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AXION HALO AROUND A BINARY SYSTEM OF DWARF STARS [1]

The gravitational field of a clump of ultralight axion like particles (ALPs) in its core with a rotating binary system of dwarf stars is computed. It is established that the induced quadrupole mass moment of the clump is controlled parametrically by the M_a/M mass ratio of the axion clump and the binary core.

Keywords: axion like particles, axion halo, dwarf star.

1. Introduction

Evidence of the existence of stars composed from gravitating light scalar particles represents durable challenge to astrophysical research [1, 2]. Various aspects of stationary equilibrium configurations where gravitational attraction is compensated by the kinetic pressure of the constituents were investigated by many authors [3–9]. Dynamics of the scalar star formation has been explored by the kinetic simulation of ensembles of gravitationally interacting free particles [10].

Particularly interesting is the research direction where the galactic halo formed by ultralight constituents is built from superpositions of quantum waves. In this case, the kinetic pressure compensating the gravitational attraction has quantum origin, and it would represent a quantum coherent phenomenon on the largest known scale. The original proposition [11] has been baptised as ψDM by Schive *et al.* [12] emphasizing the role of quantum uncertainty countering gravitation below the Jeans scale. Applying this balance requirement to dwarf spheroidal galaxies, a lower limit for the mass of the superlight dark matter particles was deduced. More recently, some progress has been achieved in self-consistent determination of the quantum superposition reproducing the observed dark matter halo density profile of dwarf spheroidal galaxies [13–15].

With the advent of black hole observations research has intensified on overdensities of axion like particles (ALP) producing primordial black holes (BH) in an era preceding the inflation. Such objects would not evaporate till today, if their mass is larger than $10^{-15}\ M_\odot$. In the gravitational collapse of axions also the emergence of BH pairs has non-zero chance [16]. Around this kind of binary BH centers of gravitational force the surrounding axion minihalo might further condensate, eventually producing a scalar star [17].

The equilibrium ALP configuration around a single BH is spherically symmetric. In the case of nonrelativistic motion of the halo particles the gravitationally bound axion clump forms a so-called gravitational atom. Higher energy configurations with nonzero angular momentum might also arise dynamically. One scenario considers a second BH falling on a gravitational atom, which resonantly induces transitions to configurations of nonzero quadrupole (and, possibly, also higher) moments [18, 19]. Such transitions would produce characteristic observable effects in the gravitational waves emitted by the system.

In this note I wish to discuss the interaction of ALPs with another gravitationally bound compact system, binaries of dwarf stars. Systematic search for brown dwarfs has been started in the 1990s with observing transiting light curves arising during the passage of brown dwarfs in front of light emitting stars. Very soon binary systems consisting of an ordi-

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nary white dwarf star and an accompanying brown dwarf were discovered. About 5–6% of the known brown dwarfs has a lighting star companion [20]. From statistical analyses, one estimates the separation of the partners in the range of 1.5–1000 au. The mass ratio of the members peaks around unity. The even more difficult observation of a system consisting of two brown dwarfs with ∼1 au separation has been announced very recently [21]. The masses of the partners were estimated to lie in the interval of 8–20 $M_{\rm Jupiter}$. The period of the rotation lies between 5 to 9 years. The corresponding power of gravitational radiation is $\approx 10^{11}$ erg/s by a simple textbook estimate [22], hoplessly low for present instruments. More encouraging is a very spectacular recent report on a rather massive ($M_{BD} \approx 80\ M_{\rm Jupiter}$) brown dwarf transiting in front of a low mass star ($M_* \approx 0.13\ M_\odot$). They are very tightly bound with a period of ∼2 hours [23]. The Keplerian separation is less than the size of our Sun. In this case, the simple estimate of the intensity of gravitational radiation gives nearly 4% of the electromagnetic radiation power of the Sun. These discoveries motivate us to investigate the structure and dynamical features of ALP clumps around a binary brown dwarf core.

