https://doi.org/10.15407/ujpe66.9.811

M.E. BOSIN, T.G. DRYGACH, V.M. RUSSKIN

Municipal Institution “Kharkiv Humanitarian and Pedagogical Academy”
of the Kharkiv Regional Council
(7, Rustaveli Lane, Kharkiv, 61001, Ukraine)

MATHEMATICAL GENERALIZATION OF EXPERIMENTAL RESULTS ON THE DEVELOPMENT OF SINGLE TWIN LAYERS IN METAL MATERIALS

A mathematical model has been proposed for the development of single twin layers in metal crystals under various loading regimes and various conditions. The model parameters depend on the geometric characteristics of the twin layer, the physical characteristics of the crystal, the Burgers vector, and the motion velocity of twin dislocations. Methods for the determination of the phenomenological parameters from experimental data were developed. In some cases, a comparison of the parameter values calculated in the framework of the proposed mathematical model with those obtained from experimental data was made, which demonstrated their satisfactory consistency. The proposed model can be useful for the development of a quantitative theory of twinning.

K e y w o r d s: mathematical model, model parameters, twin, twin boundary, dislocation structure, creep mode, active load, pulsating load, alternating load, hardening, Bauschinger effect, forest dislocations, initial conditions.

1. Introduction

The basics of twinning theory were laid in works [1–3]. Further, this theory was somewhat developed in works [4, 5]. As a result, in those works, the dislocation model for the motion of the twin layer boundary was created, the main parameters of twin dislocations were determined, the location of twin dislocations at the twin boundary was described, the relationships for the calculation of the stress, displacement, and deformation fields in a vicinity of the twin boundary were obtained, a physical description was given for a number of phenomena observed near the twin boundaries, the energy of motion of twin dislocations along the twin boundary was described, and the equilibrium condition for the incoherent twin boundary was obtained.

At the same time, there is a huge body of experimental works dealing with the twinning, where a large number of interesting phenomena associated with the behavior of twin layers under various conditions were revealed. In particular, these are the Bauschinger effect, which is observed for zinc, bismuth, and beryllium, as well as alloys on their basis [6–8]; the effect of hardening loss by the twin boundaries under pulsating loads [9–14]; and some features in the behavior of twins at the creeping [15, 16] and under a shock load [17]. Those phenomena and effects were not described in the theoretical works mentioned above, because the principal experimental works dealing with them were published a little later. At the same time, theoretical works [1–5] undoubtedly formed a necessary platform for constructing, firstly, a phenomenological theory and, afterward, a physical one for the behavior of twin layers in crystals, including the described phenomena and effects.

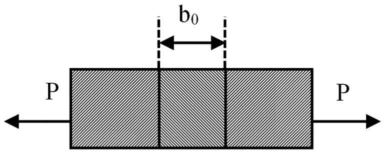

Fig. 1. Particular arrangement of a twin layer in the parent crystal

The aim of this work was to create a phenomenological model for the development of single twin layers in metal crystals in various load modes and to compare the relevant conclusions with experimental creep curves obtained for twin boundaries, in particular, with the behavior of the latter under alternating and pulsating loads. Given that the results of such a comparison are satisfactory, it is of interest to clarify the physical meaning of phenomenological parameters, which is a necessary condition for the phenomenological model to "move" toward the physical theory.

2. Phenomenological Model

Figure 1 illustrates a particular case of the twin layer location in the parent crystal. The particularity consists, first, in the fact that the twin is plane-parallel (a large number of twins are wedge-shaped, and the top of the twin wedge can be inside the crystal); and, second, the twin boundaries can be perpendicular to the facets of the specimen only at a certain orientation of the latter. The initial twin thickness is denoted in Fig. 1 as b_0. After the load P has been applied (as is shown in Fig. 1), the twin thickness increases by Δb. Let us denote the relative thickening of the twin as

$$\varepsilon = \frac{\Delta b}{b_0}. \tag{1}$$

In view of the properties of twin boundaries both to accumulate hardening under the loading and to lose it, i.e. both the deformation irreversibility and reversibility, which is observed at the development of twins [18, 19], we express the relative thickening of the twin in the form

$$\varepsilon = \varepsilon_{\rm e} + \varepsilon_{\rm p}, \tag{2}$$

where $\varepsilon_{\rm e}$ is the elastic part of the thickening, which disappears after the stress removal, and $\varepsilon_{\rm p}$ is the plastic (irreversible) part. For $\varepsilon_{\rm e}$, by analogy with Hooke's law, we write

