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VIOLATION OF γ IN BRANS–DICKE GRAVITY [1]

The Brans Class I solution in Brans–Dicke gravity is a staple in the study of gravitational theories beyond General Relativity. Discovered in 1961, it describes the exterior vacuum of a spherical Brans–Dicke star and is characterized by two adjustable parameters. Surprisingly, the relationship between these parameters and the properties of the star has not been rigorously established. In this article, we bridge this gap by deriving the *complete exterior solution of Brans Class I, expressed in terms of the total energy and total pressure of the spherisymmetric gravity source. The solution allows for the* exact *derivation of* all *post-Newtonian parameters in Brans–Dicke gravity for far field regions of a spherical source. Particularly for the γ parameter, instead of the conventional result $\gamma_{\rm PPN} = \frac{\omega+1}{\omega+2}$, we obtain the analytic expression $\gamma_{\rm exact} = \frac{\omega+1+(\omega+2)\,\Theta}{\omega+2+(\omega+1)\,\Theta}$, where Θ is the ratio of the total pressure $P_\parallel^* + 2P_\perp^*$ and total energy E^* contained within the mass source. Our* non-perturbative *γ formula is valid for all field strengths and types of matter comprising the mass source. Consequently, observational constraints on γ thus set* joint *bounds on ω and Θ, with the latter representing a global characteristic of the mass source. More broadly, our formula highlights the importance of pressure (when $\Theta \neq 0$) in spherical Brans–Dicke stars, and potentially in stars within other modified theories of gravitation.*

Keywords: Brans–Dicke gravity, static spherically symmetric solution, energy-momentum tensor.

1. Introduction

Brans–Dicke gravity is the second most studied theory of gravitation besides General Relativity. It represents one of the simplest extensions of gravitational theory beyond GR [1]. It is characterized by an additional dynamical scalar field ϕ which, in the original vision of Brans and Dicke in 1961, acts like the inverse of a variable Newton 'constant' G. The scalar field has a kinetic term, governed by a (Brans–Dicke) parameter ω in the following gravitation action

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\,\Big[\Phi\,\mathcal{R} - \frac{\omega}{\Phi}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi\Big]. \qquad (1)$$

In the limit of infinite value for ω, the kinetic term is generally said to be 'frozen', rendering Φ being a constant value everywhere. In this limit, if the field ϕ approaches its (non-zero) constant value in the rate $\mathcal{O}\left(1/\omega\right)$, the term $\frac{\omega}{\Phi}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi$ would approach zero at the rate $\mathcal{O}\left(1/\omega\right)$ and hence become negligible compared with the term $\Phi\,\mathcal{R}$, effectively recovering the classic Einstein–Hilbert action. [2]

Together with its introduction [1], Brans also identified four classes of exact solutions in the static spherically symmetric (SSS) setup [2]. The derivation of the Brans solutions was explicitly carried out by Bronnikov in 1973 [4]. Of the four classes, only the Brans Class I is physically meaningful, however. It

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[1] This work is based on the results presented at the XII Bolyai–Gauss–Lobachevskii (BGL-2024) Conference: Non-Euclidean Geometry in Modern Physics and Mathematics.

[2] It has been shown that, for non-static and/or in the presence of singularity, the rate of convergence is $\mathcal{O}\left(1/\sqrt{\omega}\right)$. This topic is beyond the scope of this article however, as we shall only consider a static and regular case here. For more information, we refer the reader to our recent work [3], where we also reviewed the literature on the $\mathcal{O}\left(1/\sqrt{\omega}\right)$ anomaly.

can recover the Schwarzschild solution in its parameter space.

For comparison with observations or experiments, Brans derived the Robertson (or Eddington–Robertson–Schiff) β and γ post-Newtonian (PN) parameters based on his Class I solution:

$$\beta_{\rm PPN} = 1, \tag{2}$$

$$\gamma_{\rm PPN} = \frac{\omega+1}{\omega+2}. \tag{3}$$

The γ parameter is important as it governs the amount of space-curvature produced by a body at rest and can be directly measured via the detection of light deflection and the Shapiro time delay. The parametrized post-Newtonian (PPN) γ formula recovers the result $\gamma_{\rm GR} = 1$ known for GR in the limit of infinite ω, in which the BD scalar field becomes constant everywhere. Current bounds using Solar System observations set the magnitude of ω to exceed $40,000$ [6].

