 14.080 | – |
| $15/2^+_8$ | 14.541 | 14.234 | 14.375 | – |
| $15/2^+_9$ | 14.598 | 14.655 | 14.440 | – |
| $19/2^+_1$ | 14.623 | 14.593 | – | – |
| $17/2^+_5$ | 14.771 | 14.766 | – | – |
| $17/2^+_6$ | 15.039 | 14.986 | 14.980 | – |
| $15/2^+_{10}$ | 15.090 | 15.021 | 15.450 | – |
| $19/2^+_2$ | 15.786 | 15.735 | 15.900 | – |
| $17/2^+_7$ | 16.376 | 16.157 | 16.320 | – |
| $21/2^+_1$ | 16.844 | 16.837 | – | – |
| $17/2^+_8$ | 17.081 | 16.665 | 16.600 | – |
| $17/2^+_9$ | 17.156 | 17.029 | – | – |
| $19/2^+_3$ | 17.270 | 17.099 | – | – |
| $17/2^+_{10}$ | 17.455 | 17.330 | – | – |
| $19/2^+_4$ | 17.537 | 17.405 | – | – |
| $19/2^+_5$ | 18.143 | 18.088 | – | – |
| $19/2^+_6$ | 19.144 | 18.862 | – | – |
| $19/2^+_7$ | 19.600 | 19.245 | – | – |
| $19/2^+_8$ | 20.314 | 19.787 | – | – |
| $21/2^+_3$ | 20.545 | 20.042 | – | – |
| $23/2^+_1$ | 21.582 | 21.310 | – | – |
| $21/2^+_4$ | 21.710 | 21.492 | – | – |
| $19/2^+_9$ | 21.815 | 21.509 | – | – |
| $19/2^+_{10}$ | 22.134 | 21.665 | – | – |
| $21/2^+_5$ | 23.567 | 23.365 | – | – |
| $21/2^+_6$ | 24.179 | 23.881 | – | – |
| $21/2^+_7$ | 24.558 | 24.098 | – | – |
| $21/2^+_8$ | 25.118 | 24.668 | – | – |

*End of Table 2*

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| $23/2_2^+$ | 26.347 | 25.455 | – | – |
| $21/2_9^+$ | 27.628 | 26.971 | – | – |
| $21/2_{10}^+$ | 28.131 | 27.700 | – | – |
| $23/2_3^+$ | 29.162 | 28.712 | – | – |

*Table 3.* **Reduced transition probability $B(E2)$ for $^{21}$Na nucleus versus the experimental data**

| Transition levels | Theoretical $B(E2)$, e$^2$ fm$^4$ | | Experimental $B(E2)$, e$^2$ fm$^4$ |
|---|---|---|---|
| | USDA results | USDB results | |
| $5/2_1^+ \to 3/2_1^+$ | 60.79 | 61.64 | 169.344 |
| $7/2_1^+ \to 3/2_1^+$ | 24.85 | 42.08 | 91.185 |
| $1/2_1^+ \to 3/2_1^+$ | 2.462 | 3.027 | – |
| $5/2_2^+ \to 3/2_1^+$ | 1.184 | 0.9352 | 173.686 |
| $3/2_2^+ \to 3/2_1^+$ | 3.190 | 2.461 | – |
| $1/2_2^+ \to 3/2_1^+$ | 5.600 | 5.950 | – |
| $7/2_1^+ \to 5/2_1^+$ | 38.52 | 40.18 | 69.474 |
| $1/2_2^+ \to 5/2_1^+$ | 7.733 | 6.314 | – |
| $9/2_1^+ \to 5/2_1^+$ | 31.46 | 31.86 | – |
| $5/2_2^+ \to 5/2_1^+$ | 3.723 | 3.612 | – |
| $3/2_2^+ \to 5/2_1^+$ | 1.342 | 2.057 | – |
| $7/2_2^+ \to 5/2_1^+$ | 0.5195 | 0.4402 | – |
| $9/2_2^+ \to 5/2_1^+$ | $0.8687\times10^{-4}$ | $0.3721\times10^{-4}$ | – |
| $9/2_1^+ \to 7/2_1^+$ | 20.02 | 19.69 | – |
| $5/2_2^+ \to 7/2_1^+$ | 7.248 | 6.184 | – |
| $11/2_1^+ \to 7/2_1^+$ | 35.35 | 36.01 | – |
| $9/2_2^+ \to 7/2_1^+$ | 0.2637 | 0.3040 | – |
| $7/2_2^+ \to 7/2_1^+$ | 0.01035 | 0.0003364 | – |
| $5/2_2^+ \to 1/2_1^+$ | 18.13 | 16.68 | – |
| $3/2_2^+ \to 1/2_1^+$ | 25.13 | 24.55 | – |
| $5/2_2^+ \to 9/2_1^+$ | 0.05698 | .0.1910 | – |
| $9/2_1^+ \to 11/2_1^+$ | 16.13 | 16.80 | – |
| $7/2_2^+ \to 9/2_1^+$ | 0.1511 | 0.3830 | – |
| $9/2_2^+ \to 9/2_1^+$ | 3.233 | 0.6196 | – |
| $13/2_1^+ \to 9/2_1^+$ | 22.51 | 21.83 | – |
| $7/2_2^+ \to 11/2_1^+$ | 2.181 | 2.829 | – |
| $9/2_2^+ \to 11/2_1^+$ | 2.793 | 0.01232 | – |
| $13/2_2^+ \to 11/2_1^+$ | 7.846 | 7.936 | – |
| $11/2_2^+ \to 11/2_1^+$ | 0.05176 | 0.01123 | – |