$$\varepsilon_{\rm e} = \frac{\sigma}{G_2}, \tag{3}$$

where G_2 plays the role of the elastic modulus for a single twin. For $\varepsilon_{\rm p}$, using the idea of "viscous resistance forces" resulting in the irreversibility of the twin boundary displacement and by analogy with the well-known Newton's law (the strain of the resistance to a plastic shear is proportional to the strain rate), we may write that

$$\frac{d\varepsilon_{\rm p}}{dt} \equiv \dot{\varepsilon}_{\rm p} = \frac{\sigma}{\eta}, \tag{4}$$

where η is a parameter with the dimension of the ordinary viscosity coefficient. As the twin grows, the value of this parameter changes owing to the variation of the dislocation structure between the twin boundaries and, as a result, the variation of the "viscous resistance" forces. Therefore,

$$\eta = \eta(\varepsilon_{\rm p}). \tag{5}$$

Let us expand function (5) in the Maclaurin series and temporarily confine it to its linear part,

$$\eta\,(\varepsilon_{\rm p}) \approx \eta_0 + \frac{d\eta}{d\varepsilon_{\rm p}}\varepsilon_{\rm p}, \tag{6}$$

where η_0 is the value of η at $\varepsilon_{\rm p} = 0$. From Eqs. (6) and (4), we find

$$\sigma = \left(\eta_0 + \frac{d\eta}{d\varepsilon_{\rm p}}\varepsilon_{\rm p}\right)\dot{\varepsilon}_{\rm p} = \eta_0\dot{\varepsilon}_{\rm p} + \frac{d\eta}{d\varepsilon_{\rm p}}\frac{d\varepsilon_{\rm p}}{dt}\varepsilon_{\rm p} = \\ = \eta_0\dot{\varepsilon}_{\rm p} + \dot{\eta}\varepsilon_p. \tag{7}$$

The parameter η has the dimension of the elastic modulus; so, let us introduce the notation

$$\dot{\eta} = G_1. \tag{8}$$

From Eqs. (2), (3), (7), and (8), we obtain the following differential equation that couples σ and ε:

$$\sigma + \tau_\varepsilon\dot{\sigma} = G(\varepsilon + \tau_\sigma\dot{\varepsilon}), \tag{9}$$

where

$$\tau_\varepsilon = \frac{\eta}{G_1 + G_2}, \quad G = \frac{G_1G_2}{G_1 + G_2}, \quad \tau_\sigma = \frac{\eta_0}{G_1}. \tag{10}$$

Equation (9) has a form similar to the equation for a "standard linear body". The quantity G in this model is the "relaxed elastic modulus"; and τ_ε and τ_σ are the stress relaxation time at a constant deformation and the deformation relaxation time at a constant stress, respectively.

By solving Eq. (9) for a given $\sigma(t)$- or $\varepsilon(t)$-dependence, we obtain the other function. Consider the $\sigma(t)$- and $\varepsilon(t)$-curves. Formally substituting $t \to t(\sigma)$ or $t \to t(\varepsilon)$ in the equation, we can plot the $\sigma - \varepsilon$ diagrams. Comparing these diagrams with the experimental $\sigma(\varepsilon)$-curves, we can determine the parameters G_1, G_2, and η_0 (see below).

Equation (9) is an approximate model of the twin development process. To improve the accuracy, we can account for the next (quadratic) term in the expansion of $\eta(\varepsilon_\mathrm{p})$ in the power series of ε_p. Then, we obtain the formula

$$\eta\left(\varepsilon_\mathrm{p}\right) \approx \eta_0 + \frac{d\eta}{d\varepsilon_\mathrm{p}}\,\varepsilon_\mathrm{p} + \frac{1}{2}\,\frac{d^2\eta}{d\varepsilon_\mathrm{p}^2}\,\varepsilon_\mathrm{p}^2 \tag{11}$$

instead of Eq. (6), and the equation

$$\sigma = \eta_0\left(\dot\varepsilon - \frac{\dot\sigma}{G_2}\right) + G_1\left(\varepsilon - \frac{\sigma}{G_2}\right) + \\ + \frac{1}{2}\,\frac{\left(\varepsilon - \frac{\sigma}{G_2}\right)^2}{\left(\dot\varepsilon - \frac{\dot\sigma}{G_2}\right)^2}\left[\xi\left(\dot\varepsilon - \frac{\dot\sigma}{G_2}\right) - G_1\left(\ddot\varepsilon - \frac{\ddot\sigma}{G_2}\right)\right] \tag{12}$$

instead of Eq. (9). Here, ξ is a new parameter. Its sense, as well as the content of other parameters, can be established by studying its dependence on various factors (the temperature, the load rate and mode, the structure, and so forth).

