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O.I. DAVYDOVSKA, V.O. NESTEROV

Institute for Nuclear Research Nat. Acad. of Sci. of Ukraine
(47, Prosp. Nauky, Kyiv 03680, Ukraine)

CROSS SECTIONS OF ELASTIC SCATTERING FOR SYSTEMS $^{16}\mathrm{O} + {}^{42}\mathrm{Ca}$, $^{16}\mathrm{O} + {}^{48}\mathrm{Ca}$ WITHIN THE MODIFIED THOMAS–FERMI METHOD WITH TAKING THE REPULSION CORE INTO ACCOUNT

Within the framework of the modified Thomas–Fermi method, the distribution densities of nucleons and nucleus-nucleus interaction potentials for the reactions $^{16}O + {}^{42}Ca$ *and* $^{16}O + {}^{48}Ca$ *have been calculated. Besides, all contributions up to the second-order terms in* $\hbar$ *in the quasi-classical expansion of the kinetic energy are taken into account, and Skyrme forces, which depend on density, are used as the nucleon-nucleon interaction. A convenient parametrization of the potentials between nuclei has been obtained, allowing them to be represented in analytic form. Using the obtained potentials, the elastic scattering cross-sections are calculated, which agree well with the available experimental data.*

K e y w o r d s: Thomas–Fermi method, reactions $^{16}\mathrm{O} + {}^{42}\mathrm{Ca}$ and $^{16}\mathrm{O} + {}^{48}\mathrm{Ca}$.

1. Introduction

The study of the fundamental properties of nuclear reactions, in particular, cross-sections of various processes, first of all, requires knowledge of the potential of nucleus-nucleus interaction [1–4]. Information about the size and shape of the nucleus-nucleus interaction potential at small distances between nuclei is of particular interest.

Qualitatively, the potential of nucleus-nucleus interaction can be represented as the sum of three parts – nuclear, Coulomb, and centrifugal. The properties of the last two components of the potential have already been studied sufficiently. But the situation with the nuclear part, unfortunately, remains much more complicated, its properties are not well defined yet. Currently, a large number of different models are used to approximate the nuclear part of the interaction potential [1–26], which can give significantly different results. Given this, information about the potential of nucleus-nucleus interaction, in particular, the height of barriers, is fundamentally important when describing nuclear reactions.

From all this variety of methods [27–38] for calculation of the nucleus-nucleus interaction potential, in our work, we chose a semi-microscopic approach, namely, the modified Thomas–Fermi method with density-dependent Skyrme forces [4, 8, 9, 11, 12, 14–26] as a nucleon-nucleon interaction. From the currently existing successful parametrizations of these forces, we choose SkP parametrization [33]. It should be noted that, in the quasi-classical distribution of the kinetic energy by powers of h, we consider all terms up to h^2. Preliminary calculations for various problems, carried out both by us and by other authors, demonstrated the sufficient accuracy of such approximation. The modified Thomas–Fermi method with Skyrme forces well describes the characteristics of the ground and excited states of atomic nuclei, for example, the binding energy, root mean square radii, *etc.* [27–33, 35].

At large distances between the nuclei, the potential of the modified Thomas–Fermi method is close to the Coulomb potential; when the distance decreases, we can observe a potential barrier, the magnitude of which is determined by the simultaneous action of Coulomb repulsion and nuclear attraction. With a

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further decrease in the distance, when the nucleon densities of the nuclei are significantly overlapped, the potential of the modified Thomas–Fermi method has a radically expressed repulsion core [8, 11, 14, 15, 18–23], the appearance of which is associated with the nuclear matter incompressibility [14, 15, 20, 23]. Note that the short-range repulsion core also exists, for example, in the Proximity potential [5].

The nucleus-nucleus interaction potentials, which have a repulsion core, were not very often used to study the process of scattering of atomic nuclei. In particular, we can mention works [14, 15, 20, 23, 39, 40], where, based on the potential with a core, it was possible to describe simultaneously the processes of both elastic scattering and subbarrier fusion of nuclei. Therefore, the study of the elastic scattering process within the framework of the modified Thomas–Fermi approach is important and relevant.

The mathematical apparatus required for the implementation of the chosen approach will be presented in Sections 2 and 3. In Section 4, we will discuss the calculation of elastic scattering cross sections and the results obtained. In Section 5 we will present our conclusions.

2. Calculation of the Potential within the Framework of the Modified Thomas–Fermi Method

Qualitatively, the nucleus-nucleus interaction potential $V(R)$ can be represented as the sum of nuclear $V_N(R)$, Coulomb $V_{\rm Coul}(R)$, and centrifugal $V_l(R)$ parts. The distance between the centers of mass of the nuclei is denoted as R:

$$V(R) = V_N(R) + V_{\rm Coul}(R) + V_l(R). \tag{1}$$

The behavior of the Coulomb and centrifugal parts has been studied quite well at present.