*Table 4.* **Reduced transition probability $B(M1)$ for $^{21}$Na nucleus versus the experimental data**

| Transition levels | Theoretical $B(M1)$, $\mu^2$ | | Experimental $B(M1)$, $\mu^2$ |
|---|---|---|---|
| | USDA results | USDB results | |
| $5/2_1^+ \to 3/2_1^+$ | 0.2123 | 0.2110 | 0.151255 |
| $1/2_1^+ \to 3/2_1^+$ | 0.3749 | 0.5078 | – |
| $5/2_2^+ \to 3/2_1^+$ | 0.7654 | 0.8190 | 27.5481 |
| $3/2_2^+ \to 3/2_1^+$ | 0.3467 | 0.4088 | – |
| $1/2_2^+ \to 3/2_1^+$ | 2.062 | 1.735 | 1.6289 |
| $7/2_1^+ \to 5/2_1^+$ | 0.4031 | 0.3856 | 0.358 |
| $5/2_2^+ \to 5/2_1^+$ | 0.05919 | 0.07862 | – |
| $3/2_2^+ \to 5/2_1^+$ | 0.7020 | 0.8900 | – |
| $7/2_2^+ \to 5/2_1^+$ | 0.3808 | 0.3881 | – |
| $9/2_1^+ \to 7/2_1^+$ | 0.5318 | 0.5226 | – |
| $5/2_2^+ \to 7/2_1^+$ | 0.1979 | 0.2191 | – |
| $9/2_2^+ \to 7/2_1^+$ | 0.1439 | 0.4205 | – |
| $7/2_2^+ \to 7/2_1^+$ | 0.3261 | 0.3074 | – |
| $1/2_2^+ \to 1/2_1^+$ | 0.3041 | 0.4111 | – |
| $7/2_2^+ \to 9/2_1^+$ | 0.3419 | 0.3789 | – |
| $9/2_2^+ \to 9/2_1^+$ | 0.3484 | 0.3373 | – |
| $9/2_2^+ \to 11/2_1^+$ | 0.02125 | 0.1971 | – |
| $9/2_1^+ \to 11/2_1^+$ | 0.5287 | 0.5289 | – |
| $13/2_1^+ \to 11/2_1^+$ | 1.133 | 1.153 | – |
| $11/2_2^+ \to 11/2_1^+$ | 3.223 | 0.3779 | – |
| $13/2_2^+ \to 11/2_1^+$ | 0.0003878 | 0.001414 | – |

11.665) MeV} were predicted with spins: $\{1/2_9^+$ and $1/2_{10}^+\}$, respectively. The experimental energies such as {(6.867, 7.834, 8.061, and 8.475) MeV} are determined with spins $\{(5/2_6^+, 7/2_6^+, 7/2_8^+$, and $5/2_9^+)\}$ theoretically. But, in the experimental data, they were undetermined as follows:- $\{(3/2^+, \underline{5/2^+}), (5/2^+, \underline{7/2^+}), (5/2^+, \underline{7/2}, 9/2^+), (3/2^+, \underline{5/2^+})\}$. The theoretical spins $\{5/2_4^+$, $(5/2_7^+$ to $13/2_2^+)$, $9/2_5^+$, $7/2_7^+$, $5/2_8^+$, $(9/2_6^+$ to $9/2_7^+)$, $13/2_3^+$, $5/2_{10}^+$, $7/2_9^+$, $(9/2_8^+$ to $11/2_6^+)$,$(9/2_9^+$ to $9/2_{10}^+)$, $15/2_2^+$, $13/2_5^+$, $11/2_7^+$, $17/2_1^+$, $11/2_9^+$, $13/2_8^+$, $(13/2_9^+$ to $15/2_9^+)$, $(17/2_6$ to $17/2_7$ ), $17/2_8^+\}$ in our calculations had been determined for the following experimental energies: {(5.776, (7.122 to 7.267) 7.412,7.876, 7.991, (8.149 to 8.261), 8.631, 8.721, 8.797, (8.822 to 9.171), (9.322 to 9.627), 9.738, 10.183, 10.221, 11.004, 11.155, 12.050, (12.330 to 14.440), (14.980 to 16.320), 16.600) MeV} which were determined. Theoretically,

