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N.M. CHEPILKO, YU.V. BOBKOV, S.A. PONOMARENKO

National Technical University of Ukraine “Igor Sikorsky Kyiv Polytechnic Institute”,
Institute of Aerospace Technologies
(37, Beresteis’kyi Ave., Kyiv 03056, Ukraine)

ON THE EXCITATION MECHANISM OF HYBRID PLASMON-POLARITONS IN SEMICONDUCTORS

The influence of direct current on the dynamics of plasmon-polaritons in semiconductors has been analyzed. It is shown that the counter motion of macroscopic continua of electrons and holes induced by an external electromotive force leads to the appearance of unstable additional hybrid bulk and surface plasmons, as well as plasmon-polaritons, which are “genetically” related to both electrons and holes. The dispersion law and the increment and decrement of the amplitudes of the dynamic variables of additional hybrid plasmons and plasmon-polaritons are demonstrated to substantially depend on the stationary motion velocity of charged particles associated with the constant direct current. It is shown that one of the solutions of the dispersion equation for surface plasmon-polaritons corresponds to “exotic” bulk plasmon-polaritons, which propagate from the air-semiconductor interface into the depth of contacting media. The instability of additional hybrid plasmons and plasmon-polaritons can be used as the basis for a simple method of excitating additional hybrid plasmons and plasmon-polaritons in semiconductors.

Keywords: electrons, holes, electric field, polarization, electric current density, plasmon frequency, spatial dispersion, dispersion equation, instability, bulk plasmons, surface plasmons, bulk polaritons, surface polaritons, increment, decrement.

1. Introduction

Currently, the considerable attention is paid to the study of plasmon and plasmon-polariton states in semiconductors and metals [1–6]. This occurs due to their promising applications, in particular, in such domains as nanophotonics and optoelectronics, plasmon biosensors, plasmon-polariton-based lasers, high-resolution imaging and optical microscopes, and radiophysics of electromagnetic waves in the terahertz range.

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An important issue in plasmonics is the search for new excitation mechanisms of plasmons and plasmon-polaritons in semiconductors and metals. Much attention is paid to the terahertz frequency range $\omega = (10^{11} \div 1.5 \times 10^{13})$ Hz, which is adjacent to the infrared frequency range. Terahertz issues are currently actively discussed in a considerable number of specialized publications, in particular, in works [7–13]. For instance, the effect of plasmon instability, which arises in a ballistic field-effect transistor due to the counter motion of electrons as a result of their bounce from a potential barrier, was analyzed in work [7] in the hydrodynamic approximation. The cited authors concluded that such a phenomenon can be used to generate far-infrared electromagnetic radiation.

In work [8], the interaction of drifting electrons with optical phonons in semiconductors was analyzed. The cited author considered three physical systems, namely, a three-dimensional electron gas in a bulk sample, a two-dimensional electron gas in a quantum well, and a two-dimensional electron gas in a quantum well under a metal electrode. It was shown that the interaction of electrons with optical phonons can lead to the instability of electron subsystem. The author also concluded that such a phenomenon can be used as a generator or amplifier of terahertz electromagnetic radiation.

In work [9], a theoretical model of terahertz instability of optical phonons was developed, which arises due to the interaction of optical phonons with drifting solid-state plasma. This model predicts the possibility of using optical phonon instability to create active terahertz electronics devices.

In work [10], a hybrid system consisting of a dipole nanoparticle and a quantum well was considered. The cited authors showed that the electrostatic coupling between the nanoparticle and the quantum well during the drift of "two-dimensional" electrons leads to the emergence of electrical instability in the terahertz frequency range. The authors concluded that such an instability can be considered as a new mechanism for the generation of terahertz electromagnetic radiation.

In work [11], the generation of terahertz electromagnetic radiation by femtosecond lasers in the optical range was experimentally studied.

By the efforts of a large scientific team [12], the effect of terahertz electromagnetic radiation amplification was experimentally recorded, when the radiation passed through planar nanostructures based on graphene; the effect takes place due to the excitation of plasmons by an electric current. The experimental results obtained in this work open a way to the creation of terahertz radiation amplifiers based on planar plasmons.

Finally, in work [13], experimental methods for the effective generation of terahertz surface plasmon-polaritons using promising technological methods of photonic excitation of surface plasmon-polaritons were analyzed in detail.

A review of relevant publications [7–13] testifies to the challenging character of scientific research in the radiophysics of terahertz electromagnetic radiation.

The presented work is also aimed at finding new methods for generating terahertz plasmons and plasmon-polaritons using the instability of solid-state plasma. For this purpose, we will consider the effect of direct electric current on the dynamics of plasmons and plasmon-polaritons (quasiparticles), which are "genetically" related to concentration oscillations of electric charges (electrons and holes) in semiconductors. Such an influence evidently follows from the fact that the motion of a continuum of charged particles with the oscillating concentration is accompanied by at least the Doppler effect.

An analysis showed that the counter motion of macroscopic continua of electrons and holes induced by an external electromotive force that stimulates a constant electric current in a semiconductor leads to the appearance of unstable additional hybrid bulk and surface quasiparticles, which are "genetically" related to both electrons and holes.

The dispersion law and the increment and decrement of the amplitudes of the dynamic variables of additional hybrid quasiparticles (AHQPs) substantially depend on the stationary velocity of motion of charged particles in the semiconductor. The instability of AHQPs can be considered as a simple excitation mechanism of additional hybrid plasmons and plasmon-polaritons in semiconductors. Model calculations of the dispersion laws and the increment and decrement of the amplitudes of dynamic variables of AHQPs were performed in this work using germanium (Ge) and indium antimonide (InSb) as examples of semiconductors. The calculation parameters of these semiconductors were taken from the electronic reference [14] are quoted in Table.