We will use widely used expressions for them, which can be found in works [20, 23, 24]. We will calculate the nuclear part of the nucleus-nucleus interaction potential $V_N(R)$ within the framework of the modified Thomas–Fermi method, with regard for all terms up to h^2 in the semi-classical distribution of the kinetic energy [4, 8, 9, 11, 12, 14–26]. The density-dependent Skyrme forces will be responsible for the nucleon-nucleon interaction in the form of the SkP parametrization [33]. The approximation of frozen densities, within which we will work, is quite applicable for energies in the nearby of the barrier.

To obtain the nucleus-nucleus interaction potential $V_N(R)$, we need to calculate the energy of the system of two nuclei at finite $E_{12}(R)$ and infinite $E_{1(2)}$ distances from each other [9, 11]:

$$V_N(R) = E_{12}(R) - (E_1 + E_2). \tag{2}$$

Note that the energy of the system at the infinite distance is the sum of the binding energies of two separate nuclei:

$$E_{12}(R) = \int \varepsilon[\rho_{1p}(\bar{r}) + \rho_{2p}(\bar{r}, R), \rho_{1n}(\bar{r}) + \rho_{2n}(\bar{r}, R)]d\bar{r}, \tag{3}$$

$$E_{1(2)}(R) = \int \varepsilon[\rho_{1(2)p}(\bar{r}), \rho_{1(2)n}(\bar{r})]d\bar{r}. \tag{4}$$

In this formula, $\rho_{1(2)n}$ and $\rho_{1(2)p}$ denote the neutron or proton density of the nucleus 1(2) respectively, $\varepsilon[\rho_{1(2)p}(\bar{r}), \rho_{1(2)n}(\bar{r})]$ is the energy density, and R is the distance between the centers of mass of the nuclei.

The expression for the energy density, which is the sum of the kinetic, potential, and Coulomb parts, with the Skyrme forces is well known [25–27, 31, 33, 38]:

$$\begin{aligned}
\varepsilon = \varepsilon_{\rm kin} + \varepsilon_{\rm pot} + \varepsilon_{\rm Coul} = {} & \frac{\hbar^2}{2m}\tau + \frac{1}{2}t_0\left[\left(1+\frac{1}{2}x_0\right)\rho^2 - \right. \\
& \left. - \left(x_0+\frac{1}{2}\right)\times\left(\rho_n^2+\rho_p^2\right)\right] + \\
& + \frac{1}{12}t_3\rho^\alpha\left[\left(1+\frac{1}{2}x_3\right)\rho^2 - \left(x_3+\frac{1}{2}\right)\times\left(\rho_n^2+\rho_p^2\right)\right] + \\
& + \frac{1}{4}\left[t_1\left(1+\frac{1}{2}x_1\right) + t_2\left(1+\frac{1}{2}x_2\right)\right]\tau\rho + \\
& + \frac{1}{4}\left[t_2\left(x_2+\frac{1}{2}\right) - t_1\left(x_1+\frac{1}{2}\right)\right]\times \\
& \times(\tau_n\rho_n + \tau_p\rho_p) + \frac{1}{16}\left[3t_1^t\left(1+\frac{1}{2}x_1\right) - t_2\left(1+\frac{1}{2}x_2\right)\times\right. \\
& \times(\nabla\rho)^2 - \frac{1}{16}\left[3t_1\left(x_1+\frac{1}{2}\right) + t_2\left(x_2+\frac{1}{2}\right)\right]\times \\
& \times\left(\left(\nabla\rho_n\right)^2 + \left(\nabla\rho_p\right)^2\right) + \\
& + \frac{1}{2}W_0\left[J^\nabla\rho + J_n\nabla\rho_n + J_p\nabla\rho_p\right] + \varepsilon_{\rm Coul}.
\end{aligned} \tag{5}$$

In this expression, ε_{kin} is the kinetic energy density, ε_{pot} is the potential energy density, and $\varepsilon_{\text{Coul}}$ is the Coulomb energy density. Values t_0, t_1, t_2, t_3, x_0, x_1, x_2, x_3, α, W_0 are Skyrme interaction parameters. The terms, which include t_0 and t_3, correspond to zero radius forces. The term including t_0 is related to the attraction, while the term proportional t_3 is related to the repulsion and increases as the density of nuclear matter increases, preventing nuclear systems from the collapsing. The terms, which include t_1 and t_2, correspond to the forces of the finite radius of action, their value increases with the nucleon density. The parameters x_0, x_1, x_2 and x_3 represent exchange effects, as well as spin and isospin asymmetry, W_0 is the spin-orbit interaction constant.