In our analysis presented below, the orbiting binary gravitational system will be treated as a pointlike source characterized by the lowest (possibly, time-dependent) multipoles of its density distribution. An obvious condition for this is that the Compton wavelength of ALPs should be much larger than the size of the binary core. The latest brown dwarf discoveries offer a realistic ALP mass range for this to be satisfied. The radius of the Sun is $R_\odot \sim 10^6$ km, 1 au $\sim 10^8$ km. For an ALP of mass 10^{-n} eV the Compton wavelength ($1/m$) is at least 100 times larger than the characteristic size of the source in the first case for $n \geq 18$, in the second for $n \geq 20$. This mass range corresponds to the class of *ultralight* ALPs. After determining the density distribution produced by the binary source and the gravitational self-interaction of the axionlike particles, one has to check also, if the condition that the clump size R exceeds the Compton wavelength of the particle $1/m$, e.g., $mR > 1$ is fulfilled.

Below, we shall determine the profile function of the ALP clump in an approximation, where one truncates the multipole expansion of the gravitational field of the (pointlike) binary core at its quadrupole moment. The particle distribution will be composed from the lowest energy configurations of the $l = 0, 2$ angular momentum channels. For the gravitational binding energy estimates, a variational strategy [9] will be applied (see also [24, 25]). The quadrupole deviation of the ALP profile function from spherical symmetry will be determined to linear order. The temporal variation of the elements of the quadrupole tensor of the binary brown dwarf system induces time dependence into the quadrupole piece of the ALP profile. The resulting additional gravitational radiation might offer further insight into the nature of the hypothetical ultralight constituents of matter.

2. Determination of the Axion Halo Profile

Our simplified model for the binary system of two brown dwarfs consists of two $M/2$ mass objects orbiting with angular velocity ω along a circle of radius d and located in diametrically opposite positions. The gravitational potential will be truncated at quadrupole order

$$V_N(\mathbf{x}) = -\frac{G_N M}{2}\left(\frac{1}{|\mathbf{x}-\mathbf{d}|} + \frac{1}{|\mathbf{x}+\mathbf{d}|}\right) \approx$$
$$\approx -\frac{G_N}{r}\left(M + \frac{1}{r^2}\Theta_{2m}Y_{2m}(\hat{\mathbf{x}})\right), \tag{1}$$
$$\Theta_{2m} = \frac{4\pi}{5} M d^2 Y_{2,-m}(\hat{\mathbf{d}}(t)).$$

In natural units ($\hbar = c = 1$) the quadrupole moment has inverse mass scaling dimension. Choosing the plane of the orbit for the (x, y)-plane, only indices $m = 0, 2, -2$ contribute to the above sum over m. The time dependence of $\mathbf{d}$ leads to the time dependence of $\Theta_{2,\pm 2}$. One can exploit that $Y^*_{22} = Y_{2,-2}$ and $Y_{2m}(-\hat{\mathbf{d}}) = Y_{2m}(\hat{\mathbf{d}}), m = 0, 2, -2$. The unit vector $\hat{\mathbf{d}}(t)$ points to one of them from the origin, $\hat{\mathbf{x}}$ points to the direction of the observation. (The detailed structure of the binary dwarf beyond the data M, Θ_{2m} does not play any role in the discussion below.)

The energy of the axion "halo" around the binary core is given by

$$H = \int d^3x \frac{1}{2}\left[\dot{a}^2(\mathbf{x},t) + (\nabla a(\mathbf{x},t))^2 + m^2 a^2(\mathbf{x},t)\right] +$$
$$+ \int d^2x \rho_a(\mathbf{x},t) V_N(\mathbf{x},t) -$$
$$- \frac{G_N}{2}\int d^3x \int d^3y \frac{\rho_a(\mathbf{x},t)\rho_a(\mathbf{y},t)}{|\mathbf{x}-\mathbf{y}|}. \tag{2}$$

The second line of the above expression gives the energy of particles of mass density ρ_a moving in the gravitational potential V_N, while the last term corresponds to the energy of the gravitational attraction among the ALPs constituting the halo.