*Table 5.* **Reduced transition probability $B(E2)$ for $^{23}$Na nucleus versus the experimental data**

| Transition levels | Theoretical $B(E2)$, $e^2$ fm$^4$ | | Experimental $B(E2)$, $e^2$ fm$^4$ |
|---|---|---|---|
| | USDA results | USDB results | |
| $5/2^+_1 \to 3/2^+_1$ | 79.18 | 80.72 | 93.246 |
| $7/2^+_1 \to 3/2^+_1$ | 31.66 | 31.49 | 49.343 |
| $1/2^+_1 \to 3/2^+_1$ | 2.891 | 2.261 | – |
| $5/2^+_2 \to 3/2^+_1$ | 0.9640 | 1.099 | 2.525 |
| $7/2^+_2 \to 3/2^+_1$ | 0.03133 | 0.006818 | – |
| $1/2^+_2 \to 3/2^+_1$ | 4.343 | 4.253 | – |
| $7/2^+_1 \to 5/2^+_1$ | 40.98 | 43.09 | 64.495 |
| $1/2^+_1 \to 5/2^+_1$ | 19.40 | 20.39 | 11.655 |
| $9/2^+_1 \to 5/2^+_1$ | 49.29 | 50.16 | 64.884 |
| $3/2^+_2 \to 5/2^+_1$ | 4.305 | 3.780 | 6.993 |
| $5/2^+_2 \to 5/2^+_1$ | 0.4375 | 1.099 | – |
| $3/2^+_2 \to 3/2^+_1$ | 8.361 | 10.07 | – |
| $1/2^+_2 \to 5/2^+_1$ | 5.787 | 6.441 | – |
| $5/2^+_3 \to 5/2^+_1$ | 3.977 | 5.625 | 19.426 |
| $7/2^+_2 \to 5/2^+_1$ | 1.153 | 1.577 | – |
| $9/2^+_2 \to 5/2^+_1$ | 1.951 | 6.193 | – |
| $9/2^+_1 \to 7/2^+_1$ | 30.14 | 30.94 | 213.69 |
| $3/2^+_2 \to 7/2^+_1$ | 2.353 | 2.161 | – |
| $7/2^+_1 \to 5/2^+_1$ | 14.07 | 14.04 | – |
| $9/2^+_2 \to 7/2^+_1$ | 0.9859 | 3.422 | – |
| $7/2^+_2 \to 7/2^+_1$ | 3.066 | 3.526 | 2.331 |
| $7/2^+_1 \to 11/2^+_1$ | 43.95 | 42.10 | – |
| $11/2^+_2 \to 7/2^+_1$ | 1.622 | 2.533 | – |
| $3/2^+_2 \to 1/2^+_1$ | 44.23 | 45.12 | – |
| $5/2^+_2 \to 1/2^+_1$ | 40.69 | 40.73 | 89.361 |
| $5/2^+_3 \to 5/2^+_1$ | 3.977 | 5.625 | 19.426 |
| $9/2^+_2 \to 11/2^+_1$ | 6.719 | 0.04996 | – |
| $13/2^+_1 \to 11/2^+_1$ | 6.819 | 6.486 | – |
| $11/2^+_2 \to 11/2^+_1$ | 11.44 | 14.84 | – |
| $13/2^+_2 \to 11/2^+_1$ | 2.861 | 2.826 | – |
| $11/2^+_1 \to 15/2^+_1$ | 26.91 | 25.12 | – |
| $5/2^+_2 \to 9/2^+_1$ | 8.159 | 4.018 | – |
| $7/2^+_2 \to 9/2^+_1$ | 4.778 | 4.521 | – |
| $11/2^+_1 \to 9/2^+_1$ | 15.92 | 15.70 | 15.541 |
| $9/2^+_2 \to 9/2^+_1$ | 3.146 | 1.547 | 2.331 |
| $11/2^+_2 \to 9/2^+_1$ | 1.455 | 1.655 | – |
| $13/2^+_1 \to 9/2^+_1$ | 16.88 | 36.58 | – |
| $13/2^+_2 \to 9/2^+_1$ | 15.95 | 13.50 | – |
| $11/2^+_1 \to 13/2^+_1$ | 7.956 | 7.567 | – |
| $11/2^+_2 \to 13/2^+_1$ | 7.591 | 8.280 | – |
| $13/2^+_2 \to 13/2^+_1$ | 23.65 | 23.01 | – |
| $15/2^+_1 \to 13/2^+_1$ | 0.07493 | 0.04802 | – |