In particular, the effective masses of electrons, m_e^*, and holes, m_h^*, in Ge are of the same order, whereas, in InSb, the effective mass of hole, m_h^*, is two orders of magnitude larger than the effective mass of electron, m_e^*. Similar relationships are also characteristic of the electron, μ_e, and hole, μ_h, mobilities in the examined semiconductors. It turned out that the AHQP frequencies in Ge and InSb belong to the terahertz frequency ranze.

Calculations showed that the parameter ratios m_e^*/m_h^* and μ_e/μ_h affect qualitatively and quantitatively the dispersion laws and the increment and decrement of the amplitudes of the dynamic variables of AHQPs in semiconductors.

2. Dielectric Permittivity of a Semiconductor Through Which a Direct Electric Current Flows

In solid-state plasmonics, an adequate model is needed for the dielectric permittivity of the medium,

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Main parameters of germanium and indium antimonide [14]

Parameters	Germanium (Ge)		Indium antimonide (InSb)	
	Electrons	Holes	Electrons	Holes
Plasma frequency (ω_p)	1.2×10^{13} 1/s	1.6×10^{13} 1/s	3.6×10^{12} 1/s	3.2×10^{11} 1/s
Mobility (μ)	$13.01\ \frac{\text{cm}^2}{\text{stat. V}\cdot\text{s}}$	$6.34\ \frac{\text{cm}^2}{\text{stat. V}\cdot\text{s}}$	$2.6\times10^{3}\ \frac{\text{cm}^2}{\text{stat. V}\cdot\text{s}}$	$2.0\ \frac{\text{cm}^2}{\text{stat. V}\cdot\text{s}}$
Critical electrostatic field (E_{kr})	$3.0\times10^{6}\ \frac{\text{stat. V}}{\text{cm}}$	$3.0\times10^{6}\ \frac{\text{stat. V}}{\text{cm}}$	$1.5\times10^{3}\ \frac{\text{stat. V}}{\text{cm}}$	$1.5\times10^{3}\ \frac{\text{stat. B}}{\text{cm}}$

$\varepsilon=\varepsilon(\omega,\mathbf{k})$, where ω is the cyclic frequency, and $\mathbf{k}$ is the wave vector of quasiparticles. To construct the dielectric constant of a semiconductor, we choose such macroscopic quantities as the electron and hole concentrations $n_{e,h}=n_{e,h}(t,\mathbf{r})$ and the velocites of motion $\mathbf{v}_{e,h}=\mathbf{v}_{e,h}(t,\mathbf{r})$ of infinitely small macroscopic volumes filled with electrons and holes, respectively, as dynamic quantities.

In turn, the dynamic quantities $n_{e,h}$ and $\mathbf{v}_{e,h}$ determine the concentrations of negative and positive electric charges

$$\rho_{fe}=-qn_{fe},\quad \rho_{fh}=qn_{fh},\quad q=-e \tag{1}$$

and the electric current densities

$$\mathbf{j}_{fe}=-qn_{fe}\mathbf{v}_{fe},\quad \mathbf{j}_{fh}=qn_{fh}\mathbf{v}_{fh}, \tag{2}$$

which satisfy the electric charge conservation laws (the laws of continuity),

$$\Big(\nabla\cdot\mathbf{j}_{fe}\Big)+\frac{\partial\rho_{fe}}{\partial t}=0,\quad \Big(\nabla\cdot\mathbf{j}_{fh}\Big)+\frac{\partial\rho_{fh}}{\partial t}=0. \tag{3}$$

Below, we will assume that the electric charge concentrations $n_{f(e,h)}$ and the electron and hole velocities $\mathbf{v}_{f(e,h)}$ have two components, namely, the stationary components $n_{0(e,h)}$ and $\mathbf{v}_{0(e,h)}$, and the components $n_{e,h}$ and $\mathbf{v}_{e,h}$ induced by the fluctuations of the electric charge concentration According to this assumption, the dynamic variables of the problem take the following representation:

$$\begin{aligned}
&n_{fe}=n_{0e}+n_e(t,\mathbf{r}),\quad \mathbf{v}_{fe}=\mathbf{v}_{0e}+\mathbf{v}_e(t,\mathbf{r}),\\
&n_{fh}=n_{0h}+n_h(t,\mathbf{r}),\quad \mathbf{v}_{fh}=\mathbf{v}_{0h}+\mathbf{v}_h(t,\mathbf{r}),\\
&n_e\ll n_{0e}=\text{const},\quad n_h\ll n_{0h}=\text{const},\\
&\mathbf{v}_e\ll \mathbf{v}_{0e}=\text{const},\quad \mathbf{v}_h\ll \mathbf{v}_{0h}=\text{const}.
\end{aligned} \tag{4}$$

As a result of formulas (2) and (4), we obtain the following expressions for the electric current densities:

$$\begin{aligned}
&\mathbf{j}_{fe}\simeq -q(n_{0e}\mathbf{v}_{0e}+n_{0e}\mathbf{v}_e+n_e\mathbf{v}_{0e}),\\
&\mathbf{j}_{fh}\simeq q(n_{0h}\mathbf{v}_{0h}+n_{0h}\mathbf{v}_h+n_h\mathbf{v}_{0h}).
\end{aligned} \tag{5}$$

The closed system of dynamic equations for the researched problem includes the laws of conservation for the electric charges, the equations of motion for the infinitesimal macroscopic volumes filled with electrons and holes, and the electrostatic Maxwell equation,

$$\begin{cases}
\Big(\nabla\cdot\mathbf{v}_e\Big)n_{0e}+\Big(\mathbf{v}_{0e}\cdot\nabla\Big)n_e+\dfrac{\partial n_e}{\partial t}=0,\\
\Big(\nabla\cdot\mathbf{v}_h\Big)n_{0h}+\Big(\mathbf{v}_{0h}\cdot\nabla\Big)n_h+\dfrac{\partial n_h}{\partial t}=0,\\
\dfrac{\partial\mathbf{v}_e}{\partial t}+\Big(\mathbf{v}_{0e}\cdot\nabla\Big)\mathbf{v}_e=-\dfrac{q}{m_e^*}\mathbf{E},\\
\dfrac{\partial\mathbf{v}_h}{\partial t}+\Big(\mathbf{v}_{0h}\cdot\nabla\Big)\mathbf{v}_h=\dfrac{q}{m_h^*}\mathbf{E},\\
\varepsilon_0\big(\nabla\cdot\mathbf{E}\big)=-4\pi q\big(n_e-n_h\big),
\end{cases} \tag{6}$$