The assumption for a non-relativistic motion of the particles is reflected in the following parametrisation of the axion field:

$$a(\mathbf{x},t)=\frac{1}{\sqrt{2m}}\left(\psi(\mathbf{x},t)e^{-imt}+\psi^*(\mathbf{x},t)e^{imt}\right). \tag{3}$$

The slowly varying function ψ is normalized to the number of particles the halo consists of:

$$\int d^3x|\psi(\mathbf{x},t)|^2=N_a, \tag{4}$$

which implies $\rho_a(\mathbf{x},t)=m|\psi(\mathbf{x},t)|^2$. Because of the assumed slow variation of $\psi(\mathbf{x},t)$ only the first time derivative is retained in its equation of motion:

$$\dot{\psi}(\mathbf{x},t)=-\frac{1}{2m}\triangle\psi(\mathbf{x},t)+V_Nm\psi(\mathbf{x},t)-$$
$$-G_Nm^2\int d^3y\frac{|\psi(\mathbf{y},t)|^2}{|\mathbf{x}-\mathbf{y}|}\psi(\mathbf{x},t). \tag{5}$$

The quadrupole part of (1) induces a piece into the profile funcion $\sim Y_{2m}$. This piece will be determined perturbatively to leading order, therefore it will be proportional also to the dimensionless combination $m\Theta_{2m}$. In the ansatz chosen for the approximate solution of (5) a coefficient function is introduced in both angular momentum channels depending on the radial coordinate scaled by a characteristic size parameter R

$$\psi(\mathbf{x},t)=e^{i\mu t}(\psi_0(\mathbf{x})+\Delta\psi_0(\mathbf{x}))=$$
$$=e^{i\mu t}w(F_0(\xi)+\tilde{F}_{2m}(\xi)m\Theta_{2m}(d)Y_{2m}(\hat{\mathbf{x}})), \tag{6}$$
$$\xi=\frac{r}{R},\quad \hat{\mathbf{x}}=\frac{\mathbf{x}}{r},\quad r=|\mathbf{x}|.$$

The R parameter characterising the size of the axion clump will be determined variationally. w is a constant to be found from the normalisation (4).

In the calculation described below, one adopts an approximation scheme, where the quadrupole piece of the gravitational potential acts perturbatively on the profile of the axion clump relative to the spherically symmetric part of the interaction. This assumption means that in (4) we work to linear order in $\Delta\psi$. Then the normalization reads as

$$w^2R^3\left(4\pi\int d\xi\xi^2F_0^2(\xi)\right)\equiv w^2R^3C_2=N_a. \tag{7}$$

The radial dependence of the quadrupole part of the profile will be the same for all values of m: $\tilde{F}_{2m}=\tilde{F}_2$.

Our goal is to compute the additional piece of the gravitational potential of the binary star created by the ALP halo far beyond of its extension. One arrives at its expression by the following sequence of equalities (below, $\eta=y/R$):

$$\Delta V_N(\mathbf{x})=-G_N\int d^3y\frac{\rho_a(\mathbf{y})}{|\mathbf{y}-\mathbf{x}|}\approx$$
$$\approx-G_Nmw^2\int d^3y\frac{1}{|\mathbf{y}-\mathbf{x}|}\times$$
$$\times\Big[F_0^2(\eta)+2F_0(\eta)\tilde{F}_2(\eta)m\Theta_{2m}(d)Y_{2m}(\hat{\mathbf{y}})\Big]=$$
$$=-G_Nmw^2\Big[\frac{1}{r}\int d^3yF_0^2(\eta)+$$
$$+\frac{8\pi R^5}{5r^3}\int d\eta\eta^4F_0(\eta)\tilde{F}_2(\eta)m\Theta_{2m}Y_{2m}(\hat{\mathbf{r}})\Big]. \tag{8}$$