The kinetic energy density can be represented as the sum of the density of the usual Thomas–Fermi method and the gradient correction of the second order, i.e., $\tau = \tau_{\text{TF}} + \tau_2$ [8, 9, 11, 12, 25, 28, 29, 38], while $\tau = \tau_p + \tau_n$ is the sum of neutron and proton kinetic energy densities. The kinetic energy density of the usual Thomas–Fermi method [28, 29]

$$\tau_{\text{TF},n(p)} = k\rho_{n(p)}^{5/3}, \tag{6}$$

is the kinetic energy density of neutrons (protons) in the Thomas–Fermi approximation, $k = \frac{5}{3}(3\pi^2)^{2/3}$ and τ_2 is the full expression for the second-order gradient correction in terms of h [28, 29]:

$$\tau_{2q} = b_1\frac{(\nabla\rho_q)^2}{\rho_q} + b_2\nabla^2\rho_q + b_3\frac{(\nabla f_q\nabla\rho_q)}{f_q} + \\ + b_4\rho_q\frac{\nabla^2 f_q}{f_q} + b_5\rho_q\left(\frac{\nabla f_q}{f_q}\right)^2 + b_6h_m^2\rho_q\left(\frac{\mathbf{W}_q}{f_q}\right)^2, \tag{7}$$

where $b_1 = 1/36$, $b_2 = 1/3$, $b_3 = 1/6$, $b_4 = 1/6$, $b_5 = -1/12$ and $b_6 = 1/2$ are numerical coefficients, $h_m = \hbar^2/2m$. Note that the last term is related to the spin-orbit interaction. In addition,

$$\mathbf{W}_q = \frac{\delta\varepsilon(r)}{\delta\mathbf{J}_q(r)} = \frac{W_0}{2}\nabla(\rho + \rho_q), \tag{8}$$

and f_q depends on the nucleon densities and Skyrme forces parameters:

$$f_q = 1 + \frac{2m}{\hbar^2}\left[\frac{1}{4}\left[t_1\left(1 + \frac{1}{2}x_1\right) + t_2\left(1 + \frac{1}{2}x_2\right)\right]\rho + \right. \\ \left. + \frac{1}{4}\left[t_2\left(x_2 + \frac{1}{2}\right) - t_1\left(x_1 + \frac{1}{2}\right)\right]\rho_q\right], \tag{9}$$

The contribution of the usual Thomas–Fermi method term is the main one, especially in the bulk of the core, but, near the surface, the gradient corrections also start to play a prominent role.

The purpose of this work is to consider the elastic scattering reactions of $^{16}\mathrm{O} + {}^{42}\mathrm{Ca}$ and $^{16}\mathrm{O} + {}^{48}\mathrm{Ca}$. For these systems, within the framework of the modified Thomas–Fermi approach, it is necessary to calculate the nucleus-nucleus interaction potential, for which it is necessary to know the nucleon density distributions in the nuclei. Nucleon densities will be obtained within the framework of the same modified Thomas–Fermi approach with Skyrme forces (SkP parametrization [33]). Nucleon densities for $^{16}\mathrm{O}$, $^{42}\mathrm{Ca}$, and $^{48}\mathrm{Ca}$ obtained within the framework of this method are shown in Fig. 1.

Having obtained nucleon densities, using formulas (1)–(9), we can calculate the nucleus-nucleus interaction potential within the framework of the modified Thomas–Fermi method with Skyrme forces. The nuclear part of the potentials calculated by us for the $^{16}\mathrm{O} + {}^{42}\mathrm{Ca}$ and $^{16}\mathrm{O} + {}^{48}\mathrm{Ca}$ systems can be seen in Fig. 2. The potentials of the modified Thomas–Fermi method have a completely realistic form, demonstrating a significant repulsion core at small distances.

3. Analytical Representation of the Interaction Potential

For further calculations, it would be very convenient to parametrize our potential, which would allow us to work with it in an analytic form. At the same time, we note that the account for the repulsion core is important for considering elastic scattering processes. That is why the use of the well-known Woods–Saxon parametrization does not suit us. Therefore, to give a more realistic form to our potential, we will use one more term, similar to the structure of the kinetic energy of the Thomas–Fermi method, which should provide the repulsion that we need at small distances. We have already used something similar in work [21] for the double folding potential, which significantly simplifies the calculations. Thus, our parametrization of the potential will have the