In what follows, we will solve Eq. (9) – and, in some cases, Eq. (12) – for those loading modes that took place in experiments. At the first stage, we will compare the model and experimental results in order to verify the phenomenological model as a whole. The elucidation of the physical meaning of phenomenological parameters will be done in a separate work, because there is a lot of materials whose analysis will exceed the reasonable volume of this article.

3. Creep Mode

In the creep mode, $\sigma = \mathrm{const}$. Then, Eq. (9) becomes simpler,

$$\dot\varepsilon + \frac{1}{\tau_\sigma}\,\varepsilon = \frac{\sigma}{G\tau_\sigma}. \tag{13}$$

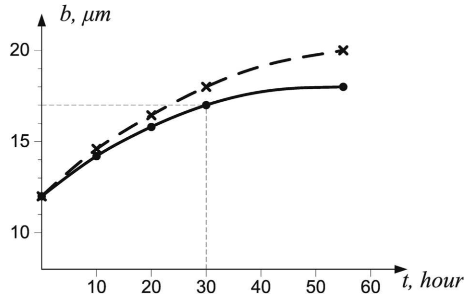

Fig. 2. Creep curves for twin boundaries in Zn crystals. The solid curve was obtained experimentally, and the dashed curve is theoretical (formula (14))

The solution of this inhomogeneous linear differential equation satisfying the initial condition $\varepsilon\,(t = 0) = 0$ looks like

$$\varepsilon = \frac{\sigma}{G}\left(1 - e^{-\frac{t}{\tau_\sigma}}\right). \tag{14}$$

Of course, it is of great interest to compare formula (14) with experimental creep curves [15, 16]. Such a comparison for zinc is presented in Fig. 2, where the experimental curve for the dependence of the twin thickness b on the creep time t of twin boundaries is shown in the case where the shear stress in the twinning plane in the twinning direction equals $\sigma = 0.75$ kgf/mm^2 [15] together with the theoretical curve [14] plotted for the parameter values $\tau_\sigma = 49.4$ h and $G = 0.69$ kgf/mm^2.

For every experimental curve $\varepsilon\,(t)$, one can select such G- and τ_σ-values for which the experimental curve will be very close to that described by Eq. (14). By changing the conditions – e.g., the density and type of forest dislocations in the crystal, the temperature, stress, and so forth – it is possible to determine the sensitivity of those parameters to every condition. By performing a complex comparison (considering the results obtained for other loading modes) of conclusions given by the mathematical model with experimental results (obtained under various conditions), the physical meaning of phenomenological parameters can be established. If the theoretical conclusions substantially differ from the experiment even at the optimal choice of parameter values, the more exact equation (12) can be used. In particular, in the case of creeping ($\sigma = \mathrm{const}$), the corresponding equation has the form

$$\ddot\varepsilon = \frac{2\dot\varepsilon^2\left(\eta_0\dot\varepsilon - \sigma\right)}{G_1\left(\varepsilon - \frac{\sigma}{G_2}\right)^2} + \frac{2\dot\varepsilon^2}{\varepsilon - \frac{\sigma}{G_2}} + \frac{\xi}{G_1}\,\dot\varepsilon. \tag{15}$$

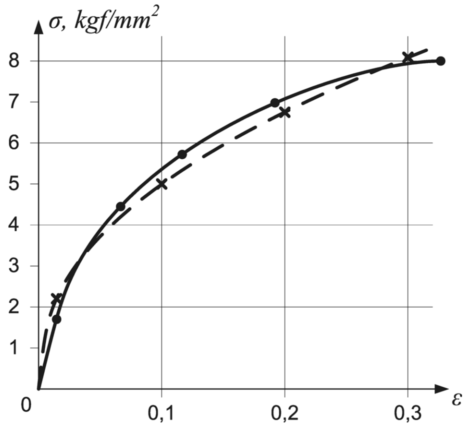

Fig. 3. Experimental (solid) and theoretical (dashed) curves of hardening for a single twin in bismuth

This equation does not contain the time t explicitly. Therefore, its order can be reduced and, if necessary, this equation can be solved without particular difficulties.