From the very last line, one reads off the contribution of the ALP-halo to the quadrupole moment of the system. The complete moment is the sum of this and the original:

$$\Theta_{2m}^{sum}=\Theta_{2m}\left(1+\frac{8\pi N_a(mR)^2}{5C_2}\int d\eta\eta^4F_0(\eta)\tilde{F}_2(\eta)\right). \tag{9}$$

Clearly, the square of the expression in the bracket will multiply the power of the gravitational radiation. Therefore, the parametric dependence of $\tilde{F}_2$ on the dimensionless quantities N_a, mR, G_Nm^2 will be decisive in estimating the effect of the halo on the gravitational power.

3. Determination of $\tilde{F}_2$

In this section, we determine $\tilde{F}_2$ which is the $l=2$ admixture to the spherically symmetric profile function $F_0(\xi)$ under the action of the quadrupole part of the gravitational potential. First, we write the operator on the right hand side of (5) as a sum:

$$H=H_0+H_I,\quad H_0=-\frac{1}{2m}\triangle-\frac{G_NMm}{r},$$

$$H_I = -\frac{G_N m}{r^3}\Theta_{2m}Y_{2m}(\hat{\mathbf{d}}(t)) - \\ - G_N m^2 \int d^3y \frac{|\psi(y)|^2}{|\mathbf{x}-\mathbf{y}|}. \tag{10}$$

The eigenvalue problem of H_0 is the gravitational analog of the hydrogen atom of quantum mechanics. The corresponding eigenvalue-eigenfunction pairs in the $l = 0, 2$ channels are denoted as μ_0, F_0 and μ_2, F_2Y_{2m}, respectively. (Be careful: the function F_2 is not the admixture $\tilde{F}_2$, we are after!)

The second term of H_I corrects the value of μ_0 in the first order of perturbation theory. In its evaluation, one can neglect, in the kernel of the operator, the $l = 2$ admixture of ψ_0 which is perturbatively of higher order. Then the following expression can be readily obtained:

$$\mu_0 N_a = w^2 \int d^3x \left[\frac{1}{2m}(\nabla F_0(\xi))^2 - \frac{G_N M m}{|\mathbf{x}|}F_0^2(\xi) - \\ - G_N w^2 m^2 \int d^3y \frac{F_0^2(\eta)}{|\mathbf{x}-\mathbf{y}|}F_0^2(\xi)\right], \tag{11}$$

where the quantities w, η, ξ were introduced in the previous section. This expression displays a more transparent dependence on the characteristic dimensionless parameter combinations N_a, mR and $G_N m^2$, when one writes the integrals in terms of the scaled variables η, ξ:

$$\mu_0 N_a = m N_a \frac{1}{C_2}\left[\frac{D_2}{2}\frac{1}{(mR)^2} - \\ - \frac{G_N m^2}{mR}\left(\frac{B_4}{C_2}N_a + \frac{M}{m}C_1\right)\right], \tag{12}$$

where the following integrals of the profile function appear:

$$C_n = 4\pi \int_0^\infty d\xi \xi^n F_0^2(\xi), \quad D_n = 4\pi \int_0^\infty d\xi \xi^n F_0'^2(\xi), \\ B_4 = 32\pi^2 \int_0^\infty d\xi \xi F_0^2(\xi) \int_0^\xi d\eta \eta^2 F_0^2(\eta). \tag{13}$$