*Table 6.* **Reduced transition probability $B(M1)$ for $^{23}$Na nucleus versus the experimental data**

| Transition levels | Theoretical $B(M1)$, $\mu^2$ | | Experimental $B(M1)$, $\mu^2$ |
|---|---|---|---|
| | USDA results | USDB results | |
| $5/2^+_1 \to 3/2^+_1$ | 0.2882 | 0.2993 | 0.37232 |
| $1/2^+_1 \to 3/2^+_1$ | 0.03059 | 0.03728 | – |
| $3/2^+_2 \to 3/2^+_1$ | 0.2639 | 0.2930 | – |
| $5/2^+_2 \to 3/2^+_1$ | 0.03309 | 0.03482 | 0.05549 |
| $1/2^+_2 \to 3/2^+_1$ | 1.602 | 1.615 | 2.0227 |
| $1/2^+_2 \to 1/2^+_1$ | 1.105 | 1.067 | – |
| $5/2^+_2 \to 3/2^+_2$ | 0.05125 | 0.05977 | 0.13604 |
| $3/2^+_2 \to 5/2^+_1$ | 0.3853 | 0.3743 | 0.3938 |
| $5/2^+_2 \to 5/2^+_1$ | 0.01103 | 0.01038 | – |
| $7/2^+_2 \to 5/2^+_1$ | 0.1501 | 0.1641 | 0.33294 |
| $9/2^+_1 \to 7/2^+_1$ | 0.5192 | 0.5224 | 0.58533 |
| $5/2^+_2 \to 7/2^+_1$ | 0.008343 | 0.01052 | 0.06265 |
| $9/2^+_1 \to 7/2^+_1$ | 0.08201 | 0.001962 | – |
| $7/2^+_1 \to 7/2^+_2$ | 0.3499 | 0.3708 | 0.4117 |
| $7/2^+_2 \to 9/2^+_1$ | 0.3040 | 0.2651 | – |
| $9/2^+_2 \to 9/2^+_1$ | 0.002866 | 0.002537 | 0.02864 |
| $11/2^+_1 \to 9/2^+_1$ | 0.2940 | 0.2912 | 0.2685 |
| $11/2^+_2 \to 9/2^+_1$ | 0.03343 | 0.03798 | – |
| $3/2^+_2 \to 1/2^+_1$ | 0.1198 | 0.1312 | 0.1969 |
| $1/2^+_2 \to 1/2^+_1$ | 1.105 | 1.067 | 1.790 |
| $9/2^+_2 \to 11/2^+_1$ | 0.004707 | 0.0004363 | – |
| $11/2^+_2 \to 11/2^+_1$ | 0.1794 | 0.2128 | – |
| $13/2^+_1 \to 11/2^+_1$ | 0.5764 | 0.6136 | – |
| $13/2^+_2 \to 11/2^+_1$ | 0.02853 | 0.01465 | – |
| $11/2^+_1 \to 13/2^+_1$ | 0.6724 | 0.7159 | – |
| $3/2^+_3 \to 5/2^+_1$ | 0.1865 | 0.2278 | 0.31146 |
| $13/2^+_2 \to 13/2^+_1$ | 0.3253 | 0.3403 | 0.31146 |
| $11/2^+_2 \to 13/2^+_1$ | 0.07864 | 0.03680 | – |

the levels $\{7/2^+_3$, $9/2^+_2$, $9/2^+_3$, $13/2^+_1$, $5/2^+_5$, $11/2^+_3$, $7/2^+_5$, $11/2^+_4$, $3/2^+_6$, ($9/2^+_6$ to $9/2^+_7$), $13/2^+_3$, ($9/2^+_9$ to $9/2^+_{10}$), $13/2^+_4$, $11/2^+_8$, $15/2^+_5$, $19/2^+_1$, $17/2^+_5$, $21/2^+_1$, ($17/2^+_9$ to $23/2^+_3$) with the energy values were expected in our calculations, but these levels are not discovered yet experimentally. Theoretically, we found that the highest energies in the two interactions are $\{(28.712,\ 29.162)$ MeV$\}$ for an angular momentum ($23/2^+_3$). But, experimentally, the highest value is 16.6 MeV. We obtained twenty-two new levels above the experimental levels.