We should emphasize that the "conventional" results (2) and (3) were derived under *the assumption of zero pressure in the gravity source.* It should be noted that these formulae can also be deduced directly from the PPN formalism for the Brans–Dicke action, without resorting to the Brans Class I solution [5, 7]. The PPN derivation relies on two crucial approximations: (i) weak field and (ii) slow motions. Regarding the latter approximation, an often under-emphasized point is that not only must the stars be in slow motion, but the microscopic constituents that comprise the stars must also be in slow motion. This translates to the requirement that the matter inside the stars exert low pressure, characterizing them as "Newtonian" stars.

The purpose of our paper is twofold. Firstly, the analytic form for the exterior vacuum contains two adjustable parameters. The issue in determining them from the energy and pressure profiles inside the mass source has not been rigorously addressed in the literature. Establishing their relationships with the mass source would typically require the full machinery of the Tolman–Oppenheimer–Volkoff (TOV) equations tailored for Brans–Dicke gravity [8]. Moreover, solving the TOV equations, even in the simpler theory of GR, generally requires numerical methods except for a few isolated, unrealistic cases such as incompressible fluids. Therefore, at first glance, deriving a specific expression for these relationships might seem elusive. Surprisingly, as we shall show in this article, this view is overly pessimistic. It turns out that the full machinery of the TOV equation is not necessary. Instead, only a subset of the field equation and the scalar equation of BD will be needed. This is because only two equations are required to fix the two free parameters of the exterior vacuum. We shall present a rigorous yet parsimonious derivation, which only became available through our recent publication [9].

Secondly, the complete solution enables the derivation of *any* PN parameters applicable for far-field regions in static spherical Brans–Dicke stars. As we shall show in this article, the derivation is *non-perturbative* and *avoids the two PPN approximations* requiring the weak field and the low pressure mentioned above.

The material presented in this article was developed during the preparation of our two recent papers [9, 10]. For a more detailed exposition of the conceptualization and technical points, we refer the reader to these papers.

2. The Field Equations and the Energy-Momentum Tensor

It is well documented [4] that, upon the Weyl mapping $\left\{\tilde{g}_{\mu\nu} := \Phi\, g_{\mu\nu},\ \tilde{\Phi} := \ln\Phi\right\}$, the gravitational sector of the BD action can be brought to the Einstein frame as $\int d^4x \frac{\sqrt{-\tilde{g}}}{16\pi}\Big[\tilde{\mathcal{R}} - (\omega+3/2)\,\tilde{\nabla}^\mu\tilde{\Phi}\tilde{\nabla}_\mu\tilde{\Phi}\Big]$. The Einstein-frame BD scalar field $\tilde{\Phi}$ has a kinetic term with a signum determined by $(\omega+3/2)$. Unless stated otherwise, we shall restrict our consideration to the normal ("non-phantom") case of $\omega > -3/2$, where the kinetic energy for $\tilde{\Phi}$ is positive.

The field equations are

$$R_{\mu\nu} - \frac{\omega}{\Phi^2}\partial_\mu\Phi\partial_\nu\Phi - \frac{1}{\Phi}\partial_\mu\partial_\nu\Phi + \Gamma^\lambda_{\mu\nu}\partial_\lambda\ln\Phi =$$

$$= \frac{8\pi}{\Phi}\left(T_{\mu\nu} - \frac{\omega+1}{2\omega+3}T\,g_{\mu\nu}\right), \tag{4}$$

$$\partial_\mu\left(\sqrt{-g}\,g^{\mu\nu}\partial_\nu\Phi\right) = \frac{8\pi}{2\omega+3}T\sqrt{-g}. \tag{5}$$

In the isotropic coordinate system which is static and spherically symmetric, the metric can be written as

$$ds^2 = -A(r)dt^2 + B(r)\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right]. \tag{6}$$