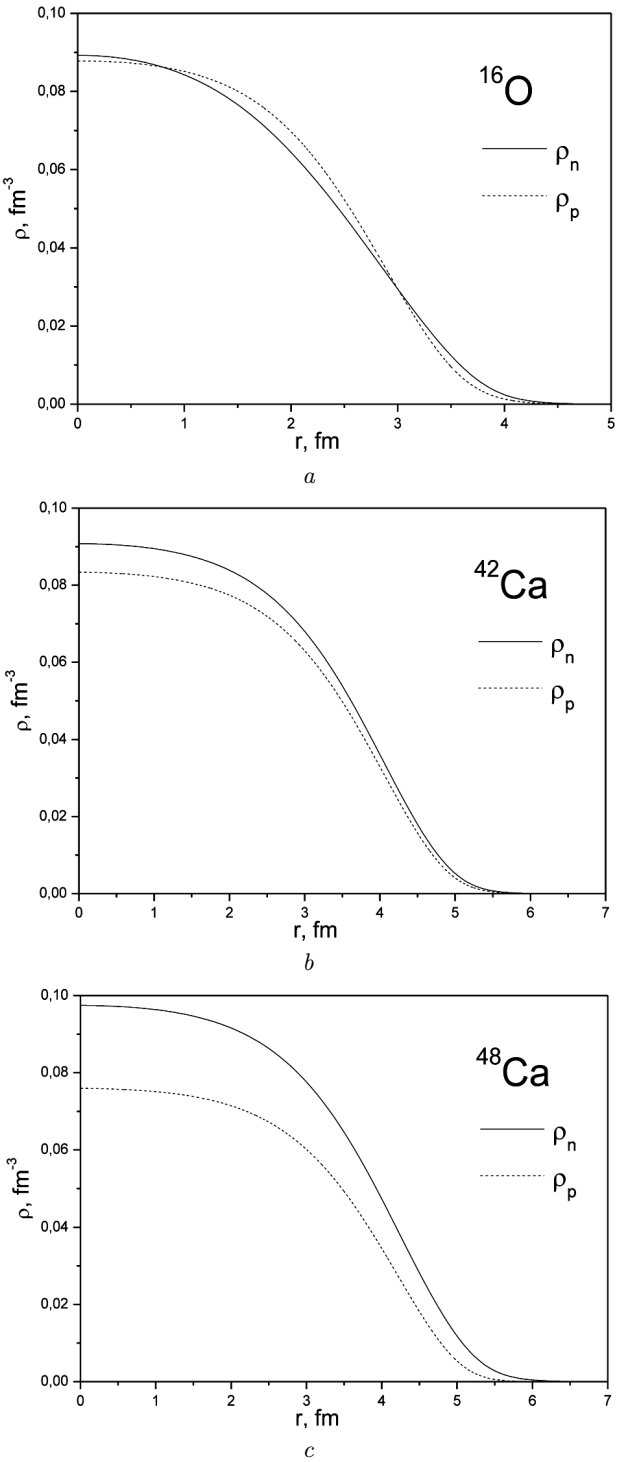

Fig. 1. Nucleon density distributions for ^{16}O (a), ^{42}Ca (b), and ^{48}Ca (c) nuclei, obtained within the framework of the modified Thomas–Fermi method

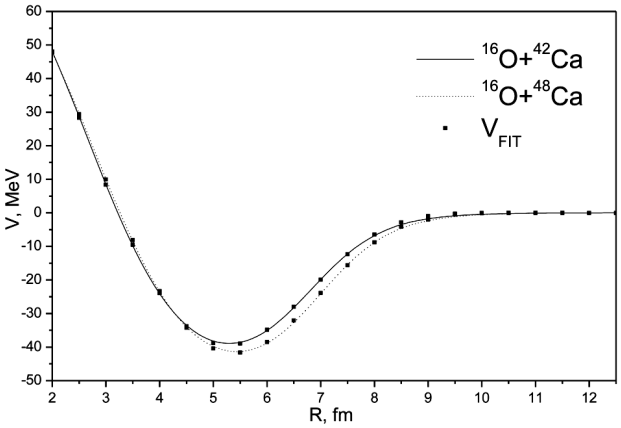

Fig. 2. Interaction potentials for the reactions $^{16}\mathrm{O} + {}^{42}\mathrm{Ca}$ and $^{16}\mathrm{O} + {}^{48}\mathrm{Ca}$, obtained in the modified Thomas–Fermi method, as well as the presentation of the corresponding potentials in analytic form (V_{FIT}) (13)

following form:

$$V_{\mathrm{FIT}}(R) = V_{\mathrm{WS}}(R) + V_{\mathrm{kin}}(R), \tag{10}$$

where V_{WS}(R) is the well-known formula for the Woods–Saxon potential

$$V_{\mathrm{WS}}(R) = \frac{-V_0}{1 + e^{(R-R_0)/d_0}}, \tag{11}$$

and $V_{\mathrm{kin}}(R)$ is the kinetic term in the form of the Thomas–Fermi method. The kinetic energy of the Thomas–Fermi method is proportional to $\rho^{5/3}$ (6), so, the repulsive term is approximated as follows, using the well-known Fermi formula for ρ:

$$V_{\mathrm{kin}}(R) = \left(\frac{V_C}{1 + e^{(R-C)/a}}\right)^{5/3}. \tag{12}$$

After that, our analytic potential takes the final form:

$$V_{\mathrm{FIT}}(R) = \frac{-V_0}{1 + e^{(R-R_0)/d_0}} + \left(\frac{V_c}{1 + e^{(R-C)/a}}\right)^{5/3}. \tag{13}$$

As a result, formula (13) contains six fitting parameters V, R, d, V_c, C, and a, the values of which are obtained by minimization for the highest quality description of the potential calculated within the framework of the modified Thomas–Fermi approach with density-dependent Skyrme forces. The potential parameters obtained for the considered reactions are presented in Table 1.

