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INSTABILITY OF A TUBULAR ELECTRON BEAM BLOWING AROUND A PLASMA SOLID-STATE CYLINDER LOCATED IN A STRONG LONGITUDINAL MAGNETIC FIELD

An electrodynamic system, where a magnetized tubular electron beam blows around a cylindrical solid-state plasma waveguide, has been theoretically studied. It is established that the hybrid bulk-surface or surface electromagnetic waves of the helicon origin are excited in the waveguide, if quasi-stationary conditions are satisfied. The waveguide eigenwaves are excited by the beam space-charge field with the matching of the longitudinal spectral components of the electric field. The non-reciprocity effect is pointed out between the waveguide eigenwaves with the structurally identical field distributions but different azimuthal directions of propagation, as well as if the direction of the external magnetic field changes. It is shown that the instability of coupled waves of the electrodynamic system takes place due to the Vavilov–Cherenkov effect.

Keywords: electron beam, space-charge wave, eigenwaves, coupled waves, beam instability, instability increment, Vavilov–Cherenkov effect.

1. Introduction

With the mastering of the submillimeter wavelength range, special attention is paid to the issues dealing with the generation of electromagnetic oscillations in systems where the fluxes of charged particles interact with slow-wave structures. Such interest is associated with a high conversion efficiency of kinetic energy of particles into electromagnetic radiation. Since the 1940s, when the traveling wave tube was invented [1], not all capabilities of such energy transformation have been exhausted. Currently, special attention is paid to electrodynamic systems where the fluxes of charged particles interact with various structures, including those containing plasma-like media [2–14]. Of particular interest are systems with magnetoactive structures composed of conducting solids with plasma-like properties. In an external magnetic field, such structures support the propagation of electromagnetic waves of the helicon type [15–25].

It should be noted that interest to open guide systems containing magnetoactive components has been renewed recently. One of the directions in this domain is the study of excitation and propagation of helicon-type waves in cylindrical plasma structures located in free space [19–21] or background magnetoactive plasma [26–29] in parallel to the external constant magnetic field. In particular, increased attention is paid to waves that play an important role in high-frequency helicon discharges [19,20]. Such discharges are accompanied by the formation of plasma-wave channels at a relatively small magnitude of external magnetic field and are considered to be very effective sources of dense low-temperature plasma. They are widely used to create active environments for gas lasers [30, 31], generate plasma in magnetic traps [32], develop new methods for accelerating particles [33], and in plasma chemical technologies [34].

In most of theoretical work dealing with the propagation of helicon-type waves in cylindrical plasma channels located in either free space or background magnetoactive plasma, a case is considered when there are no collisions in the plasma medium (see, e.g., [35–38]). The case where losses associated with particle collisions are relatively low and do not result

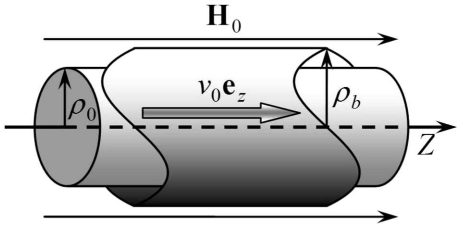

Fig. 1. Geometry of electrodynamic system

in substantial modifications of the dispersion characteristics and structure of wave fields was discussed in works [39, 40]. However, no detailed study of the wave parameters in plasma channels in the presence of collisions was carried out in those works. The case where the collision frequency of main charge carriers in the magnetized solid-state plasma cylinder is significantly lower than their cyclotron frequency was examined in work [24]. The authors of work [41] proposed a theory of helicon waves propagating in three-dimensional Weyl semimetals with conductivity determined by topological properties of charge-carrier wave functions (massless Weyl fermions) in the specimen bulk. The existence of pseudomagnetic helicons in Dirac and Weyl deformed materials was predicted in work [42].

It is worth noting that currently there is practically no strict electrodynamic analysis of the excitation of eigenwaves in open plasma waveguides, including solid-state ones. As a rule, a situation is realized in plasma-solid-state systems, where the collision frequency of charge carriers strongly exceeds the electromagnetic wave frequency and conditions for the propagation of surface and volume-surface waves [13, 43] of the helicon type [22] are provided. These waves are weakly attenuating electromagnetic waves propagating in solid-state plasma in a constant magnetic field. Undoubtedly, a research of the generation mechanisms and methods for the excitation of helicon waves is necessary, in particular, to improve available and create new sources of millimeter and submillimeter waves.

In this paper, we present a linear theory of the interaction between a magnetized tubular flow of charged particles and the field of electromagnetic eigenwaves of a coaxial solid-state plasma cylinder with high conductivity. The object of study is electromagnetic processes in an electrodynamic system, where a tubular electron beam blows around a cylindrical plasma waveguide located in a strong longitudinal uniform magnetic field. The subject of study is coupled electromagnetic waves, one of which is the space-charge wave in the electron flow, and the other the electromagnetic wave in the waveguide playing the role of a slow-wave structure. The research methods are based on the general theory of electromagnetic field in the quasi-stationary approximation, the apparatus of mathematical physics for solving boundary-value problems, and plasma electrodynamics.

The novelty of the obtained results consists in the further development of the theory of intrinsic and forced oscillations and waves of the helicon type in plasma-like structures with high conductivity. For the first time, a possibility of Cherenkov instability is shown for a magnetized electron beam that coaxially blows around a solid-state plasma cylinder arranged in a strong longitudinal magnetic field.

2. Formulation of the Problem

Consider an electrodynamic system consisting of a plasma-solid-state (semiconductor) waveguide of radius ρ_0 occupying the space region $0 \leqslant \rho \leqslant \rho_0$, $0 \leqslant \varphi \leqslant 2\pi$, and $-\infty \leqslant z \leqslant +\infty$ (the cylindrical coordinate frame) and a coaxial tubular flux of radius ρ_b of magnetized electrons blowing around the waveguide (Fig. 1). The system is located in free space in a constant magnetic field. The induction vector $\mathbf{H}_0$ of the latter is directed in parallel to the axial axis of system symmetry. Solid-state plasma has a high n-type conductivity. The equilibrium concentration of plasma conduction electrons equals N_0, and the electron concentration in the tubular beam of radial thickness a equals $N_{b0}(\rho)$. Electrons move with the average translational velocity $\mathbf{v}_0 = v_0\mathbf{e}_z$, where $\mathbf{e}_z$ is a unit vector directed along the z-axis. The charge of plasma conduction electrons is assumed to be compensated by the positively charged background of the crystal lattice of the solid (semiconductor), and the electron flux thickness a is small in comparison with other sizes of electrodynamic system; in particular, $a \leqslant 2(\rho_b - \rho_0)$. Therefore, the equilibrium electron concentration in the tubular beam can be written in the form $N_{b0}(\rho) = N_{b0}a\delta(\rho - \rho_b)$, where N_{b0} is the equilibrium electron concentration in the beam and $\delta(\rho - \rho_b)$ the Dirac delta-function.

In the linear approximation, the perturbed current density of the beam at a point described by the radius vector $\mathbf{r}$ and at the time moment t looks like

$$\mathbf{j}_b(\mathbf{r},t) = eN_{b0}(\rho)\mathbf{v}(\mathbf{r},t) + e\mathbf{v}_0 N(\mathbf{r},t) = \hat{\sigma}_b \mathbf{E}(\mathbf{r},t), \quad (1)$$

where e is the elementary charge; $N(\mathbf{r},t)$ and $\mathbf{v}(\mathbf{r},t)$ are the space- and time-dependent concentration and velocity, respectively, of beam electrons; and $\hat{\sigma}_b$ the conductivity tensor of electron beam. The strength of alternating electric field equals $\mathbf{E}(\mathbf{r},t) = \sum_{\alpha=\rho}^{z} \mathbf{e}_\alpha E_\alpha(\mathbf{r},t)$, where summation is carried out over the coordinate axes $\alpha = (\rho, \varphi, z)$. Below, the radial component of the beam current density is considered to equal zero because of the selected model of radially thin tubular beam. As a result, owing to the magnetization of electron flux, $\mathbf{v}(\mathbf{r},t) = \mathbf{e}_z v_z(\mathbf{r},t)$. In this case, the electron flux can be perturbed only in the longitudinal direction via its interaction with the electromagnetic field of eigenwaves in the plasma-solid-state waveguide.

The current density generated by conduction electrons in solid-state plasma has the form

$$\mathbf{j}_e(\mathbf{r},t) = eN_0\mathbf{u} = \hat{\sigma}\mathbf{E}(\mathbf{r},t), \quad (2)$$

where $\mathbf{u}$ is the velocity of conduction electrons, and $\hat{\sigma}$ the plasma conductivity tensor.

Let

i) the period of electromagnetic field change be substantially longer than the electron free path time in solid-state plasma;

ii) in the space region occupied by plasma (semiconductor), the bias current be negligibly small in comparison with the conduction current (due to a high plasma conductivity).

In this case, the magnetic field distribution over the plasma medium (inside the cylinder) is described by the equations of magnetostatics. Electromagnetic fields and currents satisfying the above conditions are quasi-stationary [44, 45].