In similar steps, one finds the expression of μ_2 with first perturbative order accuracy:

$$\mu_2 = \frac{m}{I_2}\left[\frac{1}{2(mR)^2}(K_2 + 6I_0) - \\ - \frac{G_N m^2}{mR}\left(\frac{M}{m}I_1 + \frac{N_a}{C_2}I_{J1}\right)\right], \tag{14}$$

with

$$I_n = \int_0^\infty d\xi \xi^n F_2^2(\xi), \quad K_2 = \int_0^\infty d\xi \xi^2 \left(F_2'(\xi)\right)^2, \\ I_{J1} = 4\pi \int_0^\infty d\xi \xi \int_0^\xi d\eta \eta^2 \left[F_2^2(\xi)F_0^2(\eta) + F_2^2(\eta)F_0^2(\xi)\right], \tag{15}$$

Here, we use the same radial profile function $F_2(\xi)$ for all 5 components of the quadrupole eigenfunction, which is chosen $wF_2(\xi)Y_{2m}(\hat{\mathbf{x}})$, for formal uniqueness.

The best estimate for the eigenvalue μ_0 corrected by the nonlinear term of H_I with a conveniently chosen zeroth order profile function $F_0(\xi)$ is found by minimizing the right hand side of (12) with respect to mR and keeping N_a, $G_N m^2$, M/m fixed [9, 24, 25]. The optimal estimates for mR and μ_0 are the following:

$$(mR)_{\rm opt} = D_2\left[G_N m^2\left(\frac{B_4}{C_2}N_a + C_1\frac{M}{m}\right)\right]^{-1}, \\ \mu_{0,\rm opt} = -\frac{m}{2C_2D_2}\left[G_N m^2\left(\frac{B_4}{C_2}N_a + C_1\frac{M}{m}\right)\right]^2. \tag{16}$$

Although, in principle, one can optimize μ_2 independently, we will be satisfied using the same scale R also for F_2.

Let us discuss the consistency of the applied approximations against the parameter range presented in the introduction. Choosing $m \sim 10^{-17}$ eV, one finds, with $M \sim M_{\rm Jupiter}$, the following order of magnitude of the values

$$G_N m^2 \sim 10^{-90}, \quad \frac{M}{m} \sim 10^{80}. \tag{17}$$

The order of magnitude of the combination of profile function integrals (e.g. D_2C_1) is at most $\mathcal{O}(10^2)$. Therefore

$$(mR)_{\rm opt} \sim \mathcal{O}(10^{-2})10^{10}, \quad |\mu_{0\rm opt}| \sim 10^{-16}\ m. \tag{18}$$

The consistency conditions $mR > 1$ and $|\mu_0| \ll m$ are thus fulfilled. The mass contained in the halo around a Jupiter-size brown dwarf binary is well approximated therefore as $N_a m$. Choosing N_a the same order of magnitude as M/m leads to $M_{\rm halo} \sim \sim M_{\rm Jupiter}$. One can quickly check that the consistency conditions are satisfied even for the high mass ($\sim 10^2 M_{\rm Jupiter}$) transiting brown dwarf candidate announced in Ref. [23].

The first term of the operator H_I which corresponds to the quadrupole part of the gravitational field of the binary core has nonzero matrix element between F_0 and F_2Y_{2m}:

$$\langle l=2,m|H_I|0\rangle = w^2\int d^3yF_2(\eta)Y^*_{2m}(\hat{\mathbf{y}})\times$$
$$\times\left(-\frac{G_Nm}{|\mathbf{y}|^3}\Theta_{2p}Y_{2p}(\hat{\mathbf{y}})\right)F_0(\eta). \quad (19)$$