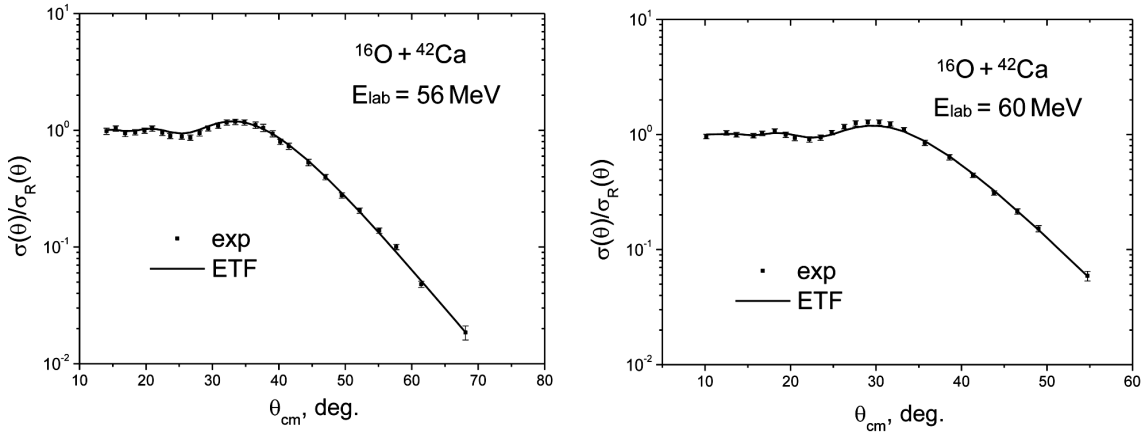

Fig. 3. Elastic scattering cross section for the $^{16}\mathrm{O} + {}^{42}\mathrm{Ca}$ system at beam energies $E_{\mathrm{lab}} = 56$ and 60 MeV, calculated within the framework of the modified Thomas–Fermi approximation with density-dependent Skyrme forces (ETF). Experimental data (exp) are taken from [41, 42]

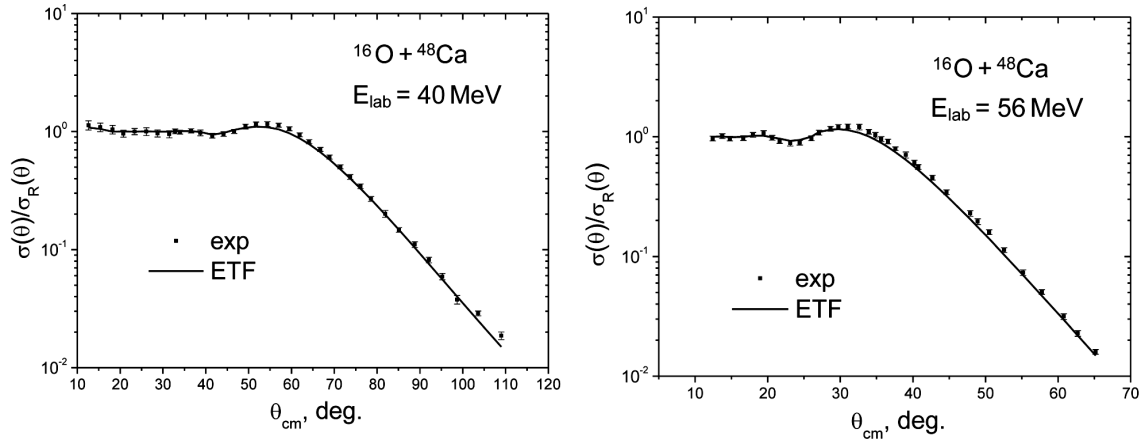

Fig. 4. Elastic scattering cross section for the $^{16}\mathrm{O} + {}^{48}\mathrm{Ca}$ system at beam energies $E_{\mathrm{lab}} = 40$ and 56 MeV, calculated within the framework of the modified Thomas–Fermi approximation with density-dependent Skyrme forces (ETF). Experimental data (exp) are taken from [43]

The quality of the approximation of the modified Thomas–Fermi approach potential with Skyrme forces using formula (13) for the $^{16}\mathrm{O} + {}^{42}\mathrm{Ca}$ and $^{16}\mathrm{O} + {}^{48}\mathrm{Ca}$ systems is demonstrated in Fig. 2. As we can see, the accuracy of the approximation is very high, the deviations are almost not noticeable on the scale of the graph. So, we can conclude that proposed formula is well suited for reproducing the realistic potentials of nucleus-nucleus interaction.

