https://doi.org/10.15407/ujpe70.6.373

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NONLINEARITIES IN MAGNETIC CONFINEMENT, IONOSPHERIC PHYSICS, AND POPULATION EXPLOSION LEADING TO PROFILE RESILIENCE

Nonlinearities play an important role in many fields. In the field of thermonuclear fusion, they are involved in questions such as profile resilience and fluid closure. A nonlinear phenomenon common to both fusion and astrophysical planets is the generation of zonal flows. These flows play a significant role in determining the level of turbulence and fluid closure in fusion. The effects of resonance broadening and nonlinearities are investigated, specifically focusing on the case of nonlinear instability that has appeared in drift waves. Similarities and differences between our systems are discussed, with population explosion and the dynamics of nonlinear systems for drift waves by different states in profile resilience described with great precision. The aim of our study is to put our fluid model for drift waves in tokamaks within the wider framework of statistical physics principles. This reinforces our belief in the broad application of our drift wave model, which encompasses current tokamaks, ITER, and the fusion pilot plant.

Keywords: magnetic confinement, nonlinearities, resonance broadening, drift waves, tokamaks, profile resilience.

1. Introduction

We have recently pointed out that nonlinearities have to be included in the description of several phenomena, where they are often left out [1–48]. We have here studied the effect of nonlinearities and their impact on profile resilience [8]. In particular, we have looked at the role of resonance broadening [1, 3, 7], since this influences the fluid closure [16, 31, 34, 35], ionospheric physics [24, 25], particle and heat pinches [11–14], and recent models for the dynamics of fast particles [33, 34, 40]. Although the linear theory [15] can be useful to list different types of instabilities, the nonlinear theory [1–4], is always needed to determine the state of saturated turbulence. Initially, rather much work was devoted to the separate studies of linear [15] and nonlinear [1–4] theories. Such work was initially mainly conducted in simple slab geometry. However, most physics relevant for transport is on a rather small scale and can then be described by the ballooning mode formalism [5]. This description could be used for most phenomena previously described in a simple geometry [3, 6].

One of the areas of interest is the fluid-like closure in magnetized plasmas, where the correct closure was already made in 1988 in the Weiland model [10]. This closure, although initially made intuitively, depends on nonlinear effects through zonal flows and is needed in order to obtain the appropriate pinch

Citation: Weiland J., Rafiq T., Schuster E. Nonlinearities in magnetic confinement, ionospheric physics, and population explosion leading to profile resilience. *Ukr. J. Phys.* **70**, No. 6, 373 (2025). https://doi.org/10.15407/ujpe70.6.373.

effects as shown in Ref. [11]. These are needed both for transport in tokamaks and for understanding the particle motion in the ionosphere [6, 9, 23, 24]. Another application for magnetized laboratory plasmas is the levitated dipole [24, 25]. A further application is for the population explosion on the Earth [32]. The crucial effect to be included here is the nonlinear frequency shift [1,3,4,9] which leads to resonance broadening. We note that models for population development [32] do not have the same base as our results for drift waves, while a non-Markovian effect [16], which will show as oscillations of the same type as those for drift waves, is important. The application to tokamak transport, where profiles of the density and temperature have been surprisingly insensitive to the exact location of sources, has been named "profile consistency" or "profile resilience" [8].

The present work is organized as follows. Section 2, describes the fundamental balance between the linear growth rate and convective $\mathbf{E}\times\mathbf{B}$ nonlinearity and elaborates on the saturation level and the diagonal element for the transport. Section 3 presents the system of three interacting waves and then delves into the stabilization of nonlinear instability through nonlinear frequency shifts. Section 4 explores the dynamics of explicit zonal flows at the correlation length, their significance in specific calculations, and the alternative of assuming absorbing boundaries for long wavelengths in the absence of active zonal flows. Section 5 highlights the significant contribution of the integration along particle orbits in advancing the theoretical frameworks. Section 6 discusses the significance of the resonance broadening in nonlinear frequency shifts and its role in changing the phase velocity of waves, taking them out of resonance with particles. Section 7 concludes by highlighting the striking similarity between the system describing the population explosion with extreme accuracy [32] and the dynamics of nonlinear systems for drift waves [9, 10], based on different states in profile resilience [8].