where ε_0 is the static permittivity of semiconductor. When writing the dynamic equations for quasiparticles in Eqs. (6), it was taken into account that the radius-vector $\mathbf{r}$ in the dynamic variables is connected with the center of mass of an infinitely small macroscopic volume filled with electric charges. Therefore, when writing down the accelerations in Eqs. (6), the substantial (material) Lagrange derivative was used, by analogy with the hydrodynamic approximation in the plasma theory [15]. The stationary velocities of electrons and holes are determined by the electrostatic field strength $\mathbf{E}_0$ (induced by an external electromotive force). Hence, $\mathbf{v}_{0(e,h)}=\mu_{e,h}\mathbf{E}_0$.

We seek solutions to the system of equations (6) in the form of monochromatic plane waves,

$$n_e, n_h, \mathbf{v}_e, \mathbf{v}_h, \mathbf{E} \sim e^{i(\omega t-(\mathbf{k}\cdot\mathbf{r}))}. \tag{7}$$

In this case, instead of the system of partial differential equations (6), we obtain a system of linear algebraic equations,

$$\begin{cases} \big(\omega-(\mathbf{k}\cdot\mathbf{v}_{0e})\big)n_e-(\mathbf{k}\cdot\mathbf{v}_e)n_{0e}=0, \\ \big(\omega-(\mathbf{k}\cdot\mathbf{v}_{0h})\big)n_h-(\mathbf{k}\cdot\mathbf{v}_h)n_{0h}=0, \\ i\Big(\omega-(\mathbf{k}\cdot\mathbf{v}_{0e})\Big)(\mathbf{k}\cdot\mathbf{v}_e)=-\dfrac{q}{m_e^*}\big(\mathbf{k}\cdot\mathbf{E}\big), \\ i\Big(\omega-(\mathbf{k}\cdot\mathbf{v}_{0h})\Big)(\mathbf{k}\cdot\mathbf{v}_h)=\dfrac{q}{m_h^*}\big(\mathbf{k}\cdot\mathbf{E}\big), \\ i\varepsilon_0\big(\mathbf{k}\cdot\mathbf{E}\big)=4\pi q\big(n_e-n_h\big). \end{cases} \tag{8}$$

Whence we find explicit expressions for the following dynamic variables of the problem:

$$\begin{cases} n_e=\dfrac{iqn_{0e}\big(\mathbf{k}\cdot\mathbf{E}\big)}{m_e^*(\omega-(\mathbf{k}\cdot\mathbf{v}_{0e}))^2}\,e^{i(\omega t-(\mathbf{k}\cdot\mathbf{r}))}, \\ n_h=-\dfrac{iqn_{0h}\big(\mathbf{k}\cdot\mathbf{E}\big)}{m_h^*(\omega-(\mathbf{k}\cdot\mathbf{v}_{0h}))^2}\,e^{i(\omega t-(\mathbf{k}\cdot\mathbf{r}))}. \end{cases} \tag{9}$$

Substituting them into the electrostatic Maxwell equation in Eqs. (8) reduces the latter to the expression

$$\left(1-\frac{{\omega_{pe}}^2}{(\omega-(\mathbf{k}\cdot\mathbf{v}_{0e}))^2}-\right.$$
$$\left.-\frac{\omega_{ph}^2}{(\omega-(\mathbf{k}\cdot\mathbf{v}_{0h}))^2}\right)\big(\mathbf{k}\cdot\mathbf{E}\big)=0, \tag{10}$$

where

$$\omega_{p(e,h)}^2=4\pi\frac{n_{0(e,h)}q^2}{\varepsilon_0 {m^*}_{e,h}}$$

are the squared plasmon frequencies of electrons and holes.

In order to clarify the physical meaning of this expression, let us write down the Maxwell wave equation for the electric field vector

$$\mathbf{E}=\mathbf{E}_1 e^{i(\omega t-(\mathbf{k}\cdot\mathbf{r}))}. \tag{11}$$

In this case, the wave equation takes the form

$$\mathbf{k}^2\mathbf{E}-\mathbf{k}\big(\mathbf{k}\cdot\mathbf{E}\big)-\frac{\omega^2}{c^2}\varepsilon\big(\omega,\mathbf{k}\big)\mathbf{E}=0. \tag{12}$$

By calculating the scalar and vector products of this equation with the vector $\mathbf{k}$, it can be decomposed into two independent equations for the longitudinal and transverse electric fields,

$$\begin{aligned} &\varepsilon\big(\omega,\mathbf{k}\big)\big(\mathbf{k}\cdot\mathbf{E}\big)=0, \\ &\Big(\mathbf{k}^2-\frac{\omega^2}{c^2}\varepsilon\big(\omega,\mathbf{k}\big)\Big)\big[\mathbf{k}\times\mathbf{E}\big]=0. \end{aligned} \tag{13}$$

Next, by comparing the equation for the longitudinal electric field in Eq. (13) with expression (10), we find an explicit expression for the dielectric constant of a semiconductor where a constant electric current flows,

$$\varepsilon\big(\omega,\mathbf{k}\big)=\left(1-\frac{{\omega_{pe}}^2}{(\omega-(\mathbf{k}\cdot\mathbf{v}_{0e}))^2}-\right.$$
$$\left.-\frac{{\omega_{ph}}^2}{(\omega-(\mathbf{k}\cdot\mathbf{v}_{0h}))^2}\right). \tag{14}$$

Note that expression (14) found for the dielectric permittivity is to some extent similar to the expression previously obtained in the theory of two-beam instability (see, for example, work [16]), which arises in the case where an electron beam passes through a stationary "cold" plasma.

