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EFFECT OF FOURTH-ORDER DISPERSION ON SOLITONIC INTERACTIONS

Solitons became important in optical communication systems thanks to their robust nature. However, the interaction of solitons is considered as a bad effect. To avoid interactions, the obvious solution is to respect the temporal separation between two adjacent solitons determined as a bit rate. Nevertheless, many better solutions exist to decrease the bit rate error. In this context, the aim of our work is to study the possibility to delete the interaction of adjacent solitons, by using a special dispersion management system, precisely by introducing both of the third- and fourth-order dispersions in the presence of a group velocity dispersion. To study the influence of the fourth- and third-order dispersions, we use the famous non-linear Schrödinger equation solved with the Fast Fourier Transform method. The originality of this work is to bring together the dispersion of the fourth, third, and second orders to separate two solitons close enough to create the Kerr-induced interaction and consequently to improve the propagation by decreasing the bit rate error. This study illustrates the influence of the fourth-order dispersion on one single soliton and two co-propagative solitons with different values of the temporal separation. Then the third order dispersion is introduced in the presence of the fourth-order dispersion in the propagation of one and two solitons in order to study its influence on the interaction. Finally, we show the existence of a precise dispersion management system that allows one to avoid the interaction of solitons.

Keywords: interaction of solitons, non-linearity, dispersion, optical fiber, transmission channel, Schrödinger equation.

1. Introduction

In optical communication systems, short impulses are used to transmit the information along an optical fiber. Due to waveguides and the material dispersion of the optical fiber that broadens the propagating pulses, there was a total waste of the information. For a long time, the dispersion was considered as a major problem at high bit rates and for a long-haul optical communication systems [1] till 1973, when Hasegawa and Tappert [2] proposed to offset the natural linear dispersion effect due to Group Velocity Dispersion (GVD) with the non-linear effect called Self-Phase Modulation (SPM).

Subsequently in 1980, Mollenauer and his collaborators [3–7] realized the proposition experimentally and gave birth to stable waves called "solitons". Since then, they have been the subject of many extensive theoretical and experimental studies [8–9], especially for their applications in optical communication systems. In counter to solitary waves whose collision would destroy their original identity, it was found in 1965 by Zabusky and Kruskal [10] that the solitons are a special category of solitary waves, because they behave like particles during the collision, by keeping their amplitude and shape and by preserving the velocity. In the soliton transmission, the necessity of carrying a massive information requires the propagation of multiple solitons at the same time in the so-called "soliton pulse train". As a carrier of information, the soliton is surnamed a bit. By definition, the more the bit rate $B = 1/T_B$ is increased, the better the communication is. But, on the other hand, the less the time interval T_B between two consecutive solitons, the higher the bit rate error. Thus, the necessary temporal separation threshold is crossed, and, consequently, harmful soliton interactions appear. This Gordon–Haus temporal jitter effect [11] has serious consequences for the optical soliton communication systems, because it increases the bit error rate, by limiting significantly the potential of the communication system [6–7].

The propagation of solitons through an optical fiber can be modeled by the famous non-linear Schrödinger (NLS) equation. For optical pulses in the picosecond regime, pulses can be described with the use of the NLS model. But, in the femtosecond regime, the higher-order effects such as the third-order dispersion, fourth-order dispersion, self-steepening, and delayed non-linear response should be taken into account. In this case, the propagation is described by the higher-order nonlinear Schrödinger (HNLS) equation derived by Kodama and Hasegawa [12, 13]. What is common between higher-order effects is that they cause a shift of the fundamental soliton in the time spectrum, by leading to a variation in the travelling speed of the fundamental soliton. The temporal shift is transcribed in the frequency spectrum as the creation of a dispersive wave, a result that was confirmed in several experimental works [5, 15].

Indeed, the NLS equation is a partial differential linear equation, and it is very difficult to solve analytically. Recently, a diversity of exact solutions of the nonlinear equation such as the rogue, lump, breather, kink, and other kinds of solitons are found with the use of many numerical methods among, which we cite the Split Step Fourier Method [16, 17], the Hirota method [18–28], the ansatz method [29–33], the inverse scattering method [34–38], the Darboux transformation based on the Lax pair [39–42], the Backlund transformation [43–48], and so on.