2. Introduction to Drift Wave Calculations, Saturation Level and Transport

The basis of our drift wave calculations is the balance between the linear growth rate and convective $\mathbf{E}\times\mathbf{B}$ nonlinearity $(\mathbf{v}_\mathrm{E}\cdot\nabla\delta T)$

$$\gamma\delta T = \mathbf{v}_\mathrm{E}\cdot\nabla\delta T. \tag{1}$$

This leads to the saturation level

$$\frac{e\phi}{T_\mathrm{e}} = \frac{\gamma}{\omega_{*\mathrm{e}}}\frac{1}{k_x L_\mathrm{n}}, \tag{2}$$

where e is the charge of an electron, ϕ is the electrostatic potential, T_e is the temperature of an electron, γ is the mode's growth rate, $\omega_{*\mathrm{e}}$ is the frequency of diamagnetic drift, k_x is the radial propagation factor, and L_n is the length of a density gradient scale. The diagonal element for the transport can be written as follows:

$$D = \frac{\gamma^3/k_x^2}{\omega_r^2+\gamma^2}. \tag{3}$$

Now, it has been found that the fastest growing mode in typical drift-wave systems may have negative energy [4]. This means that waves grow, when the energy is taken from them. This works both linearly (inverse Landau damping) and nonlinearly, with the possibility of explosive instability [4]. An example of the inverse Landau damping is the Hammett–Perkins case, as shown by Mattor and Parker [35]. We can also conclude that the nonlinear terms are destabilizing from the figure in Ref. [35]. In such cases, which seem to be typical, we have nonlinear growth [35, 36] which, if fully developed, could lead to an explosive instability [4, 32]. In such cases, a nonlinear frequency shift will develop and turn the energy into positive energy, thus, stabilizing the system.

3. System of Interacting Waves and Nonlinear Frequency Shift

We start from a system of three interacting waves j, k, l:

$$\frac{\partial u_j}{\partial t} = \gamma u_j + c_j\, u_k\, u_l \cos\Phi, \tag{4a}$$

$$\Phi = \phi_j - \phi_k - \phi_l, \tag{4b}$$

$$\frac{\partial\Phi}{\partial t} = \delta\omega + \left(\frac{u_k u_l}{u_j} - \frac{u_j u_k}{u_l} - \frac{u_j u_l}{u_k}\right)\sin\Phi, \tag{4c}$$

$$\delta\omega = \sum \alpha_\mathrm{m}|u_\mathrm{m}|^2, \tag{4d}$$

where $u = e\phi/T_e$. We note that $\delta\omega$ has a higher power in the perturbation. This is the nonlinear frequency shift, which is a change in the frequency due to the nonlinear interactions between different waves. It will dominate at high levels of perturbation.

System (4) has a constant of motion

$$u_j\,u_k\,u_l\,\sin\,\Phi + \sum \frac{\alpha_j}{c_j}|u_j|^2 = \Gamma e^{3\,\gamma\,t}, \tag{5}$$

where Γ is a constant of motion. In order to see qualitatively how the stabilization by a nonlinear frequency shift occurs, we can consider the case of equal amplitudes of perturbations, i.e., $u_j = u_k = u_l = u$. Then (5) gives the stationary state:

$$u^3 \sin\Phi + \kappa u^4 = \Gamma, \tag{6a}$$

$$\kappa = \frac{1}{4}\sum \frac{\alpha_j}{c_j}. \tag{6b}$$

We then find the maximum amplitude

$$u = \frac{1}{\kappa}. \tag{7}$$

The simplest solution of this system is for $\Gamma = 0$. It then takes the form of a soliton solution in time [4]

$$u(t) = \frac{1}{\sqrt{\kappa^2 + (t_1 - t)^2}}, \tag{8a}$$

where

$$t_1 = \frac{1}{u(0)}\sqrt{1 - \kappa^2 u^2(0)}. \tag{8b}$$

When $\Gamma \neq 0$, the solution does not meet its initial condition after the maximum, and we get an oscillatory solution. In the general dissipation-free case, we can derive our system from a Hamiltonian

$$H = \sum_j s_j\,\omega_j\,u_j^2 + 2V(u_0^2\,u_1^2\,u_2^2)^{1/2}\sin\Phi - \sum_{jk}\gamma_{jk}u_j^2u_k^2. \tag{9}$$