4. Active Loading Mode with $\dot{\sigma}$ = const

Let $\dot{\sigma} = c = \text{const}$, i.e.

$$\sigma = ct. \tag{16}$$

Then, Eq. (9) takes the form

$$\dot{\varepsilon} + \frac{1}{\tau_\sigma}\,\varepsilon = \frac{c}{G\tau_\sigma}\,(t + \tau_\varepsilon). \tag{17}$$

The solution of this equation satisfying the initial condition $\varepsilon\,(t = 0) = 0$ is the function

$$\varepsilon = \frac{c}{G}\,(\tau_\varepsilon - \tau_\sigma)\left(1 - e^{-\frac{t}{\tau_\sigma}}\right) + \frac{c}{G}\,t. \tag{18}$$

The relaxation times τ_ε and τ_σ in metals are most often related by the inequality $\tau_\varepsilon \ll \tau_\sigma$ so that $G_2 \gg G_1$. If the load duration $t \ll \tau_\sigma$, then, from Eq. (18), we obtain

$$\varepsilon = \frac{c}{2\eta_0}\,t^2. \tag{19}$$

This model result can be compared with available experimental curves [6, 20] (see Fig. 3). The solid curve demonstrates the experimental dependence $\sigma\,(\varepsilon)$ for a single twin taken from work [6], whereas the dashed curve shows the same dependence calculated using Eq. (19) after the substitution of Eq. (16) into it. The corresponding loading rate was taken from the experimental data ($c = 0.09$ kgf/(mm^2 min)), and the parameter value $\eta_0 = 1220$ kgf min was fitted to provide a satisfactory agreement between the experimental and theoretical dependences.

Substituting Eq. (16) into the more accurate Eq. (12), we obtain

$$\ddot{\varepsilon} = 2\frac{\left(\dot{\varepsilon} - \frac{c}{G_2}\right)^2}{\left(\varepsilon - \frac{c}{G_2}t\right)^2}\left[\frac{\eta_0}{G_1}\left(\dot{\varepsilon} - \frac{c}{G_2}\right) + \left(\varepsilon - \frac{c}{G_2}t\right) - \frac{c}{G_1}t\right] + \frac{\xi}{G_1}\left(\dot{\varepsilon} - \frac{c}{G_2}\right). \tag{20}$$

By introducing the auxiliary function $\varphi\,(t) = \varepsilon - \frac{c}{G_2}\,t$, Eq. (20) can be rewritten in the form

$$\ddot{\varphi} = 2\left(\frac{\dot{\varphi}}{\varphi}\right)^2\left(\frac{\eta_0}{G_1}\,\dot{\varphi} + \varphi - \frac{c}{G_1}\,t\right) + \frac{\xi}{G_1}\,\dot{\varphi}. \tag{21}$$

Recall that $\frac{c}{G_2}t = \frac{\sigma}{G_2}$ is the elastic part of a deformation. Then, the function $\varphi\,(t)$ has the meaning of the irreversible deformation part, $\varepsilon_{\text{p}} = \varepsilon - \varepsilon_{\text{e}}$.

5. Active Load Mode with $\dot{\varepsilon}$ = const

Let $\varepsilon = kt$, i.e. the rate of deformation change is equal to

$$\dot{\varepsilon} = k. \tag{22}$$

In this case, Eq. (9) takes the form

$$\dot{\sigma} + \frac{1}{\tau_\varepsilon}\,\sigma = \frac{\text{kG}}{\tau_\varepsilon}\,(t + \tau_\sigma). \tag{23}$$

The solution of this equation satisfying the initial condition $\sigma\,(t = 0) = 0$ looks like

$$\sigma = kGt + kG\,(\tau_\sigma - \tau_\varepsilon)\left(1 - e^{-\frac{t}{\tau_\varepsilon}}\right). \tag{24}$$

It can be simplified by expanding the exponential function in the Maclaurin series and keeping 2 to 3 terms in the expansion. But for this purpose, the inequality $t \ll \tau_\varepsilon$ is required, which must be provided by experimental conditions.

Now, substituting Eq. (22) into Eq. (12), we obtain

$$\ddot{\sigma} = 2\frac{G_2}{G_1}\frac{\left(k - \frac{\dot{\sigma}}{G_2}\right)^2}{\left(kt - \frac{\sigma}{G_2}\right)^2}\left[\sigma - \eta_0\left(k - \frac{\dot{\sigma}}{G_2}\right) - G_1\left(kt - \frac{\sigma}{G_2}\right)\right] - \frac{G_2}{G_1}\left(k - \frac{\dot{\sigma}}{G_2}\right)\xi. \tag{25}$$