3. Basic Equations for the System "Magnetized Tubular Electron Beam–Magnetoplasma Cylinder under Quasi-Stationary Conditions"

The interaction of the electron beam with the eigenwaves of plasma-solid-state cylinder is described by the system of Maxwell's equations and linearized equations of motion for the beam electrons and plasma conduction ones,

$$\operatorname{rot}\mathbf{E}(\mathbf{r},t) = -\frac{1}{c}\frac{\partial}{\partial t}\mathbf{H}(\mathbf{r},t), \quad (3)$$

$$\operatorname{rot}\mathbf{H}(\mathbf{r},t) = \frac{4\pi}{c}\mathbf{j}_e(\mathbf{r},t), \quad \rho \leqslant \rho_0, \quad (4a)$$

$$\operatorname{rot}\mathbf{H}(\mathbf{r},t) = \frac{1}{c}\frac{\partial}{\partial t}\mathbf{D}(\mathbf{r},t) + \frac{4\pi}{c}\mathbf{j}_b(\mathbf{r},t), \quad \rho > \rho_0, \quad (4b)$$

$$\operatorname{div}\mathbf{H}(\mathbf{r},t) = 0, \quad (5)$$

$$\operatorname{div}\mathbf{D}(\mathbf{r},t) = 4\pi e N(\mathbf{r},t), \quad \rho > \rho_0, \quad (6)$$

$$\left(\frac{\partial}{\partial t} + v_0\frac{\partial}{\partial z}\right)\mathbf{v}(\mathbf{r},t) = \\ = \frac{e}{m_0}\left(\mathbf{E}(\mathbf{r},t) + \frac{1}{c}[\mathbf{v}_0 \times \mathbf{H}(\mathbf{r},t)]\right), \quad (7)$$

$$m\nu\mathbf{u} = e\mathbf{E}(\mathbf{r},t) + \frac{e}{c}(\mathbf{u}\times\mathbf{H}_0), \quad (8)$$

where

$$\mathbf{D}(\mathbf{r},t) = \sum_{\alpha=\rho}^{z}\mathbf{e}_\alpha D_\alpha(\mathbf{r},t), \quad \mathbf{H}(\mathbf{r},t) = \sum_{\alpha=\rho}^{z}\mathbf{e}_\alpha H_\alpha(\mathbf{r},t)$$

are the induction of alternating electric field and the strength of alternating magnetic field, respectively; ν is the frequency of conduction electron collisions in solid-state plasma; m is the effective mass of conduction electron in plasma; and m_0 is the free electron mass. The frequency of charge carrier collisions in solid-state plasma characterizes the relaxation of its perturbations associated with oscillations and collisions with crystal lattice impurities and defects. The linearized motion equation for conduction electrons (8) takes into account that $|d\mathbf{u}/dt| \ll \nu|\mathbf{u}|$. This condition is an analog of the first condition for the electromagnetic field quasi-stationarity. Beyond the solid-state plasma waveguide, owing to the absence of conduction charges in the medium, the action of bias current is considerable, which is responsible for effects associated with the finite propagation velocity of electromagnetic perturbations (4b). If plasma is homogeneous, the vector of electric field induction $\mathbf{D}(\mathbf{r},t)$ is related to the strength vector $\mathbf{E}(\mathbf{r},t)$ via the material equation

$$\mathbf{D}(\mathbf{r},t) = \int\limits_{-\infty}^{t}\int \varepsilon(t-t', \mathbf{r}-\mathbf{r}')dt'd\mathbf{r}' = \hat{\varepsilon}\mathbf{E}(\mathbf{r},t)),$$

where $\varepsilon(t-t', \mathbf{r}-\mathbf{r}')$ is the influence function that characterizes the efficiency of field action transfer in time and space, and $\hat{\varepsilon}$ is the tensor of complex dielectric permittivity of the medium. If the spatial dispersion of the medium is absent (if $\mathbf{r} = \mathbf{r}'$), we have

$\varepsilon(t-t', \mathbf{r}-\mathbf{r}') = \hat{\varepsilon}\delta(t-t')$. In the vacuum beyond the electron beam – i.e., at $\rho > \rho_0$ and $\rho \neq \rho_b$ – we have $\mathbf{D}(\mathbf{r},t) \equiv \mathbf{E}(\mathbf{r},t)$ because the elements of the tensor $\hat{\varepsilon}$ look like $\varepsilon_{\alpha\beta} \equiv \delta_{\alpha\beta}$, where the subscripts α and β correspond to directions along the corresponding axes (ρ, φ, and z) of the cylindrical coordinate frame, and $\delta_{\alpha\beta}$ is the Kronecker delta.

The effective mass of charge carriers, the collision frequency, and the influence of the external magnetic field govern the properties of the plasma conductivity tensor. From equations (2) and (8), it follows that the conductivity tensor of solid-state plasma has the form

$$\hat{\sigma} = \begin{pmatrix} \sigma_{\rho\rho} & \sigma_{\rho\varphi} & 0 \\ \sigma_{\varphi\rho} & \sigma_{\varphi\varphi} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix},$$

where

$$\sigma_{\rho\rho}=\sigma_{\varphi\varphi}=\frac{e^2 N_0 \nu}{m(\omega_H^2+\nu^2)},$$

$$\sigma_{\rho\varphi}=-\sigma_{\varphi\rho}=\frac{e^2 N_0 \omega_H}{m(\omega_H^2+\nu^2)},$$

$$\sigma_{zz}=\frac{e^2 N_0}{m\nu},$$

and

$$\omega_H=\frac{eH_0}{mc},$$

where c is the light speed, is the cyclotron frequency of conduction electrons. Inside the waveguide, $\rho \leqslant \rho_0$, in the absence of spatial dispersion, the electric field induction equals

$$\mathbf{D}(\mathbf{r},t) = \hat{\varepsilon}\mathbf{E}(\mathbf{r},t) = \\ = \sum_{\alpha=\rho}^{z} \mathbf{e}_\alpha \sum_{\beta=\rho}^{z} \left[\varepsilon_l \delta_{\alpha\beta} E_\beta(\mathbf{r},t) + 4\pi \int_{-\infty}^{t} \sigma_{\alpha\beta} E_\beta(\mathbf{r},t')dt' \right],$$

where ε_l is the dielectric permittivity of crystal lattice [46]. Recall that in the framework of the quasi-stationary approximation, the bias current density in solid-state plasma

$$\frac{1}{4\pi}\frac{\partial}{\partial t}\sum_{\alpha=\rho}^{z} \mathbf{e}_\alpha \sum_{\beta=\rho}^{z} \varepsilon_l \delta_{\alpha\beta} E_\beta(\mathbf{r},t) = \frac{1}{4\pi}\frac{\partial}{\partial t}\varepsilon_l \mathbf{E}(\mathbf{r},t)$$

is neglected. In the case $\mathbf{E}(\mathbf{r},t) = \mathbf{E}(\mathbf{r})\exp(-i\omega t)$, where $i^2 = -1$ and ω is a characteristic inverse time of electromagnetic process (it corresponds to the cyclic frequency of eigenwave in the plasma cylinder), the elements of the tensor of complex dielectric permittivity $\hat{\varepsilon}$ have the form [47]

$$\varepsilon_{\alpha\beta} = \varepsilon_\beta \delta_{\alpha\beta} + i\frac{4\pi\sigma_{\alpha\beta}}{\omega}.$$

From the equation of motion for the electron beam (7), it follows that

$$\left(\frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z}\right) v_z(\mathbf{r},t) = \frac{e}{m_0} E_z(\mathbf{r},t), \quad (9)$$

$$E_\rho(\mathbf{r},t) = \frac{v_0}{c} H_\varphi(\mathbf{r},t), \quad E_\varphi(\mathbf{r},t) = -\frac{v_0}{c} H_\rho(\mathbf{r},t). \quad (10)$$

The operator $(\partial/\partial t + v_0 \partial/\partial z)$ in (9) determines the longitudinal varying component of the vector of the beam electron velocity $v_z(\mathbf{r},t)$. The action of the inverse operator $(\partial/\partial t + v_0 \partial/\partial z)^{-1}$ reflects the resonance manifestation of the Vavilov–Cherenkov effect. Note that if the values of the components of the electromagnetic field beyond the plasma-solid-state waveguide are taken into account (they follow from (3) and (4b), and are given below), conditions (10) are satisfied only in the case of resonance Vavilov–Cherenkov interaction of the electron flux with the fields of eigenwaves in the plasma cylinder.

From equations (4b) and (6) follows the continuity equation for the electronic flux,

$$e\frac{\partial}{\partial t} N(\mathbf{r},t) + \operatorname{div} \mathbf{j}_b(\mathbf{r},t) = 0.$$

Whence, taking into account (1), we obtain

$$\left(\frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z}\right) N(\mathbf{r},t) = -N_{b0}(\rho)\frac{\partial}{\partial z} v_z(\mathbf{r},t), \quad (11)$$

This relationship describes the varying fraction of electron concentration $N(\mathbf{r},t)$. The action of the inverse operator $(\partial/\partial t + v_0 \partial/\partial z)^{-1}$ on it reflects the resonance perturbation of the electron flux realized in the case of the Vavilov–Cherenkov effect. From the material equation (1) and taking (9) and (11) into account, it follows that the electron beam in the electrodynamic system has a unidirectional conductivity σ_b along the z axis, which is determined from the equation

$$\left(\frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z}\right) \sigma_b E_z(\mathbf{r},t) = \frac{e^2 N_{b0}(\rho)}{m_0}\frac{\partial}{\partial t} E_z(\mathbf{r},t).$$

Whence one can see that the conductivity of the electron beam has a resonance character, which the effect of Vavilov–Cherenkov interaction of electrons with the fields of eigenwaves in the plasma cylinder is responsible for.

In what follows, we omit the indication of the functional dependence of the electromagnetic field strengths and their components on the spatiotemporal variables $(\mathbf{r}, t)$ in the mathematical expressions; i.e., we mean that $\mathbf{E}(\mathbf{r}, t) \equiv \mathbf{E}$ and $\mathbf{H}(\mathbf{r}, t) \equiv \mathbf{H}$. The dependence on the radial variable ρ will be indicated if necessary.