For more details, we refer to Ref. 4 pages 135 and 153. However, our main result is that we reach a steady state, where we define

$$\cos\Phi = \mu, \tag{10}$$

$$u = \frac{\gamma}{c\mu}. \tag{11}$$

We are free to choose c in such a way that (11) fulfills (2) with our present definition of u. However, a typical situation as found from (6a) is that $\sin\Phi$ is close to 0, which means that $\cos\Phi = +/-1$. Thus, the magnitude of the coupling is the same in unstable and stable cases. This means that our estimate of the turbulence level continues to be given by (2). However, the effective energy sign oscillates so that dissipative kinetic effects are averaged out. This means that our stabilization of a nonlinear instability due to a nonlinear frequency shift maintains our previous saturation level for $\mathbf{E}\times\mathbf{B}$ stabilization of a linear instability. This saturation level has been found to be in good agreement with experiments, as seen from the general agreement of our model with experiment [42, 43].

However, in Ref. [11] we also included the full quasilinear transport, which includes pinch effects. The frequency dependence of (3) is a non-Markovian effect. In order to have pinch effects leading to inward fluxes, we also need nonadiabatic electrons. This was accomplished by including electron trapping, as shown in Ref. [11]. It gave possibility for both particle and thermal pinches. In particular, the particle pinch has been much explored in connection with the levitated dipole [24, 25] and ionospheric phenomena [3, 4]. It was further tested with good agreement for a tokamak experiment (Tore Supra) [28]. A similar agreement was also found with QualiKiz [19, 27, 28], a model based on quasilinear theory, but fitted to fully nonlinear simulations. We had further experimental support for poloidal spinup in internal transport barriers [20], nonlinear upshift in internal transport barriers, and the L-H-transition [23, 44]. We also note that (3) was obtained by Connor and Pogutse for collisional drift waves [17].

A remaining question is the validity of (3), when we have nonlinear growth at low amplitudes, as found in Refs. [17, 35, 36]. This will give rise to the initial phase of explosive instability, as discussed in Ref. [4]. We know from Ref. [4] that such an instability will be stabilized by a nonlinear frequency shift, as also seen in Refs. [35, 36]. Here, we get oscillations after the initial saturation (compare Refs. [4, 35], [32, 36]), and this averages out the effect of higher nonlinearities. The nonlinear frequency shift works to change the sign of the wave energy, or dissipation. Thus, we expect (3) to be valid on average. Further, we note that, for the population explosion, we only have the initial phase of stabilization by a nonlinear frequency shift. Thus, the growth rate is due to a balance between quadratic and cubic nonlinearity. This becomes very accurate, since local and system-dependent quantities usually enter through the linear growth rate.

We finally note the interesting similarity to several papers [45–47], where a fixed linear frequency mismatch in a three-wave interaction leads to limit cycles similar to those in Figure 1. The main difference

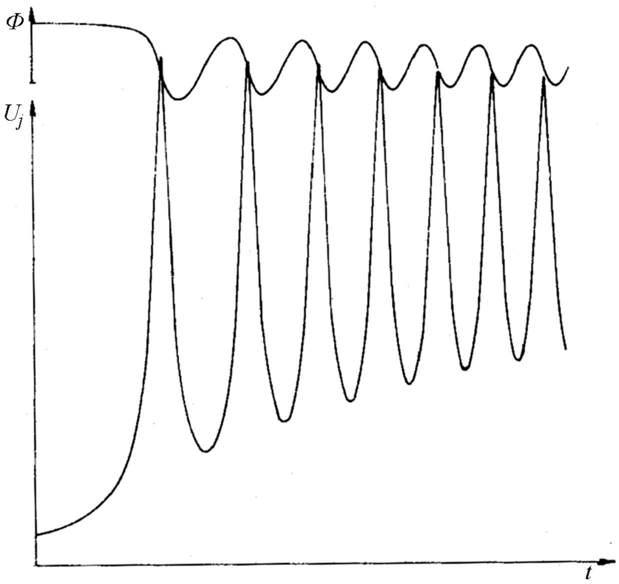

Fig. 1. Stabilization of an explosive instability, from Ref. 4. The oscillations are due to oscillations in Φ due to an oscillating nonlinear frequency shift. The oscillations in Φ lead to oscillations in cosΦ giving the periodic behavior of the amplitude