It is straightforward to verify, from Eqs. (4)–(6), that the most general form for the energy-momentum tensor (EMT) in this setup is

$$T_\mu^\nu = \mathrm{diag}\big(-\epsilon,\, p_\parallel,\, p_\perp,\, p_\perp\big), \tag{7}$$

where the energy density ϵ, the radial pressure $p_\parallel$ and the tangential pressure $p_\perp$ are functions of r. Note that the EMT is anisotropic, if $p_\parallel \neq p_\perp$. The trace of the EMT is

$$T = -\epsilon + p_\parallel + 2p_\perp. \tag{8}$$

3. The Brans Class I Vacuum Solution Outside a Star

It is known that the scalar–metric for the vacuum is the Brans Class I solution (which satisfies Eqs. (5)–(4) for $T_{\mu\nu} = 0$) [2]. In the isotropic coordinate system (6), the solution reads [2]

$$\begin{cases} A = \left(\dfrac{r-k}{r+k}\right)^{\frac{2}{\lambda}}, \\ B = \left(1+\dfrac{k}{r}\right)^4 \left(\dfrac{r-k}{r+k}\right)^{2-2\frac{\Lambda+1}{\lambda}}, \qquad \text{for } r \geqslant r_*, \\ \Phi = \left(\dfrac{r-k}{r+k}\right)^{\frac{\Lambda}{\lambda}} \end{cases} \tag{9}$$

where r_* is the star's radius, and

$$\lambda^2 = (\Lambda+1)^2 - \Lambda\left(1 - \frac{\Lambda}{2}\omega\right). \tag{10}$$

Since λ and Λ are linked by (10), this solution involves two independent parameters, which one chooses to be $(k,\, \Lambda)$.

4. The Field Equations in the Interior

For the region $r \leqslant r_*$, substituting metric (6) and the BD field $\Phi(r)$ into Eq. (5) and the 00-*component* of Eq. (4) and using the EMT in Eq. (7), the functions $A(r)$, $B(r)$, $\Phi(r)$ satisfy the 2 following ordinary differential equations (ODEs):

$$\left(r^2\sqrt{AB}\Phi'\right)' = \frac{8\pi}{2\omega+3}\Big[-\epsilon + p_\parallel + 2p_\perp\Big] r^2\sqrt{AB^3}, \tag{11}$$

$$\left(r^2\Phi\sqrt{\frac{B}{A}}A'\right)' = 16\pi\Big[\epsilon + \frac{\omega+1}{2\omega+3}\big(-\epsilon + p_\parallel + 2p_\perp\big)\Big] r^2\sqrt{AB^3}. \tag{12}$$

These equations offer the advantage of having both their left hand sides in exact derivative forms. Let us integrate Eqs. (11) and (12) from the star's center, viz. $r = 0$, to a coordinate $r > r_*$. The (A, B, Φ) functions are then given by (9) at r. For $r > r_*$, both $r^2\sqrt{AB}\Phi'$ and $r^2\Phi\sqrt{\frac{B}{A}}A'$ terms that enter the left hand sides of (11) and (12) are $r-$independent, since the right hand sides of these equations vanish in the *exterior* vacuum. On the other hand, regularity conditions inside the star impose $\Phi'(0) = A'(0) = 0$ (i.e. no conic singularity) and finite values of the fields themselves. The calculation yields

$$\frac{k\,\Lambda}{\lambda} = \frac{4\pi}{2\omega+3}\int_0^{r_*} dr\, r^2\sqrt{AB^3}\Big[-\epsilon + p_\parallel + 2p_\perp\Big] \tag{13}$$

and

$$\frac{k}{\lambda} = \frac{4\pi}{2\omega+3}\int_0^{r_*} dr\, r^2\sqrt{AB^3} \times \Big[(\omega+2)\epsilon + (\omega+1)\big(p_\parallel + 2p_\perp\big)\Big]. \tag{14}$$

Let us note that $r^2\sqrt{AB^3}$ is the square root of the determinant of the metric, up to the $\sin\theta$ term. (Accordingly, the integrals in the right hand sides of Eqs. (13) and (14) are invariant through radial coordinate transformations, since the combination $r^2\sqrt{AB^3}\sin\theta$ is equivalent to $\sqrt{-g}$.) We then can define the energy's and pressures' integrals by

$$E^* = 4\pi\int_0^{r_*} dr\, r^2\sqrt{AB^3}\,\epsilon, \tag{15}$$

$$P_\parallel^* = 4\pi\int_0^{r_*} dr\, r^2\sqrt{AB^3}\,p_\parallel, \tag{16}$$

$$P_\perp^* = 4\pi\int_0^{r_*} dr\, r^2\sqrt{AB^3}\,p_\perp. \tag{17}$$

Inserting in (13) and (14), we obtain

$$\frac{k}{\lambda} = E^*\left[\frac{\omega+2}{2\omega+3} + \frac{\omega+1}{2\omega+3}\,\Theta\right] \tag{18}$$

and

$$\Lambda = \frac{\Theta - 1}{\omega + 2 + (\omega+1)\,\Theta}, \tag{19}$$

in which the dimensionless parameter Θ is defined as

$$\Theta := \frac{P_\parallel^* + 2\,P_\perp^*}{E^*}. \tag{20}$$

Together with (9) and (10), these expressions provide a complete expression for the exterior spacetime and scalar field of a spherical BD star. To the best of our knowledge, this prescription was not made explicitly documented in the literature till our recent works [9, 10].