4. Calculations of the Elastic Scattering Cross Sections

We will calculate the cross sections of elastic scattering within the framework of the optical model. As the real part of the potential, we use the potentials obtained in the modified Thomas–Fermi method, approximated using formula (13). The corresponding approximation parameters are presented in Table 1. The imaginary part of the potential has the

Table 1. **Parameters of the analytic representation of the potential for the considered reactions**

Reaction	V_0, MeV	R_0, fm	d_0, fm	V_c, MeV$^{3/5}$	C, fm	a, fm
$^{16}\mathrm{O} + {}^{42}\mathrm{Ca}$	49.1634	6.7586	0.6847	20.6223	3.2221	1.0838
$^{16}\mathrm{O} + {}^{48}\mathrm{Ca}$	51.0870	6.9515	0.6767	20.1222	3.3725	1.0768

Table 2. **Parameters of the imaginary part of the potential (14) for the reaction $^{16}O + ^{42}Ca$**

E_{lab}, MeV	W_W, MeV	r_W, fm	d_W, fm	W_S, MeV	r_S, fm	d_S, fm
56	21.68126	1.100	0.300	9.231324	1.259115	0.615223
60	21.9798	1.100	0.300	9.30105	1.262264	0.641428

Table 3. **Parameters of the imaginary part of potential (14) for the reaction $^{16}O + ^{48}Ca$**

E_{lab}, MeV	W_W, MeV	r_W, fm	d_W, fm	W_S, MeV	r_S, fm	d_S, fm
40	22.01157	1.1106	0.301	9.400213	1.26300	0.65000
56	24.99059	1.1709	0.499	9.55938	1.26400	0.65418

following form [2, 4]:

$$W(R) = -\frac{W_W}{1+\exp\left[\frac{R-r_W(A_1^{1/3}+A_2^{1/3})}{d_W}\right]} - \frac{W_S \exp\left[\frac{R-r_S(A_1^{1/3}+A_2^{1/3})}{d_S}\right]}{d_S\left\{1+\exp\left[\frac{R-r_S(A_1^{1/3}+A_2^{1/3})}{d_S}\right]\right\}^2}. \quad (14)$$

Here, the parameters W_W, R_W, d_W, W_S, r_S, d_S are the strength, radius and diffusiness of the volume (W) and surface (S) terms of the imaginary potential. This form of the imaginary part of the potential is widely known and is often used in the description of nuclear reactions.

In this work, we consider elastic scattering reactions for $^{16}O + ^{42}Ca$ at beam energies $E_{lab} = 56$, 60 MeV, as well as $^{16}O + ^{48}Ca$ at beam energies $E_{lab} = = 40$, 56 MeV. As the real part of the potential, we used our potential calculated within the framework of the modified Thomas–Fermi method, approximated using (13). For the imaginary part, the parameters W_W, R_W, d_W, W_S, r_S, d_S were determined by fitting to best reproduce the experimental data from the elastic scattering cross sections. The parameters obtained in this way are presented in Tables 2 and 3.

Our calculated elastic scattering cross sections for $^{16}O + ^{42}Ca$ system at beam energies $E_{lab} = 56$, 60 MeV, and for $^{16}O + ^{48}Ca$ at energies $E_{lab} = 40$, 56 MeV are presented in Figs. 3 and 4. The cross-sections in the figures are normalized to the Rutherford cross section. From the figures, we can see that our calculated elastic scattering cross sections agree well with the available experimental data, which were taken from works [41–43].

5. Conclusions

In this work, we calculated the nucleus-nucleus interaction potentials within the framework of the modified Thomas–Fermi approach for the $^{16}O + ^{42}Ca$ and $^{16}O + ^{48}Ca$ systems. The density-dependent Skyrme forces were used as the nucleon-nucleon interaction, namely, SkP parametrization. Note that the nucleon distribution densities were obtained using the same approach. The obtained potentials have a completely realistic form, at small distances demonstrating the presence of a radically expressed repulsion core, which is very important when considering elastic scattering processes.

A successful form of parametrization is proposed, which well describes the realistic potentials of nucleus-nucleus interaction, in particular, obtained within the framework of the modified Thomas–Fermi method with Skyrme forces.

Based on calculated nucleus-nucleus interaction potentials, elastic scattering processes for the $^{16}O + ^{42}Ca$ and $^{16}O + ^{48}Ca$ systems at different energies were studied. The elastic scattering cross sections were obtained. Note that the expression of the real part of the potential for each reaction at different energies remained the same, the fitting was performed due to the imaginary part. Our calculation cross sections are in good agreement with the available experimental data.

1. R. Bass. *Nuclear Reactions with Heavy Ion* (Springer-Verlag, 1980).
2. G.R. Satchler. *Direct Nuclear Reactions* (Clarendon Press, 1983) [ISBN-13: 978-0198512691, ISBN-10: 0198512694].
3. P. Frobrich, R. Lipperheide. *Theory of Nuclear Reactions* (Clarendon Press, 1996) [ISBN: 9780198537830].
4. V.Yu. Denisov, V.A. Plujko. *Problems of Nuclear Physics and Physics of Nuclear Reactions* (Kyiv University Publ. Center, 2013) (in Russian) [ISBN: 978-966-439-673-5].
5. J. Blocki, J. Randrup, W.J. Swiatecki, C.F. Tsang. Proximity forces. *Ann. Phys.* **105**, 427 (1977).