Let us introduce the auxiliary function

$$\psi\,(t) = kt - \frac{\sigma}{G_2}. \qquad (26)$$

It has the same meaning as the function $\varphi\,(t)$ introduced above. Namely, this is a difference between the total deformation and its elastic part. But since the plastic deformation behaves itself differently in the modes $\dot{\sigma} = \mathrm{const}$ and $\dot{\varepsilon} = \mathrm{const}$, the functions $\psi\,(t)$ and $\varphi\,(t)$ are substantially different. Taking Eq. (26) into account, Eq. (25) can be written as follows:

$$\ddot{\psi} = 2\frac{\dot{\psi}^2}{\psi^2}\left(\frac{\eta_0}{G_1}\,\dot{\psi} + \frac{G_1 + G_2}{G_1}\,\psi - \frac{G_2}{G_1}\,kt\right). \qquad (27)$$

If function (24) agrees badly with the experiment, then Eq. (27) has to be solved with the initial condition $\sigma\,(t = 0) = 0$ or, equivalently, $\psi\,(t = 0) = 0$. The resulting dependence $\sigma(t)$ must be compared with the corresponding experimental curve. This comparison, firstly, gives information about the effectiveness of the model and, secondly, makes it possible to determine the parameters G_1, G_2, η_0, and ξ.

6. Pulsating Load Mode

For pulsations, let us choose the mode with $\dot{\sigma} = \mathrm{const} = c$ at the loading and $\dot{\sigma} = \mathrm{const} = c'$ at the unloading. As it was in experiments [9–13], let the loading rate be much lower than the unloading one: $c \ll c'$. The amplitude of the stress pulsations is denoted as σ_0. By solving Eq. (9) sequentially – first, for the loading with the rate $\sigma = c$; then, for the unloading with the rate $\sigma = c'$; then, for the reloading with the same rate c and the unloading with the same rate c', and so forth – we obtain a diagram similar to that obtained experimentally in work [20]: in every cycle, some shift of the twin boundaries is observed, which decreases as the number of cycles increases. For every repeated cycle, the relative displacement of the twin boundaries is equal to

$$\beta_k = \varepsilon' \exp\left(-k\frac{G_1\sigma_0}{\eta_0 c}\right), \qquad (28)$$

where $\varepsilon' = \varepsilon_0 - \frac{\sigma_0}{G_2}$ is the relative shift of twin boundaries in the first loading cycle minus the elastic part $\frac{\sigma_0}{G_2}$. We emphasize that, in essence, this phenomenon is a result of the hardening loss by the twin boundaries at stress pulsations, which was studied experimentally [9–13]. Furthermore, the fact that the proposed phenomenological model contains this effect,

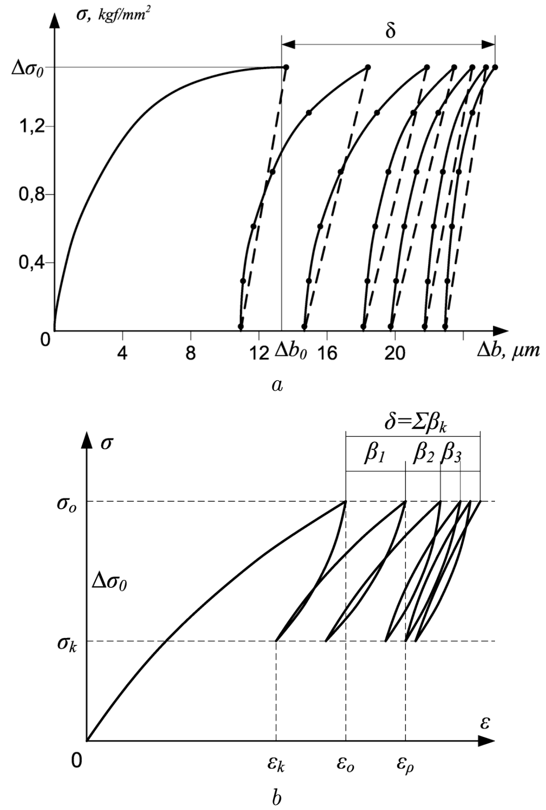

Fig. 4. Model dependence $\sigma\,(\varepsilon)$ for single twins (a). Diagram "stress versus the displacement of twin boundaries" for the pulsating loading on a bismuth single crystal (b). The loading rate is 0.044 kgf/(mm$^2\cdot$ min), the time interval between the cycles is 10 min

speaks in favor of the model. Experiments [9–12] testify that the most stable characteristic of this effect (the loss and subsequent recovery of the hardening by the twin boundaries in the course of stress pulsations) is the quantity $\delta = \sum \beta_k$. This series converges, and the sum can be easily calculated:

$$\delta = \sum_{k=1}^{\infty} \beta_k = \varepsilon' \frac{1}{\exp\left(\frac{G_1\sigma_0}{\eta_0 c}\right) - 1}. \qquad (29)$$

The result obtained agrees satisfactorily with the experiment. Figure 4 illustrates the described effect as one of the model conclusions (panel a) and as the experimental fact (panel b)[10].