3. Bulk and Surface Plasmons in a Semiconductor through Which a Direct Electric Current Flows

In solid-state plasmonics, plasmons are "genetically" related to longitudinal fluctuations of the electric charge concentration [1]. Therefore, in this case, the electric field of bulk plasmons will be determined by the electrostatic Maxwell equation in Eqs. (13). The condition $\varepsilon\big(\omega,\mathbf{k}\big)=0$ for a nontrivial solution of this equation is the dispersion equation for bulk plasmons in the following form:

$$1-\frac{{\omega_{pe}}^2}{\big(\omega-(\mathbf{k}\cdot\mathbf{v}_{0e})\big)^2}-\frac{{\omega_{\mathrm{ph}}}^2}{\big(\omega-(\mathbf{k}\cdot\mathbf{v}_{0h})\big)^2}=0. \tag{15}$$

Equation (15) can be easily transformed into the fourth-order polynomial equation

$$\begin{aligned} &\big(\omega-(\mathbf{k}\cdot\mathbf{v}_{0e})\big)^2\big(\omega-(\mathbf{k}\cdot\mathbf{v}_{0h})\big)^2- \\ &-{\omega_{pe}}^2\big(\omega-(\mathbf{k}\cdot\mathbf{v}_{0h})\big)^2- \\ &-\omega_{ph}^2\big(\omega-(\mathbf{k}\cdot\mathbf{v}_{0e})\big)^2=0, \end{aligned} \tag{16}$$

which has four solutions in the general case.

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For numerically analyzing the solutions of Eq. (16), let us rewrite it in terms of dimensionless variables,

$$\bar{\omega}_1^2\bar{\omega}_2^2 - \left(\bar{\omega}_2^2 + a^2\bar{\omega}_1^2\right) = 0, \tag{17}$$

where

$$\bar{\omega}_1 = \bar{\omega} - \bar{\omega}_\kappa, \quad \bar{\omega}_2 = \bar{\omega} + b\bar{\omega}_\kappa,$$

$$\bar{\omega} = \frac{\omega}{\omega_{pe}}, \quad a = \frac{\omega_{ph}}{\omega_{pe}}, \quad b = \frac{|\mathbf{v}_{0h}|}{|\mathbf{v}_{0e}|},$$

$$\bar{\omega}_\kappa = (\mathbf{u}_0 \cdot \boldsymbol{\kappa}), \quad \mathbf{u}_0 = \frac{\mathbf{v}_{0e}}{c}, \quad \boldsymbol{\kappa} = \frac{c\mathbf{k}}{\omega_{pe}},$$

$$\mathbf{v}_{0e} = \mu_{0e}\mathbf{E}_0, \quad \mathbf{v}_{0h} = -\mu_{0h}\mathbf{E}_0.$$

In the case $\mathbf{E}_0 = 0$, dispersion equation (17) simplifies to

$$\bar{\omega}^2\left[\bar{\omega}^2 - (1 + a^2)\right] = 0, \tag{18}$$

and its nontrivial solutions look like

$$\bar{\omega}_{1,2} = \pm\sqrt{1 + a^2}. \tag{19}$$

The general numerical solutions of Eq. (17) are plotted in Figs. 1 and 2.

It is obvious that the dispersion branches $\omega_{1,2}$ in Figs. 1 and 2 agree with solutions (19) of Eq. (18). At the same time, the dispersion branches $\omega_{3,4}$ correspond to the complex conjugate solutions of Eq. (17), which testifies to the instability of the corresponding additional AHQPs in the semiconductor. Moreover, the amplitude will increase for the oscillations of the dynamic variables of the DGKNs with the cyclic frequency ω_3 and decrease for those with the frequency ω_4. The increments and decrements of the DGKNs $\gamma_{3,4}$ will be determined by the expressions $\gamma_{3,4} = \mathrm{Im}(\omega_{3,4})$.

This interpretation of the obtained solutions of Eq. (17) seems reasonable. The dispersion branches $\omega_{1,2}$ correspond to plasmons that are "genetically" related to electrons and holes, respectively. As for the dispersion branches $\omega_{3,4}$, they correspond to the AHQPs that are genetically related to both electrons and holes.

A comparative analysis of the dispersion branches of the AHQPs in germanium and indium antimonide shows that, as the difference between the plasmon frequencies $\omega_{p(e,h)}$ of electrons and holes increases, the hybridization effect of concentration fluctuations

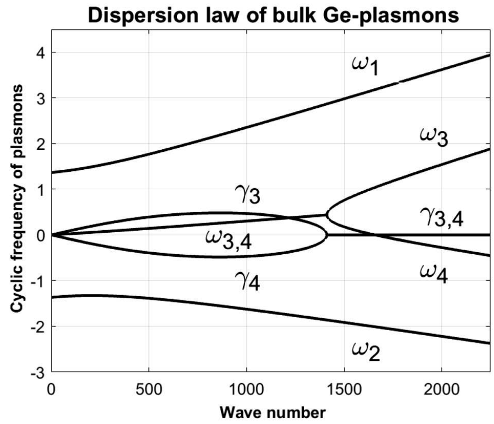

Fig. 1. Dispersion branches of bulk plasmons in Ge calculated for the dimensionless variables $\bar{\omega}$, $a = 0.866$, $b = 0.487$, $u_0 = 1.3 \times 10^{-3}$

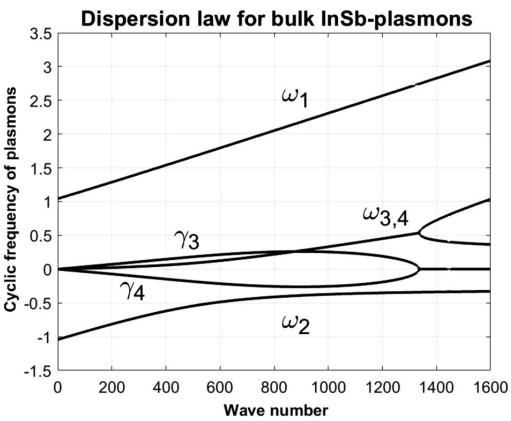

Fig. 2. Dispersion branches of bulk plasmons in InSb calculated for the dimensionless variables $\bar{\omega}$, $a = 0.089$, $b = 0.769 \times 10^{-3}$, $u_0 = 1.3 \times 10^{-4}$

of opposite electric charges decreases. As a result, the values of the cyclic frequencies $\omega_{3,4}$ become closer.