is that we consider a nonlinear frequency shift due to cubic nonlinear effects. Nonlinear frequency shifts play a major role in plasma turbulence, as first shown by Dupree [1]. This was demonstrated in a general renormalization of plasma turbulence, where nonlinear frequency shifts act as a nonlinear friction, leading to resonance broadening. We later showed that this process results in the saturation of quasilinear energy transfer between waves and particles, effectively eliminating linear wave energy transfer mechanisms such as Landau damping or magnetic drift resonances [16]. However, this is not the case where a resonance exists between waves and an external source, such as during the neutral beam heating or nuclear reactions as demonstrated in [40]. It is also interesting to compare this with the Mattor–Parker system [35, 36], which includes a nonlinear frequency shift, but represents the coherent limit of resonance broadening. In all these cases, the resonance broadening leads to a fluid-like closure, where waves move out of a resonance with particles.

Another case where nonlinear frequency shifts are important is given by the stabilization of explosive instabilities [4]. Since we have found that the Mattor–Parker system is nonlinearly unstable, when only quadratic nonlinearities are considered, we have explored its similarity to the system studied in [4]. As it turns out, the stabilization of explosive instabilities by nonlinear frequency shifts results in a scenario, where the sign of the wave-particle interaction changes after the stabilization. This implies that the limit cycle averages out the dissipative wave-particle interaction, as previously predicted in [16].

4. The Importance of Boundary Conditions in K-Space

As pointed out above, we have a similarity in the boundary conditions in k-space between Ref. [10] and Ref. [17]. These models both use reactive fluid models. An important aspect of the drift wave turbulence is that it cascades both toward shorter and longer wavelengths [6]. The cascade toward shorter wavelengths is a common feature of 3D turbulence, and this is absorbed by viscosity. However, the cascade toward longer wavelengths is critical in a finite system. Here, we need a sink for the turbulence, which absorbs the cascade toward long wavelengths [6]. This sink is caused by another nonlinearity, which is the generation of zonal flows [9, 21, 27, 28]. In the absence of this sink, the energy is accumulated at the longest wavelengths possible in the system, which leads to the excessive transport. The increased transport in the presence of dissipation was already observed in Ref. [9]. Thus, we need strong zonal flows, and these are strongest in reactive systems [10, 17]. Thus, the level of transport depends sensitively on the fluid-like closure. With a reactive closure, we get a fluid description. We note, in particular, the very strong sensitivity of zonal flows to the type of dissipation used in gyrofluid closures [22], where the Dimits nonlinear upshift [22] could not be reproduced when the dissipation was included. In a fluid description, a quasilinear description is usually valid (Refs. [17, 37]). However, in a kinetic description, we need the strongly nonlinear resonance broadening. The reason for why we need strongly nonlinear effects in a kinetic description but not in a fluid description is the vastly different magnitudes of the velocities. However, in fluid theory, we also need to include zonal flows, as shown in Refs. [20] and [21]. Actually, explicit zonal flows at the correlation length are needed in calculations of Dimits shift, spinup in internal transport barriers, and the L-H transition. When zonal flows are not active at the correlation length, we can just assume an absorbing boundary for long wavelengths, as done in, e.g., Ref. [11].

5. Orbit Integration

The integration along particle orbits has played an important role in the development of our theory. The first example is the derivation of linear and nonlinear gyrokinetic equations [26]. The first nonlinear gyrokinetic equation was actually derived by Frieman and Chen [41] by averaging the local orbits. We then derived a nonlinear gyrokinetic equation by the orbit integration [26]. This latter derivation is actually shorter, but both derivations lead to the Hasegawa–Mima equation [6] in the appropriate limit. In Ref. [37], we also showed, by the orbit integration, that the linear part of the eigenfrequency is typically obtained with high accuracy in a quasilinear treatment, as also found in Ref. [17]. Now, the imaginary part of the frequency can be due either to inhomogeneities in configuration space or wave particle resonances in kinetic theory. The latter reason will vanish in the long time asymptotic part of the velocity dispersion, i.e., the deviation of the velocity square from its initial condition. Thus, there will be no more energy transfer between resonant particles and waves.