According to Eq. (29), the quantity δ should increase with the growth of c, if σ_0 is fixed, and with the decrease of σ_0, if c is fixed. If the unloading and the next loading are separated by the time T (the

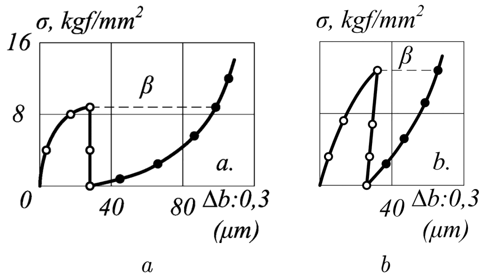

Fig. 5. Diagrams “stress versus the displacement of twin boundaries” for the Zn + 0.6%Cu alloy with various concentrations of pyramidal dislocations: 10^3 (a) and 10^7 cm^{-2} (b)

rest time), then the value of ε' – as well as the values of ε'', ε''', ... after the second, third, and so forth loading cycles – will be smaller owing to the deformation relaxation after the complete unloading of the specimen.

7. Alternating Load Mode

Let us apply Eq. (9) to consider the behavior of twin boundaries under the alternating load of the specimen. In contrast to the case analyzed above, let the second loading be performed with the rate $\dot{\sigma} = -c$, i.e. the loading with the opposite sign, $\sigma = -ct$, is applied (the time is reckoned from the reloading beginning). For the initial condition $\varepsilon\,(t = 0) = \varepsilon$, we obtain

$$\varepsilon = \left[\varepsilon' + \frac{c}{G}\left(\tau_\varepsilon - \tau_\sigma\right)\right] e^{-\frac{t}{\tau_\sigma}} + \frac{c}{G}\left(\tau_\sigma - \tau_\varepsilon - t\right). \qquad (30)$$

At the time $t = \frac{\sigma_0}{c} \equiv t_0$, the stress $-\sigma_0$ will be attained, as well as the strain ε_2, which can be calculated by substituting $t = \frac{\sigma_0}{c}$ into expression (30).

If plotting the curve $\varepsilon\,(\sigma)$ at the opposite-sign loading in the positive direction of coordinate axes, as is done to illustrate the Bauschinger effect [6, 7], then the deformation value will be

$$\varepsilon_2' = 2\varepsilon' - \varepsilon_2 \qquad (31)$$

at $\sigma = \sigma_0$.

As the magnitude of Bauschinger effect, the difference between the reloading curves in the forward and reverse directions at $|\sigma| = \sigma_0$ is usually adopted. Denoting it as β_B, we obtain

$$\beta_B = 2\varepsilon'\left(1 - e^{-\frac{t_0}{\tau_\sigma}}\right). \qquad (32)$$

Making the substitution $t_0 = \frac{\sigma_0}{c}$, we can write

$$\beta_B = 2\varepsilon'\left(1 - e^{-\frac{\sigma_0}{c\tau_\sigma}}\right). \qquad (33)$$

Expressions (32) and (33) make it possible to estimate the magnitude of Bauschinger effect at twinning in the framework of the proposed phenomenological consideration.

To simplify the calculations, let us consider the limiting case where $\tau_\varepsilon \ll \tau_\sigma$ and $t \ll \tau_\sigma$. Then, from Eq. (32), we find

$$\beta_B \approx 2\varepsilon'\frac{G_1}{\eta_0}t_0, \qquad (34)$$

and, from Eq. (19), we have

$$t_0 \approx \sqrt{\frac{2\eta_0\varepsilon_0}{c}}. \qquad (35)$$

Substituting Eq. (35) into Eq. (34), we obtain

$$\beta_B \approx 2\varepsilon' G_1\sqrt{\frac{2\varepsilon_0}{c\eta_0}}. \qquad (36)$$

If $\varepsilon' \approx \varepsilon_0$ (since $\varepsilon_\mathrm{e} \ll \varepsilon_\mathrm{p}$ and $\varepsilon_\mathrm{e} \ll \varepsilon$ at $G_2 \gg G_1$), we can write, instead of Eq. (36), that

$$\beta_B \approx 2\sqrt{2}\frac{G_1}{\sqrt{\eta_0 c}}\,\varepsilon_0^{3/2}. \qquad (37)$$

The approximate formula (37) is more convenient while analyzing the dependences of β_B on various factors.