For a perfect fluid, $p_\parallel = p_\perp \equiv p$, thence $P_\parallel^* = P_\perp^* \equiv P$. The equations (10), (13) and (14) fully determine the exterior solution (9) once the integrals (15)–(17) are known, with these integrals being fixed by the stellar internal structure model. This explicitly determines the particles' motion outside the star, in both the remote and close to the star regions.

5. The (β, γ, δ) PN Parameters

In remote spatial regions, a static spherically symmetric metric in isotropic coordinates can be expanded as [7]:

$$ds^2 = -\left(1 - 2\,\frac{M}{r} + 2\beta\,\frac{M^2}{r^2} + ...\right)dt^2 + \left(1 + 2\gamma\,\frac{M}{r} + \frac{3}{2}\delta\,\frac{M^2}{r^2} + ...\right)\left(dr^2 + r^2 d\Omega^2\right), \tag{21}$$

in which β and γ are the Robertson (or Eddington–Robertson–Schiff) parameters, whereas δ is the second-order PN parameter (for both light and planetary like motions). It is straightforward to verify that the Schwarzschild metric yields

$$\beta_{\text{Schwd}} = \gamma_{\text{Schwd}} = \delta_{\text{Schwd}} = 1. \tag{22}$$

The metric in Eq. (6) can be re-expressed in the expansion form

$$ds^2 = -\left(1 - \frac{4}{\lambda}\frac{k}{\rho} + \frac{8}{\lambda^2}\frac{k^2}{r^2} + ...\right)dt^2 + \left(1 + \frac{4}{\lambda}(1+\Lambda)\frac{k}{r} + \frac{2}{\lambda^2}(4(1+\Lambda)^2 - \lambda^2)\frac{k^2}{r^2} + ...\right)\times \times\left(dr^2 + r^2 d\Omega^2\right). \tag{23}$$

Comparing Eq. (21) against Eq. (23) and setting

$$M = 2\,\frac{k}{\lambda}, \tag{24}$$

we obtain

$$\beta_{\text{exact}} = 1, \tag{25}$$

$$\gamma_{\text{exact}} = 1 + \Lambda, \tag{26}$$

$$\delta_{\text{exact}} = \frac{1}{3}\left(4(1+\Lambda)^2 - \lambda^2\right), \tag{27}$$

where we have used the subscript "exact" as emphasis. Note that Λ directly measures the deviation of the γ parameters from GR ($\gamma_{\text{GR}} = 1$). From Eq. (19), Λ *depends on both ω and Θ*. Finally, we arrive at

$$\gamma_{\text{exact}} = \frac{\omega + 1 + (\omega+2)\,\Theta}{\omega + 2 + (\omega+1)\,\Theta}, \tag{28}$$

which can also be conveniently recast as

$$\gamma_{\text{exact}} = \frac{\gamma_{\text{PPN}} + \Theta}{1 + \gamma_{\text{PPN}}\,\Theta} \tag{29}$$

by recalling that $\gamma_{\text{PPN}} = \frac{\omega+1}{\omega+2}$. To our knowledge, the closed-form expression (28) for γ was absent in the literature, until our recent works [9, 10].

Regarding δ:

$$\delta_{\text{exact}} = \frac{1}{[\omega + 2 + (\omega+1)\,\Theta]^2}\left[\left(\omega^2 + \frac{3}{2}\omega + \frac{1}{3}\right) + \left(2\omega^2 + \frac{19}{3}\omega + \frac{13}{3}\right)\Theta + \left(\omega^2 + \frac{25}{6}\omega + \frac{13}{3}\right)\Theta^2\right]. \tag{30}$$

Figure shows contour plots of γ_{exact} and δ_{exact} as functions of γ_{PPN} (i.e, $\frac{\omega+1}{\omega+2}$) and Θ. In addition, with the aid of Eqs. (18) and (20), Eq. (24) produces the active gravitational mass