During the last decades, many researches have been accomplished with the aim to minimize the effects of soliton interactions and improve optical soliton communication systems. The obvious solution consists in separating the adjacent solitons by more than six temporal widths of the pulse, but since exceeding the threshold/limits is necessary, one cannot rely on the time interval.

Lately, researchers proposed several solutions to avoid collisions, such as the injection of solitons with the good choice of the phase [49], different amplitudes [50], introduction of the Raman effect [51], introduction of the third-order dispersion [52], and many other propositions. Thus, the suppression of soliton collisions with the use of the gathering of the fourth- and third-order dispersions was never done before.

This paper is divided into the following sections: in Section 2, we review the effect of fourth-order dispersion (FOD) on one soliton [53–57] without the third-order dispersion, and then we add this term in order to study the effect of the gathering GVD, TOD, and FOD on one single soliton. Later on, we study the influence of the fourth-order dispersion alone on two adjacent solitons. Then we introduce the third-order dispersion for a a single soliton, and then we study its influence on two solitons and add the fourth-order dispersion with the aim of deleting the interaction. In Section 3, we make conclusion for the obtained results.

2. Results

2.1. Impact of the fourth-order dispersion

The propagation of solitons by with regard for the second-, third-, and fourth-order dispersions is described by the equation

$$\frac{\partial A(Z,T)}{\partial Z} + \frac{i}{2}\beta_2 \frac{\partial^2 A}{\partial T^2} - \frac{1}{6}\beta_3 \frac{\partial^3 A}{\partial T^3} - \\ -\frac{i}{24}\beta_4 \frac{\partial^4 A}{\partial T^4} = +[|A|^2 A]. \tag{1}$$

The principle of the Split Step Fourier Method, which we use in this paper, is to consider the nonlinear Schrödinger equation that can be written in the following form:

$$\frac{\partial A}{\partial Z} = (\hat{N} + \hat{D})A. \tag{2}$$

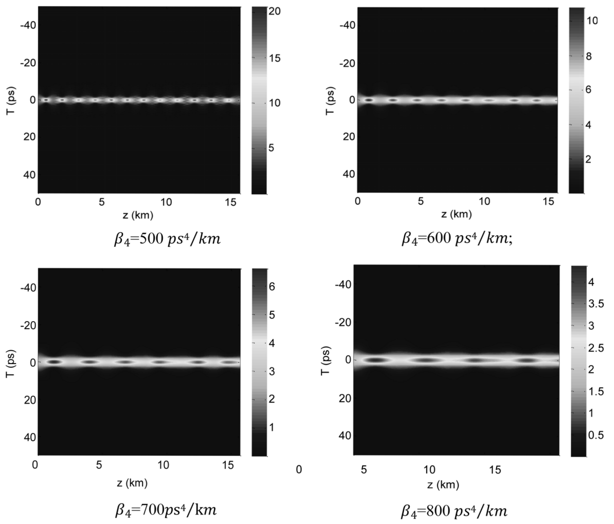


Fig. 1. The effect of FOD on two co-propagative solitons

Here, $\hat{N}$ and $\hat{D}$ are two nonlinear and linear operators, respectively. In view of Eq. (1), the operators can be written as follows:

$$\hat{N} = i\gamma[|A|^2] \tag{3}$$

and

$$\hat{D} = \frac{i}{2}\beta_2\frac{\partial^2}{\partial T^2} - \frac{1}{6}\beta_3\frac{\partial^3}{\partial T^3} - \frac{1}{24}\beta_4\frac{\partial^4}{\partial T^4}. \tag{4}$$

Generally, the dispersion and the nonlinearity act simultaneously along the fiber length. However, the Split Step Fourier method is based on an approximation, which consists in saying that, over very short propagation distances h, the operators $\hat{D}$ and $\hat{N}$ switch. The linear and non-linear effects act independently of one another and alternately. Solving the equation leads to two steps: the dispersive step and the nonlinear one.