Thus, the proposed phenomenological model also involves the Bauschinger effect observed for single twins in various metal crystals [6–8]. This circumstance can help one in understanding the meaning of phenomenological parameters, because the magnitude of Bauschinger effect turned out sensitive to the density of forest dislocations in crystals (see Fig. 5).

8. Determination of Phenomenological Parameters

Hence, in the framework of the proposed phenomenological consideration, the mechanical behavior of twin boundaries at various loading regimes can be described with the help of three parameters: G_1, G_2, and η_0 (the parameter ξ appears, if passing from

Eq. (9) to Eq. (12)). Those parameters are determined by the characteristics of twin boundaries themselves, as well as by the defect structure of their boundaries. Therefore, they must depend on the incoherence degree of twin boundaries, the type and density of forest dislocations in crystals, the presence of point-defect clusters, and so on.

The easiest way to determine the parameter η_0 is to compare the experimental dependence $\varepsilon(T)$ measured during the loading period with the approximate formula (19), which includes this parameter only. The corresponding procedure showed that the parameter η_0 is sensitive to the density of forest dislocations in crystals. In particular, when the initial density of a pyramidal forest in Zn crystals increased from 10^3 to 10^7 cm^{-2}, the η_0-value grew from 9×10^3 to 4.5×10^4 MPa$\cdot$s. This fact makes it possible to suggest that the viscous deceleration of twin boundaries is associated to a great extent with the presence of forest dislocations in the crystal.

Knowing the value of the parameter η_0, the parameter G_1 can be found from the measured values of the quantity δ or β_B making use of formulas (29) and (36). The order of magnitude of the parameter G_1 is about 10 MPa for twins in Bi and Zn crystals.

Finally, the parameter G_2 can be easily estimated using the reverse shift of twin boundaries after the crystal unloading, i.e. on the basis of the value for the elastic part of the relative deformation, ε_e, via formula (3). The order of magnitude for G_2 is about 10^2 MPa for Bi and about 10^3 MPa for Zn, which confirms the condition $G_2 \gg G_1$ used above.

Thus, the proposed model can predict the behavior of twin boundaries at various loading modes. In particular, it demonstrates the effect of loss and subsequent recovery of the hardening at pulsating loads, as well as the Bauschinger effect described in the literature, which manifests itself in the case of alternating loads.

9. Conclusions

The phenomenological model proposed in this work for the development of single twin layers in metal crystals can satisfactorily describe the following phenomena:

- the creep of twin boundaries;
- the motion of twin boundaries in the active load mode, in particular, at $\dot{\sigma} = \text{const}$;
- the loss and subsequent recovery of the hardening by the twin boundaries under pulsating loads;
- the Bauschinger effect at the twin boundaries.

As a result, this model can be useful while creating a quantitative theory of twinning. The next step in this direction undoubtedly consists in elucidating the meaning of phenomenological model parameters on the basis of dislocation concepts created in theoretical works dealing with the twinning [1–5]. In this work, the procedure aimed at determining the values of the phenomenological model parameters on the basis of experimental data was also described.

The authors express their sincere gratitude to Prof. I.I. Papirov for useful discussion.

1. I.M. Lifshits. On the macroscopic description of the crystal twinning phenomenon. *Zh. Èksp. Teor. Fiz.* **18**, 1134 (1948) (in Russian).
2. A.H. Cotrell, B.A. Bilby. A mechanism for the growth of deformation twins in crystals. *Phil. Mag.* **42**, 573 (1951).
3. A.M. Kosevich, L.A. Pastur. On the dislocation model of twinning. *Fiz. Tverd. Tela* **3**, 1291 (1961) (in Russian).
4. O. M. Ostrikov. Dislocation macroscopic model of a wedge-shaped twin. *Vesti P.O. Sukhoi GGTU* **2**, 10 (2006) (in Russian).
5. O. M. Ostrikov. Account of the shape of the wedge-shaped twin boundaries in the macroscopic dislocation model. *Fiz. Met. Metalloved.* **106**, 471 (2008) (in Russian).
6. V.I. Bashmakov, M.E. Bosin, I.I. Papirov. Cyclic Bauschinger effect at the boundaries of mechanical beryllium twins. *Fiz. Met. Metalloved.* **35**, 220 (1973) (in Russian).
7. V.I. Bashmakov, M.E. Bosin. Influence of forest dislocations on the Bauschinger effect at the twinning of zinc single crystals. *Izv. Vyssh. Ucheb. Zaved. Fiz.* **5**, 101 (1971) (in Russian).
8. M.E. Bosin, I.I. Papirov, N.G. Yakovenko. Investigation of twinning processes in beryllium and its alloys with copper. *Izv. Vyssh. Ucheb. Zaved. Fiz.* **4**, 120 (1975) (in Russian).
9. V.I. Bashmakov, M.E. Bosin. Softening of twin boundaries in bismuth and zinc crystals under repeated loadings. *Ukr. J. Phys.* **15**, 1036 (1971) (in Russian).
10. V.I. Bashmakov, M.E. Bosin. Properties of single twins in metal crystals under pulsating loadings. *Fiz. Tverd. Tela* **13**, 2475 (1971) (in Russian).
11. V.I. Bashmakov, M.E. Bosin, F.F. Lavrentiev, I.I. Papirov. Doping effect on strain hardening of twin layers in bismuth and beryllium crystals under stress pulsations. *Probl. Prochn.* **1**, 80 (1974) (in Russian).
12. V.I. Bashmakov, M.E. Bosin, S.P. Shinkarenko. Dependence of the mobility of incoherent twin boundaries on the amplitude of pulsating stress. *Ukr. J. Phys.* **18**, 1021 (1973) (in Ukrainian).