6. W.D. Myers, W.J. Swiatecki. Nucleus-nucleus proximity potential and superheavy nuclei. *Phys. Rev. C* **62**, 044610 (2000).
7. V.Yu. Denisov. Nucleus-nucleus potential with shell-correction contribution and deep sub-barrier fusion of heavy nuclei. *Phys. Rev. C* **89** (4), 044604 (2014).
8. V.Yu. Denisov, V.A. Nesterov. Potential of interaction between nuclei and nucleon-density distribution in nuclei. *Phys. Atom. Nucl.* **69**, 1472 (2006).
9. V.Yu. Denisov. Interaction potential between heavy ions. *Phys. Lett. B* **526**, 315 (2002).
10. H.J. Krappe, J.R. Nix, A.J. Sierk. Unified nuclear potential for heavy-ion elastic scattering, fusion, fission, and ground state masses and deformations. *Phys. Rev. C* **20**, 992 (1979).
11. V.Yu. Denisov, W. Norenberg. Entrance channel potentials in the synthesis of the heaviest nuclei. *Eur. Phys. J. A* **15**, 375 (2002).
12. V.Yu. Denisov. Nucleus-nucleus potential with shell-correction contribution. *Phys. Rev. C* **91**, 024603 (2015).
13. A. Winther. Dissipation, polarization and fluctuation in grazing heavy-ion collisions and the boundary to the chaotic regime. *Nucl. Phys. A* **594**, 203 (1995).
14. V.Yu. Denisov, O.I. Davidovskaya. Repulsive core potential and elastic heavy-ion collisions. *Yad. Fiz.* **73**, 429 (2010).
15. V.Yu. Denisov, O.I. Davidovskaya. Repulsive core potential and elastic heavy-ion collisions. *Ukr. J. Phys.* **54**, 669 (2009).
16. K.A. Brueckner, J.R. Buchler, M.M. Kelly. New theoretical approach to nuclear heavy-ion scattering. *Phys. Rev. C* **173**, 944 (1968).
17. J. Fleckner, U. Mosel. Antisymmetrization effects in heavy ion potentials. *Nucl. Phys. A* **277**, 170 (1977).
18. O.I. Davidovskaya, V.Yu. Denisov, V.A. Nesterov. Nucleus-nucleus potential with repulsive core and elastic scattering. Part 1. Nucleus-nucleus interaction potential. *Nucl. Phys. At. Energy.* **11** (1), 25 (2010).
19. O.I. Davidovskaya, V.Yu. Denisov, V.A. Nesterov. Nucleus-nucleus potential with repulsive core and elastic scattering. Part 2. The elastic scattering cross sections with and without core. *Nucl. Phys. At. Energy.* **11** (1), 33 (2010).
20. V.Yu. Denisov, O.I. Davidovskaya. Elastic scattering of heavy ions and nucleus-nucleus potential with a repulsive core. *Bull. Russ. Acad. Sci. Phys.* **74**, 611 (2010).
21. O.I. Davidovskaya, V.Yu. Denisov, V.A. Nesterov. Effective nucleus-nucleus potential with the contribution of the kinetic energy of nucleons, and the cross-sections of elastic scattering and subbarrier fusion. *Ukr. J. Phys.* **62**, 473 (2017).
22. V.A. Nesterov. Effect of the Pauli exclusion principle and the polarization of nuclei on the potential of their interaction for the example of the ^{16}O + ^{16}O system. *Phys. At. Nucl.* **76**, 577 (2013).
23. V.Yu. Denisov, O.I. Davidovskaya. Elastic ^{16}O + ^{16}O scattering and nucleus-nucleus potential with a repulsive core. *Ukr. J. Phys.* **55**, 861 (2010).
24. O.I. Davydovska, V.Yu. Denisov, V.O. Nesterov. Nucleusnucleus potential, the elastic scattering and subbarrier fusion cross sections for the system ^{40}Ca + ^{40}Ca. *Nucl. Phys. At. Energy.* **19**, 203 (2018).
25. O.I. Davydovska, V.Yu. Denisov, V.A. Nesterov. Comparison of the nucleus-nucleus potential evaluated in the double-folding and energy density approximations and the cross-sections of elastic scattering and fusion of heavy ions. *Nucl. Phys. A* **989**, 214 (2019).
26. V.O. Nesterov, O.I. Davydovska, V.Yu. Denisov. Calculation of the cross-sections of sub-barrier fusion and elastic scattering of heavy ions using the modified Thomas–Fermi approach with the Skyrme force. *Nucl. Phys. At. Energy.* **20** (4), 349 (2019).
27. P. Ring, P. Schuck. *The Nuclear Many-Body Problem* (Springer-Verlag, 1980).
28. M. Brack, C. Guet, H.B. Hakanson. Self-consistent semiclassical description of average nuclear properties – a link between microscopic and macroscopic models. *Phys. Rep.* **123**, 275 (1985).
29. M. Brack, R.K. Bhaduri. *Semiclassical Physics* (Addison-Wesley Publ. Co, 1997) [ISBN-10: 0813340845; ISBN-13 : 978-0813340845].
30. V.M. Strutinsky, A.G. Magner, V.Yu. Denisov. Density distributions in nuclei. *Z. Phys. A* **322**, 149 (1985).
31. J. Dobaczewski, W. Nazarewicz, P.G. Reinhard. Pairing interaction and self-consistent densities in neutron-rich nuclei. *Nucl. Phys. A* **693**, 361 (2001).
32. D. Vautherin, D.M. Brink. Hartree–Fock calculations with Skyrme's interaction. I. Spherical nuclei. *Phys. Rev. C* **5**, 626 (1972).
33. J. Bartel, P. Quentin, M. Brack, C. Guet, H.B. Hakansson. Towards a better parametrization of Skyrme-like effective forces: A critical study of the SkM force. *Nucl. Phys. A* **386**, 79 (1982).
34. S.A. Fayans, S.V. Tolokonnikov, E.L. Trykov, D. Zawischac. Nuclear isotope shifts within the local energy-density functional approach. *Nucl. Phys. A* **676**, 49 (2000).
35. J.W. Negele. The mean-field theory of nuclear structure and dynamics. *Rev. Mod. Phys.* **54**, 913 (1982).
36. T.H.R. Skyrme. The effective nuclear potential. *Nucl. Phys.* **9**, 615 (1959).
37. H. Feshbach. The optical model and its justification. *Annu. Rev. Nucl. Sci.* **8**, 49 (1958).
38. H. Friedrich, L.F. Canto. Effective nucleus-nucleus potentials derived from the generator coordinate method. *Nucl. Phys. A* **291**, 249 (1977).
39. O.I. Davydovska, V.A. Nesterov, V.Yu. Denisov. The nucleus-nucleus potential within the extended Thomas–