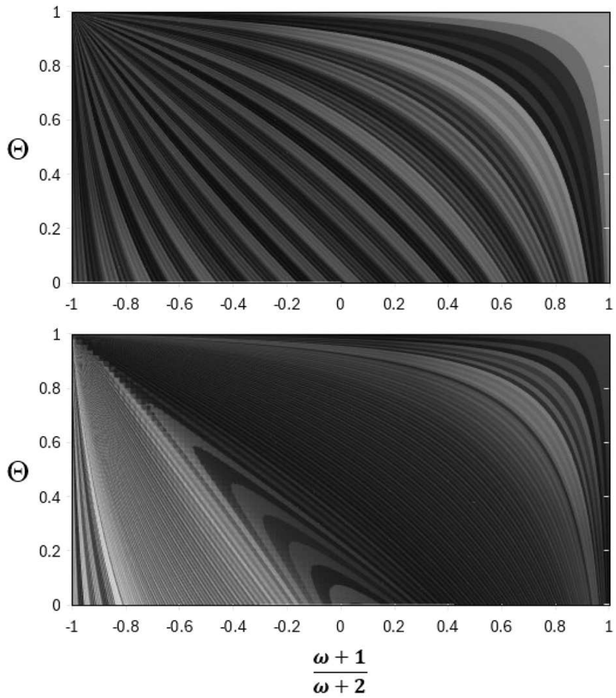
$$M = \frac{2\omega+4}{2\omega+3}\,E^* + \frac{2\omega+2}{2\omega+3}\,\left(P_\parallel^* + 2P_\perp^*\right), \tag{31}$$

where the contribution of pressure to the active gravitational mass is evident [11, 12].

6. Degeneracy at Ultra-High Pressure

For $\Theta \to 1^-$, both γ and δ go to 1, their GR counterpart values. Generally speaking, for $\Theta \to 1^-$, since $\Lambda \to 0$ and $\lambda \to 1$ regardless of ω (provided that $\omega \in (-3/2, +\infty)$), the value of k approaches

$$k \to \frac{\omega+2}{2\omega+3}\,E^* + \frac{\omega+1}{2\omega+3}\,\left(P_\parallel^* + 2P_\perp^*\right). \tag{32}$$



Contour plots of γ_{exact} (upper panel) and δ_{exact} (lower panel) in terms of Θ and $\frac{\omega+1}{\omega+2}$, for the range of $\Theta \in [0,1]$ and $\omega \in \in (-3/2,+\infty)$, the latter corresponding to $\frac{\omega+1}{\omega+2} \in (-1,1)$. A measured $\gamma_{\text{exact}} \approx 1$ could mean $\frac{\omega+1}{\omega+2} \approx 1$ (i.e., $\omega \gg 1$) or $\Theta \approx 1$ (i.e., ultra-relativistic matter). Contours are equally spaced in 0.01 increment. For a given contour in the upper panel, the corresponding value of γ_{exact} can be read on the abscissa axis, where the contour intersects it. By measuring *both* γ and δ, the values of ω and Θ may be determined

The $\omega-$dependence is thus absorbed into k, and the Brans Class I solution degenerates to the Schwarzschild solution

$$\begin{cases} A = \left(\dfrac{r-k}{r+k}\right)^2, \\ B = \left(1+\dfrac{k}{r}\right)^4, \\ \Phi = 1 \end{cases} \quad \text{for } r \geqslant r_*. \tag{33}$$

Therefore, ultra-relativistic Brans-Dicke stars are *indistinguishable* from their GR counterparts, as far as their exterior vacua are concerned. This fact can be explained by the following observation: for ultra-relativistic matter, the trace of the EMT vanishes, per Eq. (8). The scalar equation (5) then simplifies to $\Box\Phi = 0$ *everywhere*. Coupled with the regularity condition at the star center, this ensures a constant Φ throughout the spacetime which is now described by the Schwarzschild solution. Consequently, the scalar degree of freedom in BD gravity is suppressed in the ultra-relativistic limit. This prompts an intriguing possibility whether Birkhoff's theorem is fully restored in this limit.

7. Discussions

Formulae (28) and (30) are the essential outcome of this article:

• *Non-perturbative approach*: Our derivation is non-perturbative in nature. It makes use of the *integrability* of the 00−component of the field equation (4), along with the scalar field equation (5).