4. Boundary Conditions

From equations (5) and (4) and taking into account the Ostrogradskii–Gauss and Stokes theorems, it follows that at the surface of a solid-state plasma cylinder (at $\rho = \rho_0$), where $\mathbf{j}_b(\mathbf{r}, t) = 0$, all magnetic field components are continuous, i.e., $\mathbf{H}(\rho_0 + 0) = \mathbf{H}(\rho_0 - 0)$. The continuity of the tangential H_φ and H_z components of the field is a result of finite plasma conductivity. From equation (3) follows the continuity of the axial, E_z, and azimuthal, E_φ, components of the electric field strength at the cylinder surface (at $\rho = \rho_0$), i.e.,

$$E_z(\rho_0 - 0) = E_z(\rho_0 + 0), \quad E_\varphi(\rho_0 - 0) = E_\varphi(\rho_0 + 0).$$

According to equation (4a), $\operatorname{div} \mathbf{j}_e(\mathbf{r}, t) = 0$. The boundary condition for this equation is the continuity of the transition from the normal (radial) component of the conductivity current density $j_\rho(\rho)$ inside the cylinder to the displacement current density $\frac{1}{4\pi} \frac{\partial}{\partial t} E_\rho(\rho)$ outside it across the cylinder lateral surface. According to (2), this condition looks like

$$\sigma_{\rho\rho} E_\rho(\rho_0 - 0) + \sigma_{\rho\varphi} E_\varphi(\rho_0 - 0) = \frac{1}{4\pi} \frac{\partial}{\partial t} E_\rho(\rho_0 + 0). \quad (12)$$

Note that owing to the quasi-stationary approximation, condition (12) is equivalent to the continuity of the normal (radial) component of the electric field induction vector across the lateral surface of the cylindrical waveguide (at $\rho = \rho_b$).

At the electron beam surface (at $\rho = \rho_0$), the electric field has continuous tangential components of the strength vector $\mathbf{E}$, i.e.,

$$E_z(\rho_b - 0) = E_z(\rho_b + 0), \quad E_\varphi(\rho_b - 0) = E_\varphi(\rho_b + 0),$$

and a discontinuity in the normal component of the induction vector $\mathbf{D}(\mathbf{r}, t) \equiv \mathbf{E}$, the latter arising due to the perturbed charge of the beam. The magnetic field has a continuous normal component of the induction vector $\mathbf{H}$, i.e.,

$$H_\rho(\rho_b - 0) = H_\rho(\rho_b + 0),$$

and a discontinuity in its tangential component $\mathbf{H}_\tau$ arising owing to the perturbed beam current. Conditions for the discontinuity of the tangential magnetic-field component and the normal component of the electric field induction are obtained by integrating (4b) and (6) over an infinitesimally small beam surface $d\mathbf{S} = -d\rho dz \mathbf{e}_\varphi$ and beam volume $dV = \rho d\rho d\varphi dz$, respectively. In the case of ideal conductivity of the tubular beam in the longitudinal direction, which is due to the magnetization of the electron flux – i.e., if $\mathbf{v}(\mathbf{r}, t) = \mathbf{e}_z v_z(\mathbf{r}, t)$ – these conditions have the form

$$H_z(\rho_b + 0) - H_z(\rho_b - 0) = 0,$$

$$H_\varphi(\rho_b + 0, t) - H_\varphi(\rho_b - 0, t) =$$

$$= \frac{4\pi}{c} \lim_{\Delta\rho \to 0} \int\limits_{\rho_b - \Delta\rho}^{\rho_b + \Delta\rho} j_{bz}(\mathbf{r}, t) d\rho,$$

$$E_\rho(\rho_b + 0, t) - E_\rho(\rho_b - 0, t) =$$

$$= \frac{4\pi e}{\rho_b} \lim_{\Delta\rho \to 0} \int\limits_{\rho_b - \Delta\rho}^{\rho_b + \Delta\rho} N(\mathbf{r}, t) \rho d\rho,$$

where, according to (1), (9), and (11),

$$\left(\frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z}\right)^2 j_{bz}(\mathbf{r}, t) = \frac{e^2 N_{b0}(\rho)}{m_0} \frac{\partial}{\partial t} E_z(\mathbf{r}, t),$$

$$\left(\frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z}\right)^2 N(\mathbf{r}, t) = -\frac{e N_{b0}(\rho)}{m_0} \frac{\partial}{\partial z} E_z(\mathbf{r}, t).$$

5. Electromagnetic Field Components

According to (3) and (4a), as well as taking (2) and (8) into account, the transverse components of the electromagnetic field (E_ρ, E_φ, H_ρ, and H_φ) inside the waveguide (at $\rho \leqslant \rho_0$) are expressed in terms of the axial (longitudinal) E_z and H_z components as follows:

$$\left[\left(\frac{\partial^2}{\partial z^2}-\frac{4\pi\sigma_{zz}}{c^2}\frac{\partial}{\partial t}\right)^2+\frac{\sigma_{\rho\varphi}^2}{\sigma_{\rho\rho}^2}\frac{\partial^4}{\partial z^4}\right]E_\rho=\frac{\partial}{\partial z}\left[\left(\frac{1}{\sigma_{\rho\rho}}\frac{\partial^2}{\partial z^2}-\frac{4\pi}{c^2}\frac{\partial}{\partial t}\right)\frac{\partial}{\partial\rho}+\frac{4\pi\sigma_{\rho\varphi}}{c^2\sigma_{\rho\rho}}\frac{\partial}{\partial t}\frac{1}{\rho}\frac{\partial}{\partial\varphi}\right]\sigma_{zz}E_z+$$
$$+\frac{1}{c}\frac{\partial}{\partial t}\left[\frac{4\pi\sigma_{\rho\varphi}}{c^2\sigma_{\rho\rho}}\frac{\partial}{\partial t}\frac{\partial}{\partial\rho}-\left(\frac{1}{\sigma_{\rho\rho}}\frac{\partial^2}{\partial z^2}-\frac{4\pi}{c^2}\frac{\partial}{\partial t}\right)\frac{1}{\rho}\frac{\partial}{\partial\varphi}\right]\sigma_{zz}H_z,$$

$$\left[\left(\frac{\partial^2}{\partial z^2}-\frac{4\pi\sigma_{zz}}{c^2}\frac{\partial}{\partial t}\right)^2+\frac{\sigma_{\rho\varphi}^2}{\sigma_{\rho\rho}^2}\frac{\partial^4}{\partial z^4}\right]E_\varphi=\frac{\partial}{\partial z}\left[\left(\frac{1}{\sigma_{\rho\rho}}\frac{\partial^2}{\partial z^2}-\frac{4\pi}{c^2}\frac{\partial}{\partial t}\right)\frac{1}{\rho}\frac{\partial}{\partial\varphi}-\frac{4\pi\sigma_{\rho\varphi}}{c^2\sigma_{\rho\rho}}\frac{\partial}{\partial t}\frac{\partial}{\partial\rho}\right]\sigma_{zz}E_z+$$
$$+\frac{1}{c}\frac{\partial}{\partial t}\left[\frac{4\pi\sigma_{\rho\varphi}}{c^2\sigma_{\rho\rho}}\frac{\partial}{\partial t}\frac{1}{\rho}\frac{\partial}{\partial\varphi}-\left(\frac{1}{\sigma_{\rho\rho}}\frac{\partial^2}{\partial z^2}-\frac{4\pi}{c^2}\frac{\partial}{\partial t}\right)\frac{\partial}{\partial\rho}\right]\sigma_{zz}H_z,$$

$$\left[\left(\frac{\partial^2}{\partial z^2}-\frac{4\pi\sigma_{zz}}{c^2}\frac{\partial}{\partial t}\right)^2+\frac{\sigma_{\rho\varphi}^2}{\sigma_{\rho\rho}^2}\frac{\partial^4}{\partial z^4}\right]H_\rho=\frac{4\pi}{c}\left[\left(\frac{\partial^2}{\partial z^2}-\frac{4\pi\sigma_{zz}}{c^2}\frac{\partial}{\partial t}\right)\frac{1}{\rho}\frac{\partial}{\partial\varphi}-\frac{\sigma_{\rho\varphi}}{\sigma_{\rho\rho}}\frac{\partial^2}{\partial z^2}\frac{\partial}{\partial\rho}\right]\sigma_{zz}E_z+$$
$$+\frac{\partial}{\partial z}\left[\frac{4\pi\sigma_{\rho\varphi}}{c^2\sigma_{\rho\rho}}\frac{\partial}{\partial t}\frac{1}{\rho}\frac{\partial}{\partial\varphi}+\left(\frac{1}{\sigma_{\rho\rho}}\frac{\partial^2}{\partial z^2}-\frac{4\pi}{c^2}\frac{\partial}{\partial t}\right)\frac{\partial}{\partial\rho}\right]\sigma_{zz}H_z,$$

$$\left[\left(\frac{\partial^2}{\partial z^2}-\frac{4\pi\sigma_{zz}}{c^2}\frac{\partial}{\partial t}\right)^2+\frac{\sigma_{\rho\varphi}^2}{\sigma_{\rho\rho}^2}\frac{\partial^4}{\partial z^4}\right]H_\varphi=-\frac{4\pi}{c}\left[\left(\frac{\partial^2}{\partial z^2}-\frac{4\pi\sigma_{zz}}{c^2}\frac{\partial}{\partial t}\right)\frac{\partial}{\partial\rho}+\frac{\sigma_{\rho\varphi}}{\sigma_{\rho\rho}}\frac{\partial^2}{\partial z^2}\frac{1}{\rho}\frac{\partial}{\partial\varphi}\right]\sigma_{zz}E_z-$$
$$-\frac{\partial}{\partial z}\left[\frac{4\pi\sigma_{\rho\varphi}}{c^2\sigma_{\rho\rho}}\frac{\partial}{\partial t}\frac{\partial}{\partial\rho}-\left(\frac{1}{\sigma_{\rho\rho}}\frac{\partial^2}{\partial z^2}-\frac{4\pi}{c^2}\frac{\partial}{\partial t}\right)\frac{1}{\rho}\frac{\partial}{\partial\varphi}\right]\sigma_{zz}H_z.$$