Fermi method and the cross-sections of subbarrier fusion and elastic scattering for the systems $^{16}O + {}^{58,60,62,64}Ni$. *Nucl. Phys. A* **1002**, 121994 (2020).

40. J. Orloff, W.W. Daehnick. Elastic scattering of ^{16}O by ^{48}Ti, ^{40}Ca, ^{27}Al, ^{12}C, ^{7}Li, and ^{6}Li. *Phys. Rev. C* **3**, 430 (1971).
41. Y. Eisen, H.T. Fortune, W. Henning, D.G. Kovar, S. Vigdor, B. Zeidman Two-nucleon transfer reactions induced by ^{16}O on ^{42}Ca and ^{48}Ca. *Phys. Rev. C* **13**, 699 (1976).
42. K.E. Rehm, W. Henning, J.R. Erskine, D.G. Kovar, M.H. Macfarlane, S.C. Pieper, M. Rhoades-Brown. Inelastic scattering of ^{16}O from $^{40,42,44,48}Ca$. *Phys. Rev. C* **25**, 1915 (1982).
43. K.O. Groeneveld, L. Meyer-Schützmeister, A. Richter, U. Strohbusch. Backward – angle elastic scattering of ^{16}O on ^{40}Ca and ^{48}Ca. *Phys. Rev. C* **6**, 805 (1972).

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О.І. Давидовська, В.О. Нестеров

ПЕРЕРІЗИ ПРУЖНОГО РОЗСІЯННЯ ДЛЯ $^{16}O + {}^{42}Ca$ ТА $^{16}O + {}^{48}Ca$, ОДЕРЖАНІ НА ОСНОВІ ПОТЕНЦІАЛУ МОДИФІКОВАНОГО МЕТОДУ ТОМАСА–ФЕРМІ З УРАХУВАННЯМ КОРУ

Густини розподілу нуклонів та потенціали ядро-ядерної взаємодії для реакцій $^{16}O+{}^{42}Ca$ та $^{16}O+{}^{48}Ca$ було розраховано в рамках модифікованого методу Томаса–Фермі, з урахуванням усіх доданків до членів другого порядку по $\hbar$ у квазикласичному розкладі кінетичної енергії. В ролі нуклон-нуклонної взаємодії використовувалися сили Скірма, залежні від густини нуклонів. Знайдено параметризацію потенціалу ядро-ядерної взаємодії, яка добре описує величину потенціалу, розрахованого у рамках модифікованого підходу Томаса–Фермі з залежними від густини силами Скірма. На основі одержаних потенціалів були обчислені перерізи пружного розсіяння, що добре узгоджуються з експериментальними даними.

Ключові слова: метод Томаса–Фермі, реакції $^{16}O + {}^{42}Ca$ та $^{16}O + {}^{48}Ca$.