Let us pay attention to the fact that the increments and decrements of the AHQP $\gamma_{3,4}$ differ from zero in a confined interval of wave numbers κ. Let us also note that the short-wave limit for the values of $\gamma_{3,4}$ corresponds to extremely large values of the wave vector $\mathbf{k}$, which may be unattainable in experimental studies of the physical properties of the AHQPs.

From a practical point of view, the greatest interest is attracted by the AHQPs with the cyclic frequency ω_3. In the long-wave spectral part, the amplitudes of their dynamic variables are growing.

As for surface plasmons, hereafter, when finding their dispersion law, we assume that the vector of

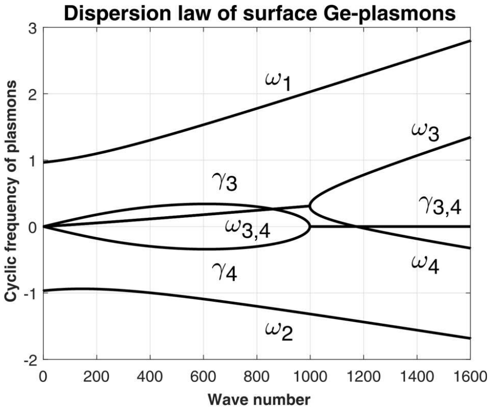

Fig. 3. Dispersion branches of surface plasmons in Ge calculated for the dimensionless variables $\bar{\omega}$, $a = 0.866$, $b = 0.487$, $u_0 = 1.3 \times 10^{-3}$

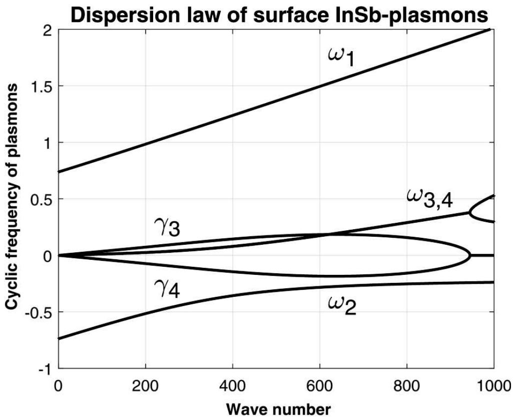

Fig. 4. Dispersion branches of surface plasmons in InSb calculated for the dimensionless variables $\bar{\omega}$, $a = 0.089$, $b = 0.769 \times 10^{-3}$, $u_0 = 1.3 \times 10^{-4}$

the electric current density is parallel to the air-semiconductor interface $z = 0$ and introduce into consideration the electric field induction $\mathbf{D}$ in a vicinity of this interface in the following form:

$$\mathbf{D} = \begin{cases} \epsilon\, \mathbf{k}_\alpha A_\alpha e^{i(\omega t - \mathbf{k}_\alpha \mathbf{r})}, & z \geqslant 0, \\ \varepsilon(\omega, \mathbf{k}) \mathbf{k}_\beta A_\beta e^{i(\omega t - \mathbf{k}_\beta \mathbf{r})}, & z \leqslant 0, \end{cases} \quad (20)$$

$$\mathbf{k}_\alpha = (\mathbf{k}, -i\alpha), \quad \mathbf{k}_\beta = (\mathbf{k}, i\beta), \quad \mathbf{k} = (k_x, k_y),$$

where the interval $z \geqslant 0$ is filled with air, and the interval $z \leqslant 0$ with a semiconductor. The parameter ϵ has the meaning of the air dielectric constant. As for the parameters α and β, they are assumed hereafter to be responsible for the region of plasmon localization near the interface $z = 0$ between two media.

The vector $\mathbf{D}$ satisfies the electrostatic Maxwell equation

$$(\nabla \cdot \mathbf{D}) = 0. \quad (21)$$

Whence we find the plasmon localization parameters α and β in a vicinity of the interface between two media: $\alpha = |\mathbf{k}|$ and $\beta = |\mathbf{k}|$. From the continuity conditions for the tangential components of the electric field and the normal components of the electric field induction across the interface $z = 0$ between two media, we find the dispersion equation for surface plasmons,

$$\epsilon + \varepsilon(\omega, \mathbf{k}) = 0, \quad \epsilon = 1. \quad (22)$$

Obviously, the construction of dispersion equation (22) is the same as in the case of dispersion equation for bulk plasmons, namely, $\varepsilon(\omega, \mathbf{k}) = 0$. Therefore, the solutions of Eq. (22) are similar to those of Eq. (15).

The general solutions of Eq. (22) reduced to dimensionless variables are plotted in Figs. 3 and 4. From a comparison of the plots in Figs. 3 and 4, on the one hand, and Figs. 1 and 2, on the other hand, we may conclude that the dynamic properties of surface plasmons localized in a vicinity of the interface $z = 0$ between two media are similar to those of bulk plasmons in a semiconductor.

Note that the amplitudes of the dynamic variables of surface plasmons with the cyclic frequency ω_3 grow exponentially as the plasmons propagate along the interface between two media.