In turn, the axial field components are determined by the solutions of the following system of equations:

$$\begin{cases}\frac{4\pi}{c}\frac{\sigma_{zz}\sigma_{\rho\varphi}}{\sigma_{\rho\rho}}\frac{\partial}{\partial z}\Delta_\perp E_z=\left[\left(\frac{\partial^2}{\partial z^2}-\frac{4\pi\sigma_{zz}}{c^2}\frac{\partial}{\partial t}\right)^2+\frac{\sigma_{\rho\varphi}^2}{\sigma_{\rho\rho}^2}\frac{\partial^4}{\partial z^4}+\left(\frac{\sigma_{zz}}{\sigma_{\rho\rho}}\frac{\partial^2}{\partial z^2}-\frac{4\pi\sigma_{zz}}{c^2}\frac{\partial}{\partial t}\right)\Delta_\perp\right]H_z,\\ \frac{\sigma_{\rho\varphi}}{\sigma_{\rho\rho}}\frac{\partial}{\partial z}\frac{1}{c}\frac{\partial}{\partial t}\Delta_\perp H_z=-\left[\left(\frac{\partial^2}{\partial z^2}-\frac{4\pi\sigma_{zz}}{c^2}\frac{\partial}{\partial t}\right)^2+\frac{\sigma_{\rho\varphi}^2}{\sigma_{\rho\rho}^2}\frac{\partial^4}{\partial z^4}+\left(\frac{\partial^2}{\partial z^2}-\frac{4\pi\sigma_{zz}}{c^2}\frac{\partial}{\partial t}\right)\Delta_\perp\right]E_z,\end{cases}\qquad(13)$$

where $\Delta_\perp=\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}$ is the transverse component of the Laplace operator $\Delta=\Delta_\perp+\frac{\partial^2}{\partial z^2}$. From (13), it follows that the components E_z and H_z are solutions of identical equations:

$$\left\{\Delta_\perp^2+\left[\left(1+\frac{\sigma_{zz}}{\sigma_{\rho\rho}}\right)\frac{\partial^2}{\partial z^2}-\frac{8\pi\sigma_{zz}}{c^2}\frac{\partial}{\partial t}\right]\Delta_\perp+\left(\frac{\partial^2}{\partial z^2}-\frac{4\pi\sigma_{zz}}{c^2}\frac{\partial}{\partial t}\right)^2+\frac{\sigma_{\rho\varphi}^2}{\sigma_{\rho\rho}^2}\frac{\partial^4}{\partial z^4}\right\}\left\{\begin{matrix}E_z\\H_z\end{matrix}\right\}=0,\qquad(14a)$$

$$\left[\Delta^2+\left(\frac{\sigma_{\rho\varphi}^2}{\sigma_{\rho\rho}^2}\frac{\partial^2}{\partial z^2}-\frac{8\pi\sigma_{zz}}{c^2}\frac{\partial}{\partial t}\right)\Delta+\frac{16\pi^2\sigma_{zz}^2}{c^4}\frac{\partial^2}{\partial t^2}\right]\left\{\begin{matrix}E_z\\H_z\end{matrix}\right\}=0.\qquad(14b)$$

Outside the waveguide ($\rho>\rho_0$) and the electron beam ($\rho>\rho_b$, where $\mathbf{j}_b(\mathbf{r},t)=0$), according to (3), (4b), (5), and (6) (the latter in the case $\rho\neq\rho_b$ reads $\operatorname{div}\mathbf{E}=0$), the perturbation of the electromagnetic field at an arbitrary spatial point and at any time moment is described by the wave equations

$$\Delta\mathbf{E}=\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{E},\quad \Delta\mathbf{H}=\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{H},$$

the solutions of which have are flat monochromatic waves. Then, according to (3) and (4b), the transverse components E_ρ, E_φ, H_ρ, and $H_\varphi$of the field are expressed in terms of the components E_z and H_z as follows:

$$\left(\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)E_\rho=\frac{\partial}{\partial z}\frac{\partial}{\partial\rho}E_z-\frac{1}{c}\frac{\partial}{\partial t}\frac{1}{\rho}\frac{\partial}{\partial\varphi}H_z,$$
$$\left(\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)E_\varphi=\frac{\partial}{\partial z}\frac{1}{\rho}\frac{\partial}{\partial\varphi}E_z+\frac{1}{c}\frac{\partial}{\partial t}\frac{\partial}{\partial\rho}H_z,$$
$$\left(\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)H_\rho=\frac{\partial}{\partial z}\frac{\partial}{\partial\rho}H_z+\frac{1}{c}\frac{\partial}{\partial t}\frac{1}{\rho}\frac{\partial}{\partial\varphi}E_z,$$
$$\left(\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)H_\varphi=\frac{\partial}{\partial z}\frac{1}{\rho}\frac{\partial}{\partial\varphi}H_z-\frac{1}{c}\frac{\partial}{\partial t}\frac{\partial}{\partial\rho}E_z.$$

In turn, the components E_z and H_z of the field are determined by the solutions of the equations

$$\left(\Delta_\perp+\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\left\{\begin{matrix}E_z\\H_z\end{matrix}\right\}=0,\qquad(15a)$$

$$\left(\Delta-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\left\{\begin{matrix}E_z\\H_z\end{matrix}\right\}=0.\qquad(15b)$$

Note that conditions (10) hold if

$$\left(\frac{\partial}{\partial t}+v_0\frac{\partial}{\partial z}\right)H_z=0\,H_z,\quad \left(\frac{v_0}{c^2}\frac{\partial}{\partial t}+\frac{\partial}{\partial z}\right)E_z=0\,E_z.$$

The first condition testifies to the resonance interaction of electrons in the beam with the fields of eigenwaves in the plasma waveguide in the case of the Vavilov–Cherenkov effect. The second condition is practically not obeyed because the nonlinear influence of the field created by space-charge waves (SCWs) in the beam on the motion of electrons is neglected.

6. Fields of Waves Excited in the Electrodynamic System

The solution of equations (14) and (15) will be obtained using the variable separation method and representing the vectors of electromagnetic field strengths in the form of sets of spatial-temporal harmonics,

$$\mathbf{E} = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathbf{E}_n(\rho) \exp\left[i\left(q_z z + n\varphi - \omega t\right)\right] dq_z d\omega,$$

$$\mathbf{H} = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathbf{H}_n(\rho) \exp\left[i\left(q_z z + n\varphi - \omega t\right)\right] dq_z d\omega,$$

where $\mathbf{E}_n(\rho)$ and $\mathbf{H}_n(\rho)$ are spectral components of the electric and magnetic fields, respectively; q_z is the longitudinal (axial) wave number; and n is the number of spatial harmonic (that coincides with the azimuthal mode index of the wave).

According to (14), at$\rho \leqslant \rho_0$, the axial spectral components $E_{zn}(\rho)$ and $H_{zn}(\rho)$ of the electromagnetic field of the waveguide eigenwaves are determined by the solutions of equations

$$\left\{\Delta_{\perp n}^2 + \left[i\omega\frac{8\pi\sigma_{zz}}{c^2} - \left(\frac{\sigma_{zz}}{\sigma_{\rho\rho}} + 1\right)q_z^2\right]\Delta_{\perp n} + \right.$$
$$\left. + \left(i\omega\frac{4\pi\sigma_{zz}}{c^2} - q_z^2\right)^2 + \frac{\sigma_{\rho\varphi}^2}{\sigma_{\rho\rho}^2}q_z^4\right\}\begin{Bmatrix}E_{zn}(\rho)\\ H_{zn}(\rho)\end{Bmatrix} = 0, \qquad (16a)$$

$$\left[\Delta_n^2 + \left(i\omega\frac{8\pi\sigma_{zz}}{c^2} - \frac{\sigma_{\rho\varphi}^2}{\sigma_{\rho\rho}^2}q_z^2\right)\Delta_n - \right.$$
$$\left. - \omega^2\frac{16\pi^2\sigma_{zz}^2}{c^4}\right]\begin{Bmatrix}E_{zn}(\rho)\\ H_{zn}(\rho)\end{Bmatrix} = 0, \qquad (16b)$$

where

$$\Delta_{\perp n} = \frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial}{\partial\rho} - \frac{n^2}{\rho^2}, \quad \Delta_n = \Delta_{\perp n} - q_z^2.$$