## 4. Reduced Electric Quadrupole $B(E2;\downarrow)$ Values and Dipole Magnetic $B(M1;\downarrow)$ Transition Probability

The transition probability is considered as one of the most sensitive parameters in determining the effective interactions. In order to elucidate this sensitivity, the reduced electric quadrupole transition probability, $B(E2)$, and the reduced magnetic dipole transition probability, $B(M1)$, are used [16]. The transition probability calculation in this present work was carried out by using the harmonic oscillator potential (HO, $b$), where ($b < 0$) for each in-band transition and by applying the (USDA) and (USDB) interactions for $^{21}$Na and $^{23}$Na nuclei at the *sd*-shell. In Table 3, for $^{21}$Na nucleus, the $B(E2)$ values give the acceptable agreement with available experimental data [17]. New values for $B(E2;\downarrow)$ were predicted in our result, where they were not seen in the experimental data.

But, in Table 4, the $B(M1)$ results $\{(5/2^+_1 \to 3/2^+_1), (1/2^+_2 \to 3/2^+_1), (7/2^+_1 \to 5/2^+_1)\}$ are expected at a good agreement with available experimental data [18]. New values of $B(M1)$ are obtained in our calculations, but they were not yet found in the experimental data.

For $^{23}$Na nucleus, the values of $B(E2)$ are presented in Table 5. We note that $\{(5/2^+_1 \to 3/2^+_1), (5/2^+_2 \to 3/2^+_1), (9/2^+_1 \to 5/2^+_1), (11/2^+_1 \to 9/2^+_1)\}$ are in good agreement with available experimental data [18]. New values of $B(E2)$ are obtained in this work as well.

In Table 6, we predict the $B(M1)$ results $\{(5/2^+_1 \to 3/2^+_1),(5/2^+_2 \to 3/2^+_1), (3/2^+_2 \to 5/2^+_1), (9/2^+_1 \to 7/2^+_1), (11/2^+_1 \to 9/2^+_1)\}$ in a relatively good agreement with available experimental data [18]. Moreover, new values of $B(M1)$ are calculated.

## 5. Conclusions

The present theoretical calculations give the values of the energy levels, by using the USDA and USDB interactions for $^{21}$Na and $^{23}$Na nuclei that have a very small nucleon number outside the closed shell. The shell model used in the present work predicts most of the energy levels and their systematics with their total angular momentum. Comparing the experimental results with the shell-model calculations shows that the level structures exhibit mainly the single-particle character.

The most important results of calculations can be summarized as follows:

- The agreement between theoretical and experimental levels is satisfactory for excitation energies and transition probabilities $B(E2;\downarrow)$.
- There are many unconfirmed experimental energy levels given by our calculations and new values for $B(E2;\downarrow)$ not indicated by the experimental data.
- The choice of the *sd* model space and the USDA and USDB effective interactions is adequate in this mass region. The theoretical calculations in the nuclear shell model by using the OXBASH code for windows reasonably agree with the experimental data. This indicates that the shell model is very good to describe the nuclear structure for $^{21}$Na and $^{23}$Na nuclei.

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## РІВНІ З ПОЗИТИВНОЮ ПАРНІСТЮ ІЗОТОПІВ $^{21,23}$Na В МОДЕЛІ ЯДЕРНИХ ОБОЛОНОК

Резюме

Досліджено рівні енергії та ймовірності переходів $B(E2;\downarrow)$ і $B(M1;\downarrow)$ для ізотопів $^{21,23}$Na з використанням USDA і USDB взаємодій в моделі *sd*-оболонки. В розрахунках передбачалося, що всі можливі багатонуклонні конфігурації визначаються $0d_{5/2}$, $1s_{1/2}$ і $0d_{3/2}$ станами над двічі магічним ядром $^{16}$O. Теоретичні передбачення рівнів добре узгоджуються з експериментальними даними. Підтверджено значення спінів і парність для нових рівнів, передбачено ймовірності переходів $B(E2;\downarrow)$ і $B(M1;\downarrow)$.