Expression (14) for the dielectric permittivity can be used to analyze plasmon states in metals. For this purpose, it is necessary to zero the plasmon frequency of holes, $\omega_{\mathrm{ph}} = 0$. As a result, we obtain the following expression for the dielectric permittivity of a metal sample through which a constant electric current flows:

$$\varepsilon(\omega, \mathbf{k}) = 1 - \frac{{\omega_{pe}}^2}{\big(\omega - (\mathbf{k} \cdot \mathbf{v}_{0e})\big)^2}. \quad (23)$$

In this case, the solutions of the dispersion equation $\varepsilon(\omega, \mathbf{k}) = 0$ for bulk metalloplasmons look like

$$\omega_{1,2} = \pm\omega_{pe} + (\mathbf{k} \cdot \mathbf{v}_{0e}). \quad (24)$$

As for surface metalloplasmons, the solutions of their dispersion equation $1 + \varepsilon(\omega, \mathbf{k}) = 0$ take, in this case, the following form:

$$\omega_{1,2} = \pm\frac{\omega_{pe}}{\sqrt{2}} + (\mathbf{k}\cdot\mathbf{v}_{0e}). \quad (25)$$

From Eqs. (23) and (25), we obtain that a direct electric current flow through a metal sample provides both bulk and surface metalloplasmons with spatial dispersion.

4. Bulk and Surface Plasmon-Polaritons in a Semiconductor through which a Direct Electric Current Flows

The dispersion equation for plasmon-polaritons, with regard for the expression for the dielectric permittivity (14), has the following form

$$\mathbf{k}^2 = \frac{\omega^2}{c^2}\left(1 - \frac{{\omega_{pe}}^2}{(\omega-(\mathbf{k}\cdot\mathbf{v}_{0e}))^2} - \frac{{\omega_{ph}}^2}{(\omega-(\mathbf{k}\cdot\mathbf{v}_{0h}))^2}\right). \quad (26)$$

This equation, after its reduction to dimensionless variables, can be easily transformed to the sixth-order polynomial equation

$$(\boldsymbol{\kappa}^2 - \bar{\omega}^2)\bar{\omega}_1^2\bar{\omega}_2^2 + \bar{\omega}^2\left(\bar{\omega}_2^2 + a^2\bar{\omega}_1^2\right) = 0 \quad (27)$$

for the frequency $\bar{\omega}$, where

$$\bar{\omega}_1 = \bar{\omega} - (\boldsymbol{\kappa}\cdot\mathbf{u}_0), \quad \bar{\omega}_2 = \bar{\omega} + b(\boldsymbol{\kappa}\cdot\mathbf{u}_0).$$

This equation has six solutions, but no analytic one. Therefore, when numerically determining solutions of Eq. (27), we confine the calculations to the practically significant long-wave interval of the wave number κ.

In the absence of a constant electric current, Eq. (27) acquires a simple form

$$\boldsymbol{\kappa}^2 - \bar{\omega}^2 + (1 + a^2), \quad (28)$$

which has the following solutions:

$$\bar{\omega}_{1,2} = \pm\sqrt{(1+a^2) + \boldsymbol{\kappa}^2}. \quad (29)$$

The general numerical solutions of Eq. (27) for Ge are plotted in Figs. 5 and 6.

Let us attract attention to the fact that the spectral intervals corresponding to the AHQP frequencies $\omega_{3,4}$ and $\omega_{5,6}$ are three orders of magnitude lower than the spectral interval corresponding to the main bulk plasmon frequencies $\omega_{1,2}$. It is obvious that the plasmon-polaritons with the frequencies $\omega_{1,2}$ agree with solutions (29) of Eq. (28).

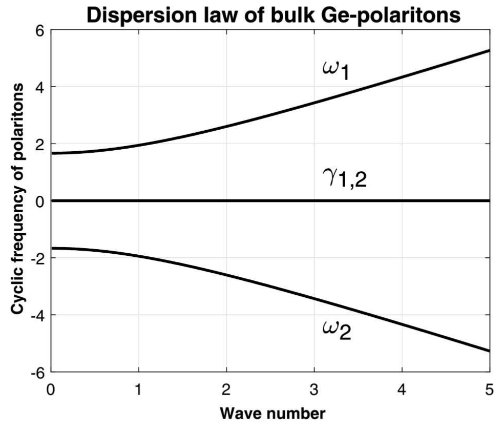

Fig. 5. Dispersion branches of main bulk plasmon-polaritons in Ge calculated for the dimensionless variables $\bar{\omega}$, $a = 0.866$, $b = 0.487$, $u_0 = 1.3 \times 10^{-3}$

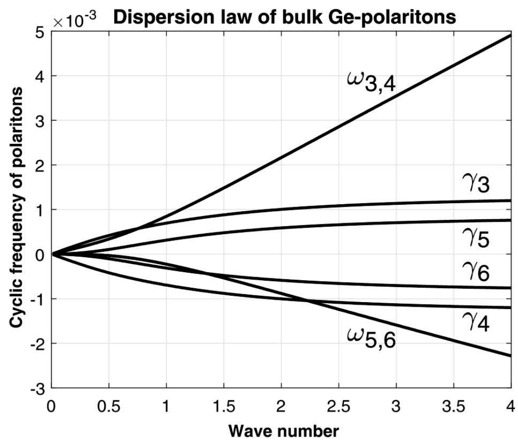

Fig. 6. Dispersion branches of additional bulk plasmon-polaritons in Ge calculated for the dimensionless variables $\bar{\omega}$, $a = 0.866$, $b = 0.487$, $u_0 = 1.3 \times 10^{-3}$

In the case of InSb, the situation is more complicated. The corresponding general numerical solutions of Eq. (27) are plotted in Figs. 7, 8, and 9. From these figures, we find that the spectral intervals of the bulk-plasmon frequencies $\omega_{1,2}$, $\omega_{3,4}$, and $\omega_{5,6}$ are significantly spaced apart. For instance, the spectral interval of $\omega_{3,4}$ has frequencies three orders of magnitude lower than the frequencies of the spectral interval with $\omega_{1,2}$, and the frequencies of the spectral interval with $\omega_{5,6}$ are four orders of magnitude lower