6. Resonance Broadening

We have mentioned resonance broadening on several occasions. Resonance broadening [1,3,7,30–32,35,36] occurs due to nonlinear frequency shifts that act to reduce the effect of wave-particle resonances. This process is active, when we keep a Maxwellian distribution but still observe that wave-particle resonances are not active [35, 36]. This is a strongly nonlinear effect that is not present in quasilinear theory. This effect has recently been added to several studies of fast particle instabilities [33, 34]. We have also recently derived a combined theory for drift wave turbulence and nonlinear friction (resonance broadening), where resonance broadening is one of the main features [40]. As expected, the resonance broadening reduces the strength of wave-particle resonances when there are no external sources in the velocity space, as usually for drift waves. A more general treatment of triplet wave dynamics with independent, unequal damping rates has recently been explored in the context of fast ion-driven modes [47], further demonstrating the role of nonlinear effects in stabilizing energetic particle-driven instabilities.

Resonance broadening is able to make a reactive fluid-like closure possible [16]. Without resonance broadening, linear Landau damping and magnetic drift resonances remain and are able to completely damp out particle pinches [11, 14]. This is also the reason for the need to make a fit to nonlinear kinetic theory in Ref. [19]. Thus, the essence of the fit of QualiKiz to nonlinear kinetic codes is to introduce resonance broadening with an empiric procedure. As mentioned above, this leads to similar results by our model and QualiKiz for the particle pinch in Tore Supra [27, 28]. Thus, a strongly nonlinear approach is needed for kinetic theory. On the other hand, the orbit integration has shown that the quasilinear approach is sufficient in fluid calculations. This was also found in Ref. [17]. Of course, we define strongly nonlinear as including an explicit nonlinear frequency shift. We also need to include zonal flows in the fluid model [20, 21]. In Ref. [40], we found that there may be a balance between resonance broadening β and an external source in the velocity space, S_v for fast particles. Thus, we expect the resonance broadening to reduce fast particle instabilities.

$$\left(\frac{\partial}{\partial t} + v\cdot\frac{\partial}{\partial r}\right) W(X,X',t,t') = \frac{\partial}{\partial v}\left[\beta v + D^{\nu}\frac{\partial}{\partial v}\right]\times$$
$$\times W(X,X',t,t') + S_{\rm v}. \tag{12}$$

Here W is the transition probability in phase space, β is the nonlinear friction (resonance broadening), D^{ν} is the nonlinear diffusivity in the velocity space, and $S_{\rm v}$ is an external source (typically, the heating) in the velocity space

$$\beta = \sum_j \beta_j |\phi_j|^2, \tag{13a}$$

$$D^{\nu} = \sum_j d_j |\phi_j|^2. \tag{13b}$$

However, at drift wave frequencies, the fast particle source term is typically a factor 100 smaller than the resonance broadening and can, thus, be neglected. We are then back in the Fokker–Planck equation derived in Ref. [16]. Thus, our fluid model is valid to within 1% in the drift wave regime.

For constant coefficients and without source (12) leads to the velocity dispersion

$$\langle \triangle v^2 \rangle = \frac{D^\nu}{\beta}(1 - e^{-\beta t}). \tag{14}$$

Showing that $\triangle v^2$ saturates for $t > 1/\beta$. This means that, after this time, there is no more energy transfer between waves and resonant particles. This fact was tested for much more general cases in Ref. [48].

We found that the resonance broadening β can counteract an external source term in the velocity space, $S_{\rm v}$, leading to a reduction in the fast particle instabilities. However, this suppression is not absolute; if the source term is in resonance with the excited waves, instabilities such as fishbones can still be driven. In the case of fusion alphas and neutral beam injection, where $S_{\rm v}$ is sharply peaked at the birth energy, fishbone modes can be preferentially excited by particles at their injection energy before redistribution effects modify their phase-space characteristics.

As pointed out above, the resonance broadening is what is needed for kinetic models to give adequate particle pinches [19]. In our fluid model, however, the resonance broadening has already turned the kinetic model into a fluid model, and, there, we do not need further strongly nonlinear effects in order to recover particle pinches [11]. Instead, our quasilinear fluid model allows us to recover the Dimits nonlinear upshift [21, 22, 31], the L-H transition [23], and the poloidal spinup in internal transport barriers [20] when we include zonal flows.