The resolution of this equation in the frequency domain is as follows:

$$\hat{A}(z+h,\omega) = A(z,\omega)\exp[h\hat{D}]. \tag{5}$$

Table 1. **Simulation parameters**

Parameter	Values	Units
Non-linear parameter γ	1.0	1/W km
Dispersion of the second order β_2	-25	ps^2/km
Fiber length L	15	km
Width of the impulse T_0	0.5	ps

With the use of the inverse Fourier transform TF^{-1}, one can write the solution in the time domain as follows:

$$A'(z+h,T) = TF^{-1}[\hat{A}(z,\omega)\exp[h\hat{D}(i\omega)]]. \tag{6}$$

The operator $\hat{N}$ is applied in the time domain:

$$A(z+h,T) = A(z,T)\exp(h\hat{N}). \tag{7}$$

Finally, the solution of the Schrödinger equation becomes:

$$A'(z+h,T) = \\ = \exp(hi\gamma[|A|^2])TF^{-1}[A'(z,\omega)\exp[hD'(i\omega)]]. \tag{8}$$

The solution of the above equation is given in [15]:

$$A = [3^*(b^2)^*A_0(\text{sech}^*(T/T_0)^2)]^{*(8/5)\cdot(dz/2)\cdot b^2}, \tag{9}$$

where

$$\delta_4 = \beta_4/(24^*|\beta_2|^*(T_0)^2), \tag{10}$$

and

$$b = 1/(40\cdot\delta_4)^2. \tag{11}$$

The simulation values are presented in Table 1.

2.1.1. The effect of FOD with GVD on a fundamental soliton

The effect of the fourth-order dispersion with the GVD and without TOD is shown in Fig. 1.

Discussion: From the results done by keeping the same distance of propagation and the same value of the second-order dispersion, we see that the fourth-order dispersion induces a periodic compression of the pulse. This period is longer, when the fourth-order dispersion is smaller. We note also that the intensity at the points, where there is a maximum of compression is higher, when the fourth-order dispersion is stronger.

2.1.2. The effect of FOD with GVD and TOD on a fundamental soliton

The effect of the fourth-order dispersion with GVD and TOD is illustrated in Fig. 2 for different values of FOD and keeping GVD and TOD constant.

Discussion: The results show that the FOD with the GVD and TOD create a decay of the fundamental soliton into two pulses after a short propagation. We can see from the figures that the two pulses get shifted the more the FOD is higher, but the solitonic wave is more shifted, than the wave of the decay. Note that the effect of the FOD on the soliton is the same as in Fig. 1: a periodic compression is proportional to FOD.

2.1.3. The effect of FOD with GVD on two fundamental solitons

We now focus on the effect of FOD on two co-propagative solitons. The simulation results are done for two co-propagative solitons with different values of a temporal separation, with the same values of GVD and FOD, and without TOD. The equation used for the following study is the following:

$$A = 3A_0^* \cdot b^{2*}\left[\operatorname{sech}(T/T_0+\tau)^2 + \right.$$
$$\left. + \operatorname{sech}(T/T_0-\tau)^2\right]^* e^{i(8/5)\cdot(dz/2)\cdot b^2}, \quad (12)$$

where

$$\delta_4 = \beta_4/(24^*|\beta_2|^*(T_0)^2), \quad (13)$$

and

$$b = 1/(40 \cdot \delta_4)^2. \quad (14)$$

Discussion: The simulation results show that the solitons split into two pulses with equal intensities after a propagation.

The more we reduce the separation, the more the waves of the decay merge with each other like in Fig. 3. But when the separation is less than $\tau = 2.5$, the attraction becomes very strong, and it leads to a collision between the two solitons before that they split under the attraction induced by FOD (see the low bottom of Fig. 3. Later on, after the attraction, the solitons split each one into two waves. But since there is an attraction between the two waves of the center, it gather them in one pulse, and we get three pulses after the collision.

2.1.4. The effect of FOD with GVD and TOD on two fundamental solitons

The propagation of two solitons in the presence of GVD, TOD, and FOD is illustrated in Table 2.

Discussion: It is shown in the figures that the shift in time induced by TOD creates a separation between the two solitons. We should note that the temporal shift is a function of the value and the sign of TOD, because the more it is higher the more the shift is stronger, but the leading and trailing pulses are not affected by the TOD in the same way. As for the sign of TOD, it affects the side of a shift as it is mentioned in the last section of the present work.