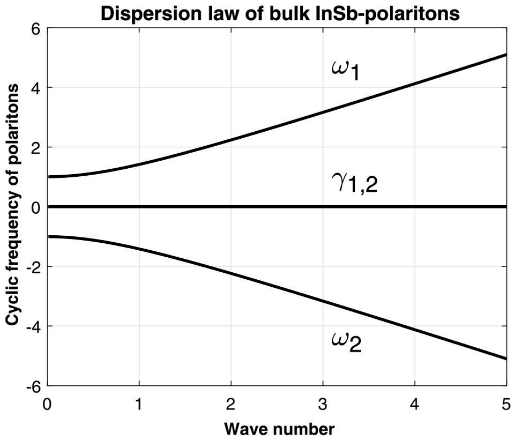

Fig. 7. Dispersion branches of main bulk plasmon-polaritons in InSb calculated for the dimensionless variables $\bar{\omega}$, $a = 0.089$, $b = 0.769 \times 10^{-3}$, $u_0 = 1.3 \times 10^{-4}$

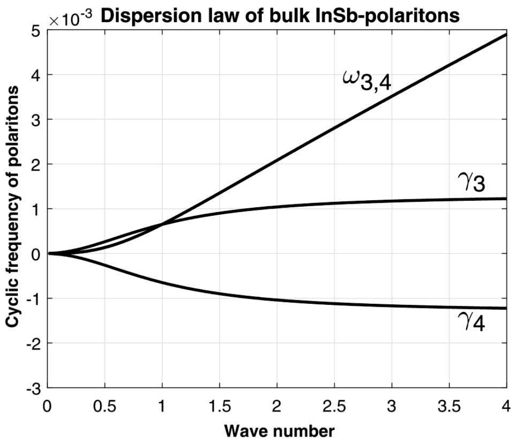

Fig. 8. Dispersion branches of additional bulk plasmon-polaritons in InSb calculated for the dimensionless variables $\bar{\omega}$, $a = 0.089$, $b = 0.769 \times 10^{-3}$, $u_0 = 1.3 \times 10^{-4}$

than those of the spectral interval with $\omega_{3,4}$. It is obvious that plasmon-polaritons with the frequency $\omega_{1,2}$ agree with solutions (29) of Eq. (28).

To summarize, we can point out that the number of unstable types of polariton AHQPs in semiconductors increases twice as much as the number of unstable types of plasmon AHQPs excited by a direct electric current. It is obvious that, in the cases of both plasmon and polariton AHQPs, these are polariton AHQPs with increasing amplitudes of dynamic variables that are of practical importance; the corresponding increments of growth equal $\gamma_{3,5} = \mathrm{Im}(\omega_{3,5})$.

Now, let us consider surface plasmon-polaritons in a vicinity of a flat air-semiconductor interface at

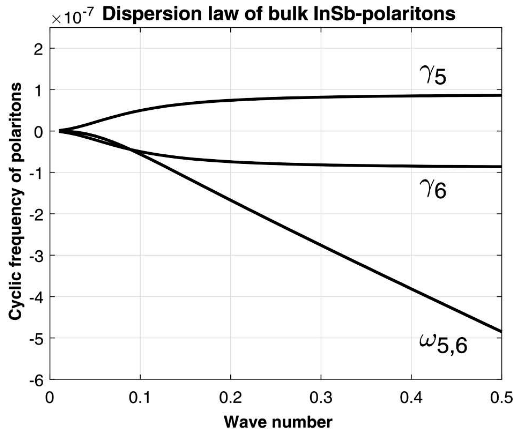

Fig. 9. Dispersion branches of additional bulk plasmon-polaritons in InSb calculated for the dimensionless variables $\bar{\omega}$, $a = 0.089$, $b = 0.769 \times 10^{-3}$, $u_0 = 1.3 \times 10^{-4}$

$z = 0$. For the intensities of the electric, $\mathbf{E}$, and magnetic, $\mathbf{H}$, fields of plasmon-polaritons, we choose the following ansatzes:

$$\mathbf{E} = \begin{cases} \mathbf{A}_\alpha e^{i(\omega t - \mathbf{k}_\alpha \mathbf{r})}, & z \geq 0, \\ \mathbf{A}_\beta e^{i(\omega t - \mathbf{k}_\beta \mathbf{r})}, & z \leq 0, \end{cases}$$
$$\mathbf{H} = \begin{cases} -\frac{1}{q}\left[\mathbf{k}_\alpha \times \mathbf{A}_\alpha\right] e^{i(\omega t - \mathbf{k}_\alpha \mathbf{r})}, & z \geq 0, \\ -\frac{1}{q}\left[\mathbf{k}_\beta \times \mathbf{A}_\beta\right] e^{i(\omega t - \mathbf{k}_\beta \mathbf{r})}, & z \leq 0, \end{cases} \quad (30)$$

where

$$(\mathbf{k}_\alpha \cdot \mathbf{A}_\alpha) = 0, \quad (\mathbf{k}_\beta \cdot \mathbf{A}_\beta) = 0, \quad (\mathbf{E} \cdot \mathbf{H}) = 0,$$
$$\mathbf{k}_\alpha = (\mathbf{k}, -i\alpha), \quad \mathbf{k}_\beta = (\mathbf{k}, i\beta), \quad q = \frac{\omega}{c}, \quad (31)$$
$$\alpha^2 = \mathbf{k}^2 - q^2\epsilon, \quad \beta^2 = \mathbf{k}^2 - q^2\varepsilon(\omega, \mathbf{k}), \quad \epsilon = 1,$$

which are consistent with the Maxwell wave equation. As in the previous section, the parameters α and β determine the localization region of plasmon-polaritons in a vicinity of the interface $z = 0$ between the two media.