7. Discussion

In conclusion, we have explained the similarity between our system describing the population explosion with high accuracy [32] and the dynamics of nonlinear systems for drift waves [9,10] by different states in the profile resilience [8]. The stabilization of the world population, projected to reach a saturation level of 10 billions, which is the estimated maximum the world can sustain, bears strong similarities to the stabilization of explosive instabilities driven by cubic nonlinearities. The resulting limit cycle oscillations arise from the inclusion of a 25-year time delay in the stabilizing cubic terms. To the best of our knowledge, the connection between explosive instabilities and saturation due to cubic nonlinear terms has not previously been associated with the mixing length saturation of drift waves. Another important aspect is that the orbit integration has shown that our quasilinear fluid approach works extremely well [32–36]. However, the resonance broadening is the main nonlinear effect that turns kinetic theory into fluid theory. Our first derivation of our model was mainly intuitive, assuming that moments without sources in the experiment would be damped out by transport. It was really not until Ref. [16] that we could see how the resonance broadening turned a fully nonlinear description into a reactive fluid model. While a quasilinear kinetic model does not have a particle pinch, a fit to the nonlinear kinetic model, introducing the resonance broadening [19,27] recovers the particle pinch. However, our fluid model recovers the particle pinch directly without additional fitting [11, 28]. Furthermore, an important aspect is that we keep $\epsilon_n = 2L_{\rm n}/R$ arbitrary. This means that we can describe L modes, as well as H modes and the L-H transition dynamically [23]. This also includes electromagnetic effects, and the H-mode barrier is typically in the second stability regime of MHD ballooning modes. This strengthens our confidence in the broad applicability of our drift wave model, which involves current tokamaks, ITER, and fusion pilot plants in addition to various ionospheric problems [25], the levitated dipole [24, 25] and the population explosion [32].

This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Fusion Energy Sciences, under Award Numbers DE-SC0013977. One of us (JW) wants to express his gratitude to the Bogolyubov Institute for Theoretical Physics, Kyiv, Ukraine, for developing the nonlinear kinetic theory on which our main derivations are based.

1. T.H. Dupree, A perturbation theory for strong plasma turbulence. *Phys. Fluids* **9**, 1773 (1966).
2. B. Coppi, M. Rosenbluth, R. Sudan. Nonlinear interactions of positive and negative energy modes in rarefied plasmas. *Ann. Phys.* **55**, 207 (1969).
3. A. Hasegawa. *Plasma Instabilities and Nonlinear Effects* (Springer, 1975).
4. J. Weiland, H. Wilhelmsson. *Coherent Non-linear Interaction of Waves in Plasmas* (Pergamon Press, 1977).
5. J.W. Connor, R.J. Hastie, J.B. Taylor. Shear, periodicity, and plasma ballooning modes. *Phys. Rev. Lett.* **40**, 396 (1978).