Therefore it generates the first order perturbative correction of the lowest energy ALP configuration. The leading quadrupole correction of the profile function $\Delta\psi_0$ is determined using the (familiar from quantum) first order perturbative relation

$$\Delta\psi_0(\mathbf{x}) = \frac{wF_2(\xi)Y_{2m}(\hat{\mathbf{x}})}{\mu_0-\mu_2}\frac{\langle l=2,m|H_I|0\rangle}{\langle l=2,m|l=2,m\rangle} =$$
$$= -w\frac{G_Nm^2}{(mR)^3}\frac{I_{20}^{(-1)}}{I_2}\frac{m}{\mu_0-\mu_2}m\Theta_{2p}(\hat{\mathbf{d}})Y_{2p}(\hat{\mathbf{x}})F_2(\xi)\equiv$$
$$\equiv w\tilde{F}_{2m}(\xi)m\Theta_{2p}(d)Y_{2p}(\hat{\mathbf{x}}) \quad (20)$$

with

$$I_{20}^{(n)} = \int_0^\infty d\eta\eta^nF_2(\eta)F_0(\eta). \quad (21)$$

4. Discussion of the Result

In this note, we computed the gravitational potential of an axion cloud with its quadrupole distortion induced by a rotating binary dwarf star system in its core:

$$\Delta V_N = -\frac{G_NN_am}{r} - \frac{G_N}{r^3}\Theta_{2p}Y_{2p}\times$$
$$\times\frac{8\pi}{5C_2}\frac{I_{20}^{(-1)}I_{20}^{(4)}}{I_2}\frac{m}{\mu_0-\mu_2}\frac{N_aG_Nm^2}{mR}. \quad (22)$$

The first term is the contribution of the axion clump to the Newton potential outside the compact object. Adding the second term to the quadrupole piece of the gravitational field of the core, we easily find the "amplification" factor of the quadrupole potential due to the axion halo:

$$Z_{\text{axion}} = 1-\frac{8\pi}{5C_2}\frac{I_{20}^{(-1)}I_{20}^{(4)}}{I_2}\frac{m}{\mu_0-\mu_2}\frac{N_aG_Nm^2}{mR}. \quad (23)$$

If one would optimize both Schrödinger-like eigenvalues μ_0 and μ_2, one would find parametrically $\mu_2-\mu_0\sim(mR)^{-2}$. The same parametric dependence is suggested by the analogy with the Balmer-formula of the hidrogen atom. Then we can write parametrically

$$Z_{\text{axion}} = 1+\text{const}\times(mR)N_a(G_Nm^2), \quad (24)$$

which by the optimized expression of mR leads to

$$Z_{\text{axion}} = 1+\text{const}\times\frac{N_am}{(B_4/C_2)N_am+C_1M}. \quad (25)$$

We can conclude that the amplification of the quadrupole moment parametrically depends mainly on the ratio $M_a/M=(N_am)/M$. If the mass of the axion clump reaches that of the core then it contributes to the gravitational radiation of the system parametrically the same amount as the core itself.

In order to present a quantitative estimate for the size of the extra gravitational power originating from the axion halo around the binary brown star system, one has to evaluate (15) and (21) with some well motivated choice of the profile functions F_0 and F_2. A physically appealing choice offered by the close formal analogy of the lowest energy configurations of the gravitational "atom" with the $1s$ and $3d$ levels of the hydrogen atom. Then the approach in Refs. [26, 27] can be followed choosing for the profile functions the following trial expressions:

$$F_0(\xi)=Q_0e^{-\xi},\quad F_2(\xi)=Q_2\xi^2e^{-\xi/3}. \quad (26)$$

The arbitrary normalisation coefficients Q_0,Q_2 do not appear in any physically meaningful quantity. Straightforward elementary integrations yield explicit values for the coefficients, but do not offer any deeper insight. This excercise is left for the readers.

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А. Паткош

АКСІОННЕ ГАЛО НАВКОЛО ПОДВІЙНОЇ СИСТЕМИ КАРЛИКОВИХ ЗІРОК

Розраховано ґравітаційне поле згустку надлегких аксіоноподібних частинок (ALP) з обертовою подвійною системою карликових зірок у його ядрі. Встановлено, що індукований квадрупольний момент маси згустку визначається параметром відношення мас M_a/M згустку аксіонів і бінарного ядра.

Ключові слова: надлегкі аксіоноподібні частинки, аксіонне гало, карликова зірка.