By comparing the rows of Table 2, we see that the shift of the solitons increases, when TOD becomes higher, but the leading and the trailing pulses are not affected in the same way, except for when TOD is equal to 7.5 ps^3/km. Here, the two pulses travel with the same speed and undergo the same shift, so they stay in parallel. Otherwise, if the TOD is positive and differs from 7.5 ps^3/km, the leading pulse is more affected by the TOD, than the trailing pulse, so it travels with a higher speed and catches the trailing pulse at a certain point after the propagation. After the collision, they will be separated again. If the TOD is negative, the trailing pulse is more affected by the

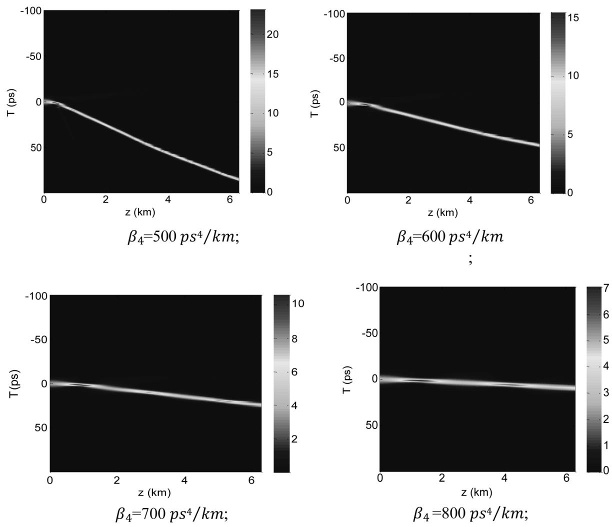


Fig. 2. The effect of FOD in the presence of GVD and TOD on two co-propagative solitons

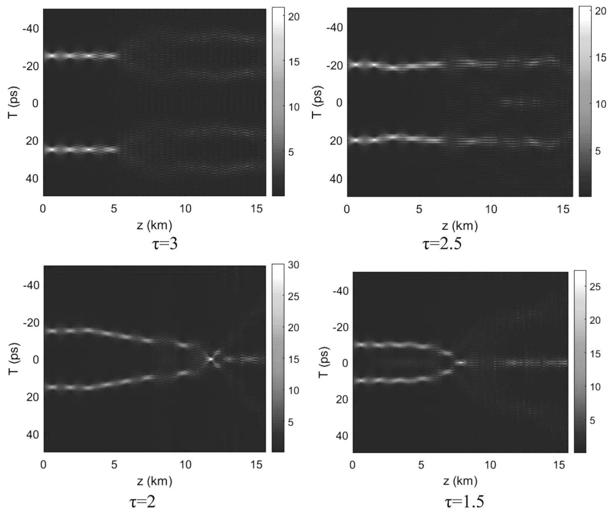
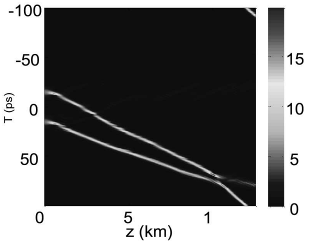
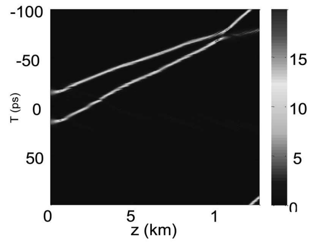
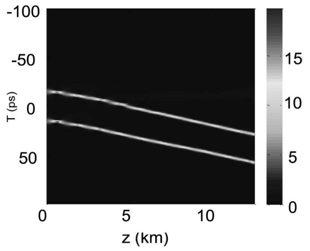
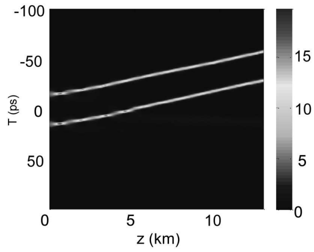
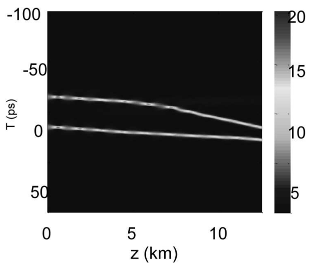
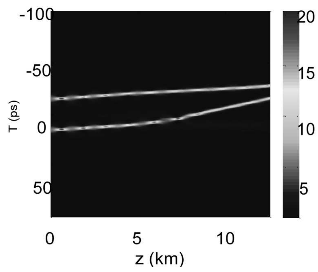


Fig. 3. The effect of FOD on two co-propagative solitons with $\beta_4 = 500$ ps^4/km for different values of the temporal separation