6. A. Hasegawa, K. Mima. Pseudo three dimensional turbulence in magnetized nonuniform plasma. *Phys. Fluids* **21**, 87 (1978).
7. J.D. Huba, K. Papadopoulos. Nonlinear stabilization of the lower hybrid drift instability by electron resonance broadening. *Phys. Fluids* **21**, 121 (1978).
8. B. Coppi. Nonclassical transport and the principle of profile consistency. *Comments Plasma Phys. Controlled Fusion* **5**, 261 (1980).
9. M. Wakatani, A. Hasegawa. A collisional drift wave description of plasma edge turbulence, *Phys. Fluids* **27**, 611 (1984).
10. J. Weiland, H. Nordman. Theory of fusion plasmas. In: *Proc. Varenna-Lausanne Workshop, Chexbres 1988, Bologna Editrice Compostori* (1988), p. 451.
11. J. Weiland, A. Jarmén, H. Nordman. Diffusive particle and heat pinch effects in toroidal plasmas. *Nucl. Fusion* **29**, 1810 (1989).
12. T.C. Luce, C.C. Petty, J.C.M. de Haas. Inward energy transport in tokamak plasmas. *Phys. Rev. Lett.* **68**, 52 (1992).
13. J. Weiland, H. Nordman. Drift wave model for inward energy transport in tokamak plasmas. *Phys. Fluids B* **5**, 1669 (1993).
14. F. Wagner, U. Stroth. Transport in toroidal devices' the experimentalist's view. *Plasma Phys. Controlled Fusion* **35**, 1321 (1993).
15. A.J. Redd, A.H. Kritz, G. Bateman, G. Rewoldt, W.M. Tang. Drift mode growth rates and associated transport. *Phys. Plasmas* **6**, 1162 (1999).
16. A. Zagorodny, J. Weiland. Statistical theory of turbulent transport (non-Markovian effects). *Phys. Plasmas* **6**, 2359 (1999).
17. J.W. Connor, O.P. Pogutse. On the relationship between mixing length and strong turbulence estimates for transport due to drift turbulence. *Plasma Phys. Controlled Fusion* **43**, 155 (2001).
18. J. Weiland, I. Holod. Drift wave transport scalings introduced by varying correlation length. *Phys. Plasmas* **12**, 012505 (2004).
19. C. Bourdelle, X. Garbet, F. Imbeaux, A. Casati, N. Dubuit, R. Guirlet, T. Parisot. A new gyrokinetic quasilinear transport model applied to particle transport in tokamak plasmas. *Phys. Plasmas* **14**, 112501 (2007).
20. J. Weiland, K. Crombe, P. Mantica, V. Naulin, T. Tala. JET-EFDA Contributors. Comparison of edge and internal transport barriers in drift wave predictive simulations. *AIP Conf. Proc.* **1392**, 85 (2011).
21. G.Q. Wang, J. Ma, J. Weiland. Zonal flows near marginal stability in drift wave transport. *Phys. Scr.* **90**, 065604 (2015).
22. A.M. Dimits, G. Bateman, M.A. Beer, B.I. Cohen, W. Dorland, G.W. Hammett, C. Kim, J.E. Kinsey, M. Kotschenreuther, A.H. Kritz *et al.* Comparisons and physics basis of tokamak transport models and turbulence simulations. *Phys. Plasmas* **7**, 969 (2000).
23. J. Weiland. Simulations of the L-H transition on experimental advanced superconducting Tokamak. *Phys. Plasmas* **21**, 122501 (2014).
24. A.C. Boxer, R. Bergmann, J.L. Ellsworth, D.T. Garnier, J. Kesner, M.E. Mauel, P. Woskov. Turbulent inward pinch of plasma confined by a levitated dipole magnet. *Nat. Phys.* **6**, 207 (2010).
25. J. Weiland. Turbulence at a pinch. *Nat. Phys.* **6**, 167 (2010).
26. J. Weiland. *Stability and Transport in Magnetic Confinement Systems* (Springer, 2012).
27. W.L. Zhong, X.L. Zou, C. Bourdelle, S.D. Song, J.F. Artaud, T. Aniel, X.R. Duan. Convective velocity reversal caused by turbulence transition in tokamak plasma. *Phys. Rev. Lett.* **111**, 265001 (2013).
28. J. Ma, G. Wang, J. Weiland, T. Rafiq, A.H. Kritz. Reversal of particle flux in collisional-finite beta tokamak discharges. *Phys. Plasmas* **22**, 012304 (2015).
29. T. Rafiq, A.H. Kritz, J. Weiland, A.Y. Pankin, L. Luo. Physics basic of Multi-Mode anomalous transport module. *Phys. Plasmas* **20**, 032506 (2013).
30. M. Kikuchi, M. Azumi. *Frontiers in Fusion Research II, Introduction to Modern Tokamak Physics* (Springer, 2015).
31. J. Weiland, A. Zagorodny. On the normalization of transport from ITG Modes. *Phys. Plasmas* **23**, 102307 (2016).
32. J. Weiland, T. Rafiq. Effects of nonlinearities in physics and demography. *Ukr. J. Phys.* **67**, 574 (2022).
33. K. Ghantous, H.L. Berk, N.N. Gorelenkov. Comparing the line broadened quasilinear model to Vlasov code. *Phys. Plasmas* **21**, 032119 (2014).
34. G. Meng, N. Gorelenkov, V. Duarte, H. Berk, R. White, X. Wang. Resonance frequency broadening of wave-particle interaction in tokamaks due to Alfvénic eigenmode. *Nucl. Fusion* **58**, 082017 (2018).
35. N. Mattor, S.E. Parker. Nonlinear kinetic fluid equations. *Phys. Rev. Lett.* **79**, 3419 (1997).
36. I. Holod, J. Weiland, A. Zagorodny. Nonlinear fluid closure: Three-mode slab ion temperature gradient problem with diffusion. *Phys. Plasmas* **9**, 1217 (2002).
37. J. Weiland. The role of zonal flows in reactive fluid closures. *Plasma Sci. Technol.* **20**, 074007 (2018).
38. J. Weiland, J. Zagorodny. Drift wave theory for transport in tokamaks. *Rev. Mod. Plasma Phys.* **3**, 8 (2019).
39. J. Weiland, A. Zagorodny, T. Rafiq. Theory for transport in magnetized plasmas *Phys. Scr.* **95**, 105607 (2020).
40. J. Weiland, T. Rafiq, E. Schuster. Fast particles in drift wave turbulence. *Phys. Plasmas* **30**, 042517 (2023).
41. E.A. Frieman, L. Chen. Nonlinear gyrokinetic equations for low-frequency electromagnetic waves in general plasma equilibria. *Phys. Fluids* **25**, 502 (1982).
42. G. Bateman, A.H. Kritz, J.E. Kinsey, A.J. Redd, J. Weiland. Predicting temperature and density profiles in tokamaks. *Phys. Plasmas* **5**, 1793 (1998).
43. E. Doyle, W. Houlberg, Y. Kamadam *et al.* Chapter 2: Plasma confinement and transport. *Nucl. Fusion* **47**, S18 (2007).
44. T. Rafiq, J. Weiland. Self-consistent core-pedestal ITER scenario modeling. *Nucl. Fusion* **61**, 116005 (2021).