Equations (16a) and (16b) are reduced to the forms

$$\left(\Delta_{\perp n} + \kappa_1^2\right)\left(\Delta_{\perp n} + \kappa_2^2\right)\begin{Bmatrix}E_{zn}(\rho)\\ H_{zn}(\rho)\end{Bmatrix} = 0, \qquad (17)$$

$$\left(\Delta_n + q_1^2\right)\left(\Delta_n + q_2^2\right)\begin{Bmatrix}E_{zn}(\rho)\\ H_{zn}(\rho)\end{Bmatrix} = 0, \qquad (18)$$

where $\kappa_{1,2}^2$ and $q_{1,2}^2$ are eigenvalues of the operators $\Delta_{\perp n}$ and Δ_n, which are defined by the solutions of characteristic equations

$$\kappa^4 - \kappa^2\left[i\omega\frac{8\pi\sigma_{zz}}{c^2} - \left(\frac{\sigma_{zz}}{\sigma_{\rho\rho}} + 1\right)q_z^2\right] +$$
$$+ \left(i\omega\frac{4\pi\sigma_{zz}}{c^2} - q_z^2\right)^2 + \frac{\sigma_{\rho\varphi}^2}{\sigma_{\rho\rho}^2}q_z^4 = 0, \qquad (19a)$$

$$q^4 - q^2\left(i\omega\frac{8\pi\sigma_{zz}}{c^2} - \frac{\sigma_{\rho\varphi}^2}{\sigma_{\rho\rho}^2}q_z^2\right) - \omega^2\frac{16\pi^2\sigma_{zz}^2}{c^4} = 0 \qquad (19b)$$

with respect to κ^2 and q^2, respectively, since

$$\Delta_{\perp n}\begin{Bmatrix}E_{zn}(\rho)\\ H_{zn}(\rho)\end{Bmatrix} = -\kappa^2\begin{Bmatrix}E_{zn}(\rho)\\ H_{zn}(\rho)\end{Bmatrix}$$

and

$$\Delta_n\begin{Bmatrix}E_{zn}(\rho)\\ H_{zn}(\rho)\end{Bmatrix} = -q^2\begin{Bmatrix}E_{zn}(\rho)\\ H_{zn}(\rho)\end{Bmatrix}.$$

Whence we have

$$\kappa_{1,2}^2 = \frac{1}{2}q_z^2\frac{\sigma_{\rho\varphi}^2}{\sigma_{\rho\rho}^2}\left(1 - \frac{8\pi\sigma_{zz}\sigma_{\rho\rho}^2}{q_z^2c^2\sigma_{\rho\varphi}^2} \mp \right.$$
$$\left. \mp \sqrt{1 - i\frac{16\pi\sigma_{zz}\sigma_{\rho\rho}^2}{q_z^2c^2\sigma_{\rho\varphi}^2}}\right) - q_z^2, \qquad (20a)$$

$$q_{1,2}^2 = \kappa_{1,2}^2 + q_z^2. \qquad (20b)$$

In order to physically understand the nature of eigenwaves in the magnetized plasma waveguide, consider some cases where plasma is almost collision-free ($\nu \to 0$) or strongly colliding with the collision frequency of main charge carriers substantially lower than their cyclotron frequency ($\nu \to \infty$, $\nu \ll |\omega_H|$). In the former case, equations (19) are simplified to the forms

$$\kappa^2q_z^2 - \frac{16\pi^2\sigma_{zz}^2\sigma_{\rho\rho}^2\omega^2}{\sigma_{\rho\varphi}^2c^4} + q_z^4 = 0$$

and

$$q^2q_z^2 - \frac{16\pi^2\sigma_{zz}^2\sigma_{\rho\rho}^2\omega^2}{\sigma_{\rho\varphi}^2c^4} = 0,$$

respectively, and in the latter one to the forms

$$\kappa^4+\kappa^2\left(2q_z^2+\frac{q_z^2\sigma_{\rho\varphi}^2}{\sigma_{\rho\rho}^2}\right)+q_z^4\left(1+\frac{\sigma_{\rho\varphi}^2}{\sigma_{\rho\rho}^2}\right)=0$$

and

$$q_z^4+\frac{q^2q_z^2\sigma_{\rho\varphi}^2}{\sigma_{\rho\rho}^2}=0,$$

respectively. Hence, if $\nu\to 0$, we have $\kappa^2=q_0^2-q_z^2$ and $q=q_0$, where

$$q_0=\frac{4\pi\sigma_{zz}\sigma_{\rho\rho}\omega}{\sigma_{\rho\varphi}q_zc^2}=\frac{4\pi eN_0\omega}{H_0q_zc}$$

is the helicon wave number [22]. In this case, (8) takes the form of the equation of motion for charged particles with zero effective mass ($m=0$), which in a loss-free system generate processes of magnetic field oscillation, i.e., perturbations of the external magnetic field $\mathbf{H}_0$ of helicon origin [23, 24, 38]. Due to the ideal conductivity of the waveguide medium, current can run on its surface. In this case, the magnetized free electrons in the waveguide can move rectilinearly only along the cylinder generatrix. Therefore, the surface current has only one component along the longitudinal coordinate axis z. In this case, as follows from equations (5) and (4) and taking into account the Ostrogradskii–Gauss and Stokes theorems, the magnetic field components H_ρ and H_z are continuous at the cylinder surface ($\rho=\rho_0$), whereas the component H_φ has the discontinuity $H_\varphi(\rho_0+0)-H_\varphi(\rho_0-0)=4\pi j/c$ as a result of the longitudinal surface current with the density j_{sz} [23].

In the case $\nu\to\infty$, we have $\kappa_1^2=q_z^2$ and $\kappa_2^2=\overline{q}_0^2-q_z^2$, where

$$\overline{q}_0^2=-\frac{q_z^2\sigma_{\rho\varphi}^2}{\sigma_{\rho\rho}^2}=-\frac{q_z^2\omega_H^2}{\nu^2}$$

is the squared wave number that characterizes the perturbed state of the magnetic field. Negative values of κ_1^2 and κ_2^2 mean the electrostatic character of the field.

Taking into account the finiteness of the quantities $E_{zn}(\rho)$ and $H_{zn}(\rho)$ at the cylinder axis ($\rho=0$), the solutions of (16)–(18) inside the waveguide ($\rho\le\rho_0$) have the form

$$E_{zn}(\rho)=A_{n1}^EZ_n(\kappa_1\rho)+A_{n2}^EZ_n(\kappa_2\rho),$$

$$H_{zn}(\rho)=A_{n1}^HZ_n(\kappa_1\rho)+A_{n2}^HZ_n(\kappa_2\rho),$$

where

$$Z_n(\kappa_{1,2}\rho)=\begin{cases}J_n(\kappa_{1,2}\rho), & \kappa_{1,2}^2>0\ (\kappa_{1,2}^2\in\mathbb{C}),\\ I_n(\kappa_{1,2}\rho), & \kappa_{1,2}^2<0,\end{cases}$$

$J_n(x)$ the Bessel function of the first kind and n-th order, $I_n(x)$ the modified Bessel function of the first kind and n-th order (the Infeld function), $\kappa_{1,2}$ are the transverse (radial) wave numbers (20), and $A_{n1,n2}^{E,H}$ are arbitrary constants. According to (13), there are relations between the constants A_{n1}^E and A_{n1}^H and between the constants A_{n2}^E and A_{n2}^H,

$$A_{n1}^E=i\gamma_1\frac{\omega}{c}A_{n1}^H,\quad A_{n2}^E=i\gamma_2\frac{\omega}{c}A_{n2}^H,$$

where $\gamma_{1,2}=\left(1\mp\sqrt{1-4iq_0\sigma_{\rho\rho}/q_z\sigma_{\rho\varphi}}\right)/(2q_0)$. Then, the transverse wave numbers (20) equal

$$\kappa_{1,2}=q_0\left(\overline{q}_0^2\gamma_{1,2}+\frac{iq_z\sigma_{\rho\varphi}}{\sigma_{\rho\rho}}\right)-q_z^2,\quad q_{1,2}^2=\kappa_{1,2}+q_z^2.$$

The transverse spectral components of the Fourier components of the electromagnetic field are determined via the axial components and have the form

$$\kappa_1^2\kappa_2^2E_{\rho n}(\rho)=\frac{1}{q_z}\frac{\omega}{c}\left[\kappa_2^2\left(b_1\frac{\partial}{\partial\rho}-q_z\frac{n}{\rho}\right)A_{n1}^HZ_n(\kappa_1\rho)+\right.$$
$$\left.+\kappa_1^2\left(b_2\frac{\partial}{\partial\rho}-q_z\frac{n}{\rho}\right)A_{n2}^HZ_n(\kappa_2\rho)\right],$$

$$\kappa_1^2\kappa_2^2E_{\varphi n}(\rho)=\frac{i}{q_z}\frac{\omega}{c}\left[\kappa_2^2\left(b_1\frac{n}{\rho}-q_z\frac{\partial}{\partial\rho}\right)A_{n1}^HZ_n(\kappa_1\rho)+\right.$$
$$\left.+\kappa_1^2\left(b_2\frac{n}{\rho}-q_z\frac{\partial}{\partial\rho}\right)A_{n2}^HZ_n(\kappa_2\rho)\right],$$

$$\kappa_1^2\kappa_2^2H_{\rho n}(\rho)=i\left[\kappa_2^2\left(q_z\frac{\partial}{\partial\rho}-a_1\frac{n}{\rho}\right)A_{n1}^HZ_n(\kappa_1\rho)+\right.$$
$$\left.+\kappa_1^2\left(q_z\frac{\partial}{\partial\rho}-a_2\frac{n}{\rho}\right)A_{n2}^HZ_n(\kappa_2\rho)\right],$$

$$\kappa_1^2\kappa_2^2H_{\varphi n}(\rho)=-\left[\kappa_2^2\left(q_z\frac{n}{\rho}-a_1\frac{\partial}{\partial\rho}\right)A_{n1}^HZ_n(\kappa_1\rho)+\right.$$
$$\left.+\kappa_1^2\left(q_z\frac{n}{\rho}-a_2\frac{\partial}{\partial\rho}\right)A_{n2}^HZ_n(\kappa_2\rho)\right],$$

where $a_{1,2}=-iq_0\gamma_{1,2}q_z\sigma_{\rho\varphi}/\sigma_{\rho\rho}$ and $b_{1,2}=(\overline{q}_0^2-q_z^2)\gamma_{1,2}+iq_z\sigma_{\rho\varphi}/\sigma_{\rho\rho}$.