Table 2. **The effect of FOD, TOD, and GVD on co-propagating solitons**

	TOD negative	TOD positive
$\beta_3 = 25$ ps^3/km	 The propagation of two solitons for $\beta_3 = -25$ ps^3/km; $\beta_4 = 500$ ps^4/km; $\beta_2 = -25$ ps^2/km; $\tau = 3$	 The propagation of two solitons for $\beta_3 = 25$ ps^3/km; $\beta_4 = 500$ ps^4/km; $\beta_2 = -25$ ps^2/km; $\tau = 3$
$\beta_3 = 7.5$ ps^3/km	 The propagation of two solitons for $\beta_3 = -7.5$ ps^3/km; $\beta_4 = 500$ ps^4/km; $\beta_2 = -25$ ps^2/km; $\tau = 3$	 The propagation of two solitons for $\beta_3 = 7.5$ ps^3/km; $\beta_4 = 500$ ps^4/km; $\beta_2 = -25$ ps^2/km; $\tau = 3$
$\beta_3 = 3$ ps^3/km	 The propagation of two solitons for $\beta_3 = -3$ ps^3/km; $\beta_4 = 500$ ps^4/km; $\beta_2 = -25$ ps^2/km; $\tau = 3$	 The propagation of two solitons for $\beta_3 = 3$ ps^3/km; $\beta_4 = 500$ ps^4/km; $\beta_2 = -25$ ps^2/km; $\tau = 3$

TOD, than the leading pulse. So, the inverse of the first scenario will occur.

On the other hand, we note that the TOD has an influence on the periodic compression induced by the FOD, and the compression is inversely proportional to the TOD. As a consequence, it is possible to improve the bit rate by means of the good choice of a dispersion management system. The addition of GVD and FOD and the proper value of TOD may be a solution to avoid the interaction of solitons.

3. Conclusion

The effect of FOD, TOD, and GVD all together on the solitonic interactions has been discussed. It is proved that the gathering of FOD, TOD, and GVD is able to delete the collisions, when their values are carefully chosen. By our simulation results, we found out that the FOD adds a periodic compression to the traditional compression induced by the GVD, when it is used for one soliton. In the case where the FOD is higher, the periodic compression is stronger. But if the FOD and GVD are used for two co-propagating solitons, we obtain a considerable attraction between them in addition to the periodic compression. This attraction is proportional to the FOD value, but it is much lower, than the attraction between two solitons owing only to GVD. So, we can say that the FOD decreases the electric attraction induced by the Kerr nonlinearity, but not to the point of canceling it.

By adding the TOD, the dispersion becomes high so that it can defeat the attraction induced by the Kerr nonlinearity between two co-propagating solitons. Nevertheless, another effect appears because of the TOD, which is the temporal shift.

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ВПЛИВ ДИСПЕРСІЇ ЧЕТВЕРТОГО ПОРЯДКУ НА ВЗАЄМОДІЮ СОЛІТОНІВ

Р е з ю м е

Солітони стали важливими для систем оптичного зв'язку завдяки їх стійкості. Однак взаємодія солітонів вважається негативним ефектом. Щоб уникнути цієї взаємодії, очевидним рішенням є забезпечити необхідний часовий інтервал між двома сусідніми солітонами, що визначає швидкість передачі бітів. Однак є кращі можливості знизити рівень помилок при передачі даних. Метою даної роботи є вивчення можливості усунути взаємодію сусідніх солітонів, використовуючи спеціальну систему управління шляхом введення дисперсій як третього, так і четвертого порядку за наявності дисперсії групової швидкості. Для вивчення впливу дисперсій четвертого та третього порядку ми використовуємо відоме нелінійне рівняння Шредінгера, розв'язане методом швидкого фур'є-перетворення. Оригінальність цієї роботи полягає у використанні спільної дії дисперсії четвертого, третього та другого порядків для розділення двох солітонів, досить близьких для виникнення взаємодії за рахунок ефекту Керра. Тим самим покращаться умови розповсюдження солітонів і зменшаться похибки у передачі даних. Дане дослідження ілюструє вплив дисперсії четвертого порядку на одиничний солітон та два солітони, що рухаються в одному напрямку з різними значеннями часового інтервалу між ними. Після цього вводиться дисперсія третього порядку за наявності дисперсії четвертого порядку для вивчення її впливу на взаємодію між солітонами. В результаті ми показуємо існування системи точного управління дисперсією, яка дозволяє уникнути взаємодії солітонів.