45. S.Y. Vyshkind, M.I. Rabinovich. The mechanism for phase stochastizationand the structure of wave turbulence in dissipative media. *Sov. Phys. JETP* **44**, 292 (1976).
46. J. Wersinger, J.M. Finn, E. Ott. Bifurcation and " strange" behavior in instability saturation by nonlinear three-wave mode coupling. *Phys. Fluids* **23**, 1142 (1980).
47. V.S. Marchenko, S.N. Reznik, Y.I. Kolesnichenko. Nonlinear dynamics of multiple Alfvén modes driven by trapped energetic ions in tokamaks: A triplet paradigm. *Phys. Plasmas* **31**, 022507 (2024).
48. I. Holod, A. Zagorodny, J. Weiland. Anisotropic diffusion across an external magnetic field and large-scale fluctuations in magnetized plasmas. *Phys. Rev. E* **71**, 046401 (2005).

Received 06.10.24

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НЕЛІНІЙНОСТІ В МАГНЕТНОМУ УТРИМАННІ, ФІЗИЦІ ІОНОСФЕРИ ТА ПРОЦЕСІ ДЕМОГРАФІЧНОГО ВИБУХУ, ЯКІ ПРИВОДЯТЬ ДО СТІЙКОСТІ ПРОФІЛЮ

Нелінійності відіграють важливу роль у багатьох явищах. У галузі термоядерного синтезу вони пов'язані з такими питаннями, як стійкість профілю та утримання плазми. Нелінійним явищем, спільним як для ядерного синтезу, так і для астрофізики, є генерація зонних потоків. Ці потоки відіграють значну роль у визначенні рівня турбулентності та утриманні плазми під час синтезу. Досліджено ефекти розширення резонансу та роль нелінійностей, зокрема, у випадку нелінійної нестійкості, яка виникає в дрейфових хвилях. Обговорюються подібності та відмінності між розглянутими нами системами, при цьому з великою точністю описано процес демографічного вибуху та динаміку нелінійних систем для дрейфових хвиль із різними станами профілю стійкості. Метою даного дослідження є врахування нашої моделі для дрейфових хвиль у токамаках у ширших рамках принципів статистичної фізики. Ми сподіваємось на застосування даної моделі дрейфових хвиль у сучасних токамаках, ITER та пілотній установці термоядерного синтезу.

Ключові слова: магнетне утримання плазми, нелінійності, розширення резонансу, дрейфові хвилі, токамак, стійкість профілю.