13. F.F. Lavrentev, M.E. Bosin. The peculiar effect of forest dislocations on single twin layer development in zinc and beryllium single crystals. *Mater. Sci. Eng.* **33**, 243 (1978).
14. M.E. Bosin. Specific features of twin boundaries motion in beryllium single crystals at the twinning and untwining. *Funct. Mater.* **4**, 557 (1997).
15. V.I. Bashmakov, M.E. Bosin, M.M. Brodskii. Boundary creeping of single twin layers in zinc crystals. *Kristallografiya* **19**, 592 (1974) (in Russian).
16. M.E. Bosin, P.L. Pakhomov. On the mechanism of the influence of the density of forest dislocations on the mobility of twin boundaries in Zn crystals. *Ukr. J. Phys.* **27**, 372 (1982) (in Russian).
17. V.N. Nikiforenko, M.E. Bosin, V.G. Korchakov, N.V. Korchakov. Mechanisms of the dislocation motion and interaction in the field of high-speed stresses. In: *Proceedings of the 3rd International Conference "Physical Phenomena in Solids", Kharkiv, January 21–23, 1997.* Edited by V.V. Ulyanov (1997), p. 144 (in Russian).
18. M.E. Bosin, F.F. Lavrentiev, V.N. Nikiforenko. Features of the motion of twin boundaries in beryllium single crystals under cyclic alternating loading. In: *Proceedings of the 3rd International Conference "Physical Phenomena in Solids", Kharkiv, January 21–23, 1997.* Edited by V.V. Ulyanov (1997), p. 193 (in Russian).
19. M.E. Bosin, F.F. Lavrentiev, V.N. Nikiforenko. Pair interation of dislocations and twin boundary motion in bismuth single crystals under alternating cyclic loading. *Funct. Mater.* **6**, 60 (1999).
20. V.I. Bashmakov, M.E. Bosin, S.P. Shinkarenko. Single twins and brittle destruction of metal crystals. *Probl. Prochn.* **12**, 44 (1973) (in Russian).

Received 14.05.20.
Translated from Ukrainian by O.I. Voitenko

М.Є. Босін, Т.Г. Дригач, В.М. Русскін

МАТЕМАТИЧНЕ УЗАГАЛЬНЕННЯ РЕЗУЛЬТАТІВ ЕКСПЕРИМЕНТІВ ЗІ СПОСТЕРЕЖЕННЯ РОЗВИТКУ ОДИНИЧНИХ ДВІЙНИКОВИХ ПРОШАРКІВ У МЕТАЛЕВИХ МАТЕРІАЛАХ

Запропоновано математичну модель розвитку одиничних двійникових прошарків у металевих кристалах при різних режимах навантаження, за різних умов. Параметри моделі залежать від геометричних характеристик двійникового прошарку, фізичних характеристик кристала, вектора Бюргерса та швидкості руху двійникових дислокацій. Розроблено методики відновлення феноменологічних параметрів з експериментальних даних. У низці випадків проведено порівняння значень параметрів, що обчислено згідно з запропонованою математичною моделлю, з тими, які отримано з даних експерименту. Порівняння показує задовільну узгодженість. Запропонована модель може бути корисною в створенні кількісної теорії двійникування.

Ключові слова: модель, параметри моделі, двійник, двійникова межа, дислокаційна структура, рівняння, режим повзучості, активне навантаження, пульсуючі навантаження, знакозмінні навантаження, зміцнення, ефект Баушингера, дислокації лісу, початкові умови.