According to (15), for $\rho > \rho_0$ and $\rho \neq \rho_0$, the axial spectral components $E_{zn}(\rho)$ and $H_{zn}(\rho)$ of the electromagnetic field of waveguide eigenwaves satisfy the equations

$$\left(\Delta_{\perp n} + \kappa_0^2\right)\begin{Bmatrix} E_{zn}(\rho) \\ H_{zn}(\rho) \end{Bmatrix} = 0, \tag{21a}$$

$$\left(\Delta_n + \frac{\omega}{c}\right)\begin{Bmatrix} E_{zn}(\rho) \\ H_{zn}(\rho) \end{Bmatrix} = 0, \tag{21b}$$

where $\kappa_0^2 = \omega^2/c^2 - q_z^2$. They are Bessel equations if $\kappa_0^2 > 0$, or modified Bessel equations if $\kappa_0^2 < 0$. Taking the finiteness of the quantities $E_{zn}(\rho)$ and $H_{zn}(\rho)$ at $\rho \to \infty$ into account, solutions (21) outside the cylinder look like

$$E_{zn}(\rho) =$$

$$= \begin{cases} \begin{cases} B_n^E H_n^{(1)}(\kappa_0\rho) + C_n^E J_n(\kappa_0\rho) \\ \text{if } \kappa_0^2 > 0\colon (\kappa_0^2 \in \mathbb{C}), \\ B_n^E K_n(|\kappa_0|\rho) + C_n^E I_n(|\kappa_0|\rho) \\ \text{if } \kappa_0^2 < 0, \end{cases} & \text{if } \rho_0 \leqslant \rho \leqslant \rho_b, \\ \begin{cases} D_n^E H_n^{(1)}(\kappa_0\rho) \\ \text{if } \kappa_0^2 > 0\colon (\kappa_0^2 \in \mathbb{C}), \\ D_n^E K_n(|\kappa_0|\rho) \\ \text{if } \kappa_0^2 < 0, \end{cases} & \text{if } \rho \geqslant \rho_b, \end{cases}$$

$$H_{zn}(\rho) =$$

$$= \begin{cases} \begin{cases} B_n^H H_n^{(1)}(\kappa_0\rho) + C_n^H J_n(\kappa_0\rho) \\ \text{if } \kappa_0^2 > 0\colon (\kappa_0^2 \in \mathbb{C}), \\ B_n^H K_n(|\kappa_0|\rho) + C_n^H I_n(|\kappa_0|\rho) \\ \text{if } \kappa_0^2 < 0, \end{cases} & \text{if } \rho_0 \leqslant \rho \leqslant \rho_b, \\ \begin{cases} D_n^H H_n^{(1)}(\kappa_0\rho) \\ \text{if } \kappa_0^2 > 0\colon (\kappa_0^2 \in \mathbb{C}), \\ D_n^H K_n(|\kappa_0|\rho) \\ \text{if } \kappa_0^2 < 0, \end{cases} & \text{if } \rho \geqslant \rho_b, \end{cases}$$

where $H_n^{(1)}(x)$ and $K_n(x)$ are the Hankel functions of the first kind and the Macdonald functions, respectively; and $B_n^{E,H}$, $C_n^{E,H}$, and $D_n^{E,H}$ are arbitrary constants.

The other spectral components of the Fourier components of the electromagnetic field of the waveguide eigenwaves outside the electron beam ($\rho > \rho_0$ and $\rho \neq \rho_b$) look like

$$\kappa_0^2 E_{\rho n}(\rho) = iq_z \frac{\partial}{\partial\rho} E_{zn}(\rho) - \frac{\omega}{c}\frac{n}{\rho} H_{zn}(\rho),$$
$$\kappa_0^2 E_{\varphi n}(\rho) = -q_z \frac{n}{\rho} E_{zn}(\rho) - i\frac{\omega}{c}\frac{\partial}{\partial\rho} H_{zn}(\rho),$$
$$\kappa_0^2 H_{\rho n}(\rho) = iq_z \frac{\partial}{\partial\rho} H_{zn}(\rho) + \frac{\omega}{c}\frac{n}{\rho} E_{zn}(\rho),$$
$$\kappa_0^2 H_{\varphi n}(\rho) = -q_z \frac{n}{\rho} H_{zn}(\rho) + i\frac{\omega}{c}\frac{\partial}{\partial\rho} E_{zn}(\rho).$$

In the case of the Vavilov–Cherenkov effect, the condition $(q_z v_0 - \omega) H_{zn}(\rho) = 0$ holds. Whence, for the space-charge wave, $\omega = q_z v_0$.

Hence, the spectral components of the waveguide eigenwaves are superpositions of two partial waves different in amplitudes and wave numbers (κ_1 and κ_2). According to [5, 13, 14, 43], the electromagnetic fields of the waveguide eigenwaves correspond to bulk waves if $\kappa_{1,2}^2 > 0$ (or $\kappa_{1,2}^2 \in \mathbb{C}$) and $\kappa_0^2 > 0$ (or $\kappa_0^2 \in \mathbb{C}$), and to bulk-surface waves, the radiation of which into free space has a static nature, if $\kappa_0^2 < 0$. The eigenwaves are hybrid; their type (E or H) is determined by the dominant axial component of electromagnetic field [13, 43, 48].

In our case, $\kappa_0^2 < 0$ because for the SCWs generated by electrons in the case of the Vavilov–Cherenkov effect, we have $q_z = \omega/v_0$ and $\omega^2/c^2 \ll q_z^2$ since $v_0 \ll c$. Therefore, the axial spectral components of the electromagnetic field, $E_{zn}(\rho)$ and $H_{zn}(\rho)$, beyond the waveguide ($\rho > \rho_0$) have the form

$$E_{zn}(\rho) = \begin{cases} B_n^E K_n(|\kappa_0|\rho) + C_n^E I_n(|\kappa_0|\rho), & \rho_0 \leqslant \rho \leqslant \rho_b, \\ D_n^E K_n(|\kappa_0|\rho), & \rho \geqslant \rho_b, \end{cases}$$
$$H_{zn}(\rho) = \begin{cases} B_n^H K_n(|\kappa_0|\rho) + C_n^H I_n(|\kappa_0|\rho), & \rho_0 \leqslant \rho \leqslant \rho_b, \\ D_n^H K_n(|\kappa_0|\rho), & \rho \geqslant \rho_b. \end{cases}$$

Due to the resonance Vavilov–Cherenkov interaction of the SCW with the fields of eigenwaves in the plasma waveguide, we have $q_0 = 4\pi e N_0 v_0 / (H_0 c)$ and $\overline{q}_0^2 = -\omega_H^2 \omega^2 / \left(\nu^2 v_0^2\right)$. Therefore, q_0 has a real value and does not depend on the frequency ω, but it is determined by the concentration of plasma conduction electrons N_0 and such external factors as the induction of the external magnetic field H_0 and the electron velocity v_0 in the beam blowing around the waveguide. The perturbations of the force lines of the magnetic field H_0 are characterized by the wave number

$\overline{q}_0$ depending on the SCW frequency ω. The quantity $\overline{q}_0$ is determined by the induction H_0, such internal plasma parameters as the collision frequency ν and the effective electron mass m, and one external parameter, the velocity v_0.

Hence, due to the blowing of the magnetized tubular electron beam around the plasma waveguide, hybrid bulk-surface (with $\kappa_{1,2}^2 > 0$) or surface (with $\kappa_{1,2}^2 < 0$) waves of helicon origin are excited in the latter. If the squares of the transverse wave numbers $\kappa_{1,2}^2$ have different signs or they are complex-valued, hybrid bulk-surface eigenwaves are excited in the waveguide. In this case, the waveguide eigenwaves are excited by the beam SCW with the coordination of the field components $E_{zn}(\rho)$. According to [49–61], the stabilization mechanism of their amplitude growing is based on the effect of synchronous capture of beam electrons by the fields of coupled waves, one of which is the SCW in the electron flux, and the other the electromagnetic wave in the waveguide playing the role of decelerating structure.