• *Parsimony*: Our derivation relies solely on the scalar field equation and the 00-component of the field equation, without the need for the full set of equations, specifically the 11- and 22-components of the field equation [3]. The additional physical assumptions employed are the regularity at the star's center and the existence of the star's surface separating the interior and the exterior.

• *Universality of results*: The final formulae, (28) and (30), hold for all field strengths and all types of matter (whether convective or non-convective, for example). We do not assume the matter comprising the stars to be a perfect fluid or isentropic.

• *Higher-derivative characteristics*: In contrast to the one-parameter Schwarzschild metric, the Brans Class I solution depends on two parameters, i.e. the solution is not only defined by its gravitational mass, but also by *a scalar mass* besides the gravitational one [4]. The exterior BD vacuum should reflect the internal structure and composition of the star. This expectation is confirmed in Eqs. (28) and (30), highlighting the role of the parameter Θ.

• *Role of pressure*: Figure shows contour plots of γ_{exact} and δ_{exact} as functions of $\frac{\omega+1}{\omega+2}$ and Θ. There are three interesting observations:

– An ultra-relativistic limit, $\Theta \simeq 1^-$, would render $\gamma_{\text{exact}} \simeq 1$, *regardless* of ω.

– For Newtonian stars, i.e., low pressure ($\Theta \approx 0$), the PPN result is a good approximation *regardless* of the field strength.

[3] Note that establishing the functional form of the Brans Class I solution still requires the full set of equations.

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– A joint measurement of γ and δ in principle can determine ω and Θ. However, due to the non-linear relationships in (28) and (30), for a given pair of $\{\gamma,\ \delta\}$, multiple solutions for $\{\omega,\ \Theta\}$ can exist. A measurement of a third PN parameter (apart from β) in principle can resolve the multiplicity problem.

8. Conclusion

We have derived the exact analytic formulae, (28) and (30), for the PN parameters γ and δ for spherical mass sources in BD gravity. The derivation relies on the integrability of the 00−component of the field equation, rendering it non-perturbative and applicable for any field strength and type of matter constituting the source. The conventional PPN result for BD gravity $\gamma_{\rm PPN} = \frac{\omega+1}{\omega+2}$ lacks dependence on the physical features of the mass source. In the light of our exact results, the $\gamma_{\rm PPN}$ should be regarded as an approximation for stars in modified gravity under low-pressure conditions. Our findings expose the limitations of the PPN formalism, particularly in scenarios characterized by high star pressure. It is reasonable to expect that the role of the pressure may extend to other modified theories of gravitation.

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ПОРУШЕННЯ γ В ҐРАВІТАЦІЇ БРАНСА–ДІККЕ

Розв'язок Бранса класу I у ґравітації Бранса–Дікке (БД) є основним у вивченні ґравітаційних теорій за межами загальної теорії відносності. Відкритий у 1961 році, цей розв'язок відображає зовнішній вакуум сферичної зірки Бранса–Дікке і характеризується двома підгоночними параметрами. Дивно, але зв'язок між цими параметрами і властивостями зірки не був точно встановлений. В даній роботі ми заповнюємо цю прогалину, виводячи повний зовнішній розв'язок Бранса класу I, виражений через загальну енергію і загальний тиск сферично симетричного джерела ґравітації. Розв'язок дозволяє точно вивести всі постньютонівські параметри ґравітації Бранса–Дікке для віддалених областей поля сферичного джерела. Зокрема, для параметра γ замість традиційного результату $\gamma_{\rm PPN} = \frac{\omega+1}{\omega+2}$ ми отримуємо аналітичний вираз $\gamma_{\rm exact} = = \frac{\omega+1+(\omega+2)\,\Theta}{\omega+2+(\omega+1)\,\Theta}$, де Θ є відношенням загального тиску $P_{\parallel}^{*} + 2P_{\perp}^{*}$ до повної енергії E^{*}, що міститься в джерелі маси. Наша непертурбативна формула для γ дійсна для всіх напруженостей полів і типів матерії, що входять до складу джерела маси. Отже, спостережувані обмеження на γ таким чином встановлюють спільні обмеження на ω і Θ, причому останній представляє загальну характеристику джерела маси. У більш широкому сенсі наша формула підкреслює важливість тиску (коли $\Theta \neq 0$)) у сферичних зірках Бранса–Дікке і, можливо, у зірках в межах інших модифікованих теорій ґравітації.

Ключові слова: ґравітація Бранса–Дікке, стаціонарний сферично симетричний розв'язок, тензор енергії-імпульсу.