7. Dispersion Equations

Note that the continuity conditions for the tangential components of the electric field strength $\mathbf{E}_\tau$ at the cylinder ($\rho = \rho_0$) and electron beam ($\rho = \rho_b$) surfaces, which are reduced to

$$E_{zn}(\rho_0 - 0) = E_{zn}(\rho_0 + 0),$$
$$E_{\varphi n}(\rho_0 - 0) = E_{zn}(\rho_0 + 0)$$

and

$$E_{zn}(\rho_b - 0) = E_{zn}(\rho_b + 0),$$
$$E_{\varphi n}(\rho_b - 0) = E_{zn}(\rho_b + 0),$$

are equivalent to the continuity conditions for the normal components of the magnetic field induction,i.e.,

$$H_{\rho n}(\rho_0 - 0) = H_{\rho n}(\rho_0 + 0)$$

and

$$H_{\rho n}(\rho_b - 0) = H_{\rho n}(\rho_b + 0),$$

respectively. In turn, condition (12) is equivalent to the continuity condition for the tangential component of the magnetic field strength $\mathbf{H}_\tau$, which is equivalent to

$$H_{zn}(\rho_0 - 0) = H_{zn}(\rho_0 + 0),$$
$$H_{\varphi n}(\rho_0 - 0) = H_{zn}(\rho_0 + 0).$$

At the beam surface, the discontinuity condition for $\mathbf{H}_\tau$ is reduced to

$$H_{zn}(\rho_b - 0) = H_{zn}(\rho_b + 0),$$
$$H_{\varphi n}(\rho_b + 0) - H_{\varphi n}(\rho_b - 0) = \frac{4\pi}{c} \lim_{\Delta\rho \to 0} \int\limits_{\rho_b - \Delta\rho}^{\rho_b + \Delta\rho} \sigma_b E_{zn}(\rho) d\rho,$$

where

$$\sigma_b = \frac{ie^2 \omega N_{b0}(\rho)}{m_0 \left(q_z v_0 - \omega\right)^2},$$

which is equivalent to the discontinuity condition for the normal component of the electric field induction,

$$E_{\varphi n}(\rho_b + 0) - E_{\varphi n}(\rho_b - 0) = \frac{4\pi e}{\rho_b} \lim_{\Delta\rho \to 0} \int\limits_{\rho_b - \Delta\rho}^{\rho_b + \Delta\rho} N(\mathbf{r}, t) \rho d\rho,$$

where

$$N(\mathbf{r}, t) = \frac{ieq_z N_{b0}(\rho) E_{zn}(\rho)}{m_0 \left(q_z v_0 - \omega\right)^2}.$$

The set of linearly independent boundary conditions at the cylinder ($\rho = \rho_0$) and electron beam ($\rho = \rho_b$) surfaces, which consists of conditions for the tangential components of the electromagnetic field, forms a quadratic system of homogeneous linear algebraic equations. A condition for the existence of its nontrivial solutions leads to a dispersion equation of electrodynamic system with coupled waves,

$$D_n(q_z, \omega) \left[\left(q_z v_0 - \omega\right)^2 + \Gamma_n(q_z, \omega) \omega_b^2 \right] = \\ = \alpha_n(q_z, \omega) \omega_b^2, \quad (22)$$

where

$$D_n(q_z, \omega) = \Delta_1^H \Delta_2^E - \Delta_2^H \Delta_1^E,$$

$$\Delta_{1,2}^H = \frac{q_z}{\kappa_{1,2}^2} \frac{\frac{\partial}{\partial\rho} Z_n(\kappa_{1,2}\rho_0)}{Z_n(\kappa_{1,2}\rho_0)} - \\ - \frac{q_z}{\kappa_0^2} \frac{\frac{\partial}{\partial\rho} K_n(|\kappa_0|\rho_0)}{K_n(|\kappa_0|\rho_0)} - \frac{n}{\rho_0} \left(\frac{b_{1,2}}{\kappa_{1,2}^2} + \frac{q_z^2 \gamma_{1,2}}{\kappa_0^2} \right),$$

$$\Delta_{1,2}^E = \frac{a_{1,2}}{\kappa_{1,2}^2} \frac{\frac{\partial}{\partial\rho} Z_n(\kappa_{1,2}\rho_0)}{Z_n(\kappa_{1,2}\rho_0)} + \\ + \frac{\gamma_{1,2}}{\kappa_0^2} \frac{\omega^2}{c^2} \frac{\frac{\partial}{\partial\rho} K_n(|\kappa_0|\rho_0)}{K_n(|\kappa_0|\rho_0)} - \frac{q_z n}{\rho_0} \left(\frac{1}{\kappa_{1,2}^2} - \frac{1}{\kappa_0^2} \right),$$

and $\omega_b^2 = 4\pi e^2 N_{b0}/m_0$ is the squared plasma frequency of beam electrons. The depression coefficient of space charge forces, $\Gamma_n(q_z,\omega)$, and the coefficient of beam coupling with the synchronous eigenmode of plasma-solid cylinder, $\alpha_n(q_z,\omega)$ [2, 62], look like

$$\Gamma_n(q_z,\omega) = \kappa_0^2 a\rho_b K_n(|\kappa_0|\rho_b) I_n(|\kappa_0|\rho_b) \times \times \left[1 - \frac{K_n(|\kappa_0|\rho_b) I_n(|\kappa_0|\rho_0)}{K_n(|\kappa_0|\rho_0) I_n(|\kappa_0|\rho_b)}\right],$$

$$\alpha_n(q_z,\omega) = \frac{a\rho_b\omega^2}{\rho_0 c^2}\frac{K_n^2(|\kappa_0|\rho_b)}{K_n^2(|\kappa_0|\rho_0)}\left(\Delta_1^H\gamma_2 - \Delta_2^H\gamma_1\right),$$

respectively. The coefficient $\Gamma_n(q_z,\omega)$ characterizes the variation of the space charge field: either in the case of transition from an infinitely thin beam to a beam with a finite cross-section or if the impact parameter changes from zero to a value of $\rho_b - \rho_0$. The coefficient $\alpha_n(q_z,\omega)$ has a maximum value if the beam is transported along the generatrices of the cylinder lateral surface – i.e., if $\rho_b = \rho_0$–and decreases with the increasing impact parameter.

Equation (22) is written in the form of a characteristic equation for the O-type traveling-wave tube [2]. In our case, it determines the resonance interaction of the beam SCW with the cylinder eigenwaves in the case of the Vavilov–Cherenkov effect.

In the absence of the electron beam in the electrodynamic system ($\omega_b = 0$, $v_0 = 0$), Eq. (22) is reduced to the dispersion equation of magnetoplasma cylinder in the quasi-stationary approximation [25],

$$D_n(q_z,\omega) = \Delta_1^H\Delta_2^E - \Delta_2^H\Delta_1^E = 0. \tag{23}$$

Depending on the values of the transverse wave numbers $\kappa_{1,2}$, its solutions are characterized the bulk-surface (if $\kappa_{1,2}^2 \in \mathbb{C}$ or $\kappa_{1,2}^2 > 0$) or surface (if $\kappa_{1,2}^2 < 0$) cylinder eigenwaves.

If $\rho_b \gg \rho_0$ (including the limiting case $\rho_b \to \infty$), the waveguide radius ρ_0 is constant, $\alpha_n(q_z,\omega) \to 0$, and the electrodynamic system has two non-interacting subsystems – a plasma-solid cylinder and an electron flux – the dispersion equation (22) splits into two equations, $D_n(q_z,\omega) = 0$ and

$$\left(q_z v_0 - \omega\right)^2 + \Gamma_n(q_z,\omega)\omega_b^2 = 0. \tag{24}$$

Equation (24) is the dispersion equation for a tubular electron beam with a finite diameter a. The coefficient $\Gamma_n(q_z,\omega)$ characterizes the variation of the space charge field when changing from an infinitesimally thin tubular beam, for which the SCW frequencies equal $\omega = q_z v_0$, to a beam with a finite cross-section and a weakly influencing cylindrical inhomogeneity of radius $\rho_0 \ll \rho_b$, for which the SCW frequencies are determined by the solutions of (24).

In the electrostatic approximation (which is equivalent to the case $c \to \infty$), we have

$$\Gamma_n(q_z,\omega) = -\Gamma_n(q_z) = q_z^2 a\rho_b K_n(|\kappa_0|\rho_b) I_n(|\kappa_0|\rho_b) \times \times \left[1 - \frac{K_n(|\kappa_0|\rho_b) I_n(|\kappa_0|\rho_0)}{K_n(|\kappa_0|\rho_0) I_n(|\kappa_0|\rho_b)}\right],$$

i.e., the coefficient Γ_n does not depend on the frequency. Then Eq. (24) becomes similar to [62],

$$\left(q_z v_0 - \omega\right)^2 - \Gamma_n(q_z)\omega_b^2 = 0,$$

and its solutions determine the frequencies of the slow (ω_-) and fast (ω_+) SCWs of the beam:

$$\omega_\mp = q_z v_0 \mp \omega_b\sqrt{\Gamma_n(q_z)}.$$

The phase velocities of those waves, $\omega_\mp/q_z$, are lower and higher, respectively, than the velocity v_0.

The solutions of the dispersion equation (23) determine the dispersion dependences $\omega_{ns}(q_z)$ for the eigenwaves of the cylindrical solid-state-plasma waveguide embedded in a strong constant coaxial magnetic field. The mode index $s = 1, 2, 3, \ldots$ is a result of the multi-valued solution of the transcendental equation (23) and corresponds to the number of field variations along the radial coordinate ρ. The beam–cylinder interaction affects the frequencies ω_{ns} of waveguide eigenwaves and the corresponding wave numbers q_{zns} with the corresponding shifts $\delta\omega$ and δq_z. Since $\omega_b \ll \omega_{ns}$, the shift magnitudes are small, $|\delta\omega| \ll \omega_{ns}$, which leads to the "swinging" of the cylinder eigenwaves with an energy exchange between them and the SCWs. In this case, according to the variational calculus, equation (22) acquires the form

$$\left(q_{zns}v_0 - \omega_{ns} + v_0\delta q_z - \delta\omega\right)^2 + \left[\Gamma_n(q_{zns},\omega_{ns}) + + \delta q_z\Gamma'_{nq_z}(q_{zns},\omega_{ns}) + \delta\omega\Gamma'_{n\omega}(q_{zns},\omega_{ns}) - - \frac{\delta q_z\alpha'_{nq_z}(q_{zns},\omega_{ns}) + \delta\omega\alpha'_{n\omega}(q_{zns},\omega_{ns})}{\delta q_z D'_{nq_z}(q_{zns},\omega_{ns}) + \delta\omega D'_{n\omega}(q_{zns},\omega_{ns})}\right]\omega_b^2 = 0,$$

where $D'_{n\omega}(q_{zns},\omega_{ns})$, $\Gamma'_{n\omega}(q_{zns},\omega_{ns})$, $\alpha'_{n\omega}(q_{zns},\omega_{ns})$ and $D'_{nq_z}(q_{zns},\omega_{ns})$, $\Gamma'_{nq_z}(q_{zns},\omega_{ns})$, $\alpha'_{nq_z}(q_{zns},\omega_{ns})$ are derivatives of the corresponding functions $D_n(q_z,\omega)$, $\Gamma_n(q_z,\omega)$, and $\alpha_n(q_z,\omega)$ with respect to the frequency ω and the wave number q_z, respectively, calculated at the resonance point(q_{zns},ω_{ns}), i.e., the intersection point of the line $\omega = q_z v_0$ and the curve $D_n(q_z,\omega) = 0$ corresponding to the index s on the plot of the dispersion dependences of the SCWs and the cylinder eigenwaves.

In the Vavilov–Cherenkov effect case – i.e., if the electron velocity v_0 satisfies the condition $q_{zns}v_0 = \omega_{ns}$ – and when transporting the tubular beam along the cylinder surface – i.e., if $\rho_b = \rho_0$ so that $\Gamma_n(q_{zns},\omega_{ns}) = 0$, $\Gamma'_{nq_z}(q_{zns},\omega_{ns}) = 0$, and $\Gamma'_{n\omega}(q_{zns},\omega_{ns}) = 0$ – we obtain

$$\left(v_0\delta q_z - \delta\omega\right)^2 = \\ = \frac{\delta q_z \alpha'_{nq_z}(q_{zns},\omega_{ns}) + \delta\omega\alpha'_{n\omega}(q_{zns},\omega_{ns})}{\delta q_z D'_{nq_z}(q_{zns},\omega_{ns}) + \delta\omega D'_{n\omega}(q_{zns},\omega_{ns})}\omega_b^2 = 0.$$

In the case $\delta q_z = \text{const}$, this equation has two complex-conjugate roots $\delta\omega_{1,2}$. One of them has a positive imaginary part, corresponding to the growth of the wave amplitude in time. The root with the negative imaginary part corresponds to a fading wave. The frequency of those waves equals $\omega_{ns} + \Re\delta\omega$. If $\delta q_z = 0$, the instability increment $\Im\delta\omega = \omega_b\Im\left[\alpha'_{n\omega}(q_{zns},\omega_{ns})/D'_{n\omega}(q_{zns},\omega_{ns})\right]$ is proportional to $\sqrt{N_{b0}}$. In the case $\Re\delta\omega < 0$, the excited waves are slow (low-frequency) because their phase velocities are lower than the velocity of beam electrons, and in the case $\Re\delta\omega > 0$ they are fast (high-frequency). In the case of a Gaussian particle energy distribution, which corresponds to real beams, the number of particles that loss their energy for the excitation of slow waves exceeds the number of particles that transfer their energy to fast waves in the waveguide. The monotronic generation mechanism of slow electromagnetic waves is implemented in the electrodynamic system, when charged beam particles are grouped around such a phase of the electromagnetic field excited by themselves that, on average, they give up their energy to the cylinder eigenwaves.

The additional attention should be paid to the fact that solutions (23) for modes with identical field distributions but different by sign azimuthal indices do not coincide. Moreover, the solutions obtained for different directions of the external magnetic field, which differ in phase by π, do not coincide as well. This circumstance means that the eigenwaves of magnetoplasma cylinder propagating in opposite azimuthal directions have different phase velocities ω_{ns}/q_{zns}. This is a manifestation of the non-reciprocity principle for the propagation of eigenwaves in a plasma cylinder located in a strong coaxial magnetic field. This nonreciprocity effect leads to the elimination of the frequency degeneration with respect to the azimuthal index. The eigenwave non-reciprocity associated with the direction of $\mathbf{H}_0$ is substantiated by the opposite rotation directions of conduction electrons in solid-state plasma in the external magnetic field.

Note also that Eqs. (22)–(24) hold for electrodynamic systems containing a cylindrical magnetoplasma solid-state resonator where quasi-stationary conditions take place. If the waveguide is confined in the longitudinal direction between ideally conductive end surfaces $z = 0$ and $z = L$, the longitudinal wave numbers of eigenmodes are $q_z \equiv q_{znsl} = \pi l/L$, where L is the resonator length, and $l \in \mathbb{N}_0$ is the axial (longitudinal) mode index corresponding to the number of field variations (half-wavelengths) along the axial coordinate z. Then the solutions of equations (23) determine the resonator eigenfrequencies ω_{nsl} corresponding to the frequencies of eigenmodes with the wave numbers q_{znsl}. It is evident that, in the case of the Vavilov–Cherenkov effect in the electrodynamic system, a uniform distribution of the electromagnetic field in the resonator along the direction z is impossible. Therefore, in our case, the axial index $l \in \mathbb{N}$. From the synchronism condition, it follows that $\omega_{nsl} = \pi l v_0/L$ and only one ns pair corresponds to a specific index l. The efficiency of the monotronic excitation mechanism of the nsl-th eigenmode of the resonator is maximum if its dimensionless length is $\theta_{nsl} = \omega_{nsl}L/v_0 + \pi/2 = \pi(l + 1/2)$. In the case of the waveguide of finite length L (without conductive end walls), the longitudinal wave numbers of resonator eigenmodes are $q_z \equiv q_{znsl} = \pi\delta/L$, where the axial mode index is $\delta \in \mathbb{R}$. The monotron mechanism of the excitation of the resonator eigenmode has the maximum efficiency if $\theta_{ns\delta} = \omega_{ns\delta}L/v_0 \approx 7.72$, which corresponds to the first generation zone: $2\pi < \theta_{ns\delta} < 9$ [63, 64].

8. Conclusions

In this work, a theory describing a radially infinitely thin tubular flux of nonrelativistic magnetized electrons blowing around a plasma-solid-state cylinder has been presented. The action of an external longitudinal magnetic field leads to a collective motion of charged plasma particles (electrons) along the force lines. Periodic fluctuation processes in the flux of plasma conduction electrons disturb the magnetic field lines via the presence of an alternating electric current. The collision-free plasma cylinder supports the propagation of helicons. Their propagation is accompanied by the appearance of a surface current running along the cylinder generatrices. Collisions of charged particles destroy the surface current.

Owing to the high conductivity of plasma in the waveguide, conditions required for the formation of quasi-stationary electromagnetic fields of eigenwaves are realized. It is found that the waves in the waveguide are formed as superpositions of partial hybrid waves of the helicon origin. It is established that hybrid bulk-surface electromagnetic waves with positive squared transverse wave numbers ($\kappa_{1,2}^2 > 0$) or surface waves with $\kappa_{1,2}^2 < 0$ that are excited in the cylinder are of the helical origin. If the squared transverse wave numbers $\kappa_{1,2}^2$ have different signs, or if they are complex numbers, hybrid bulk-surface waves are excited in the waveguide.

The effect of propagation non-reciprocity has been considered for the waveguide eigenwaves with identical field distribution structures but different, by the azimuthal coordinate, propagation directions. This effect also takes place in the case of different directions of the external magnetic field also different by their phases by π.

The waveguide eigenwaves are excited by the field of the beam space charge with the matching of the longitudinal electric field components. The instability of coupled waves in the electrodynamic system "tubular electron beam–magnetoplasma cylinder" located in a strong longitudinal magnetic field has been studied. It was shown that this instability is induced by the Vavilov–Cherenkov effect. In the case of extremely small impact parameter of the beam, the instability increment is proportional to $\sqrt{N_{b0}}$. The main mechanism of generation of slow (low-frequency) electromagnetic waves in a plasma solid-state waveguide or resonator is the monotronic mechanism.

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НЕСТІЙКІСТЬ ТРУБЧАСТОГО ЕЛЕКТРОННОГО ПУЧКА У РАЗІ ОБДУВАННЯ ПЛАЗМОВОГО ТВЕРДОТІЛЬНОГО ЦИЛІНДРА, ЯКИЙ РОЗМІЩЕНО У СИЛЬНОМУ ПОЗДОВЖНЬОМУ МАГНІТНОМУ ПОЛІ

Теоретично досліджено електродинамічну систему, в якій замагнічений трубчастий пучок електронів обдуває циліндричний плазмово-твердотільний хвилевід. Встановлено, що у разі виконання квазістаціонарних умов у хвилеводі збуджуються гібридні об'ємно-поверхневі або поверхневі електромагнітні хвилі геліконного походження. Збудження власних хвиль хвилеводу здійснюється полем просторового заряду пучка із узгодженням поздовжніх спектральних складових електричного поля. Відзначено ефект невзаємності власних хвиль хвилеводу з ідентичною структурою розподілу полів, але таких, що відрізняються поширенням в азимутальному напрямі, а також у разі зміни напряму зовнішнього магнітного поля. Показано, що нестійкість зв'язаних хвиль електродинамічної системи зумовлена ефектом Вавілова–Черенкова.

Ключові слова: електронний пучок, хвиля просторового заряду, власні та зв'язані хвилі, пучкова нестійкість, інкремент нестійкості, ефект Вавілова–Черенкова.