From the continuity conditions for the tangential components of the electric and magnetic field vectors of plasmon-polaritons [17, 18]

$$\mathbf{E} = (E_x, E_y, E_z), \quad \mathbf{H} = (H_x, H_y, 0). \quad (32)$$

across the interface $z = 0$ between two media, we find the following condition for a nontrivial relation between the vectors $\mathbf{A}_\alpha$ and $\mathbf{A}_\beta$:

$$\left(\frac{\alpha}{\epsilon}\right)^2 - \left(\frac{\beta}{\varepsilon}\right)^2 = 0. \quad (33)$$

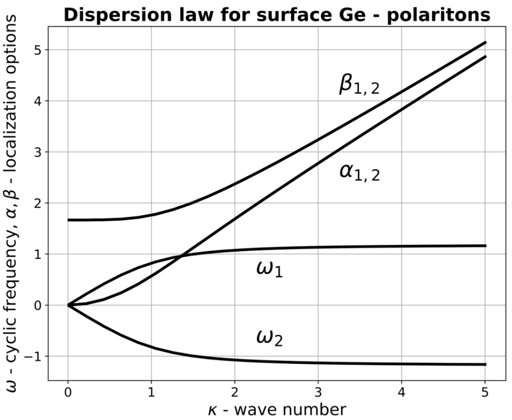

Fig. 10. Dispersion branches and localization parameters of main surface plasmon-polaritons in Ge calculated for the dimensionless variables $\bar{\omega}$, $a = 0.866$, $b = 0.487$, $u_0 = 1.3 \times 10^{-3}$

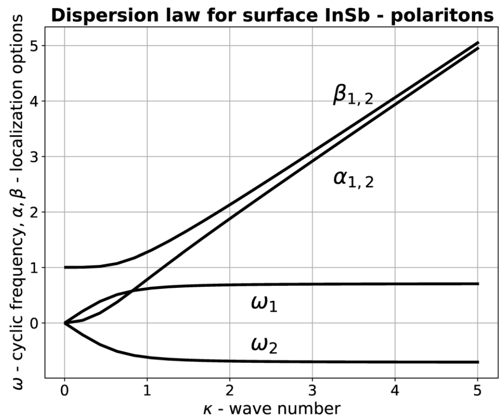

Fig. 11. Dispersion branches and localization parameters of main surface plasmon-polaritons in InSb calculated for the dimensionless variables $\bar{\omega}$, $a = 0.089$, $b = 0.769 \times 10^{-3}$, $u_0 = 1.3 \times 10^{-4}$

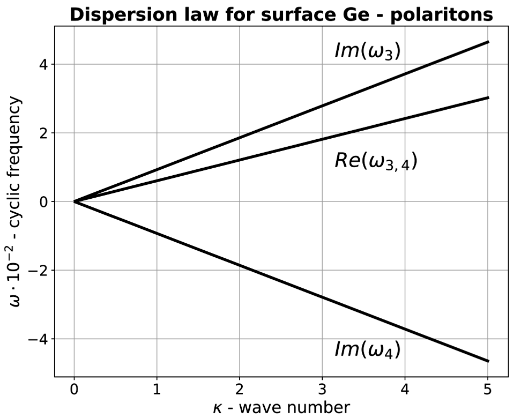

Fig. 12. Dispersion branches of additional hybrid surface plasmon-polaritons in Ge calculated for the dimensionless variables $a = 0.866$, $b = 0.487$, $u_0 = 1.3 \times 10^{-3}$

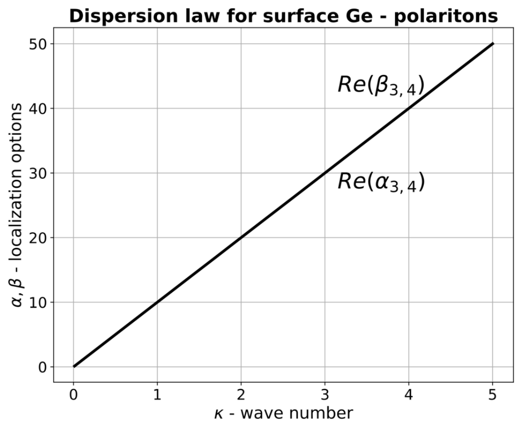

Fig. 13. Localization parameters of additional hybrid surface plasmon-polaritons in Ge calculated for the dimensionless variables $\bar{\omega}$, $a = 0.866$, $b = 0.487$, $u_0 = 1.3 \times 10^{-3}$

Algebraic transformations of expression (33) bring it to the following form:

$$\mathbf{k}^2 = q^2 \frac{\epsilon \cdot \varepsilon(\omega, \mathbf{k})}{\epsilon + \varepsilon(\omega, \mathbf{k})}. \tag{34}$$

Expression (34) has the meaning of the dispersion equation for surface plasmon-polaritons in a semiconductor through which a direct electric current flows. It is essential that surface plasmon-polaritons in this problem possess the spatial dispersion.

The localization parameters α and β of plasmon-polaritons in a vicinity of the interface $z = 0$ between two media can be reduced to the following forms with the help of Eq. (34):

$$\alpha^2 = -q^2 \frac{\epsilon^2}{\epsilon + \varepsilon(\omega, \mathbf{k})}, \quad \beta^2 = -q^2 \frac{\varepsilon(\omega, \mathbf{k})^2}{\epsilon + \varepsilon(\omega, \mathbf{k})}. \tag{35}$$

After the substitution of dimensionless variables, this equation can be easily transformed into the sixth-order polynomial equation for the frequency $\bar{\omega}$,

$$(\bar{\kappa}^2 \epsilon + \bar{\alpha}^2)\bar{\omega}_1^2 \bar{\omega}_2^2 - \bar{\alpha}^2(\bar{\omega}_2^2 + a^2 \bar{\omega}_1^2) = 0, \tag{36}$$

where

$\bar{\alpha}^2 = \bar{\kappa}^2 - \bar{\omega}^2 \epsilon,$

$\bar{\omega}_1 = \bar{\omega} - (\boldsymbol{\kappa} \cdot \mathbf{u}_0), \quad \bar{\omega}_2 = \bar{\omega} + b(\boldsymbol{\kappa} \cdot \mathbf{u}_0),$

which has six solutions. There are no analytic solutions to Eq. (35). Therefore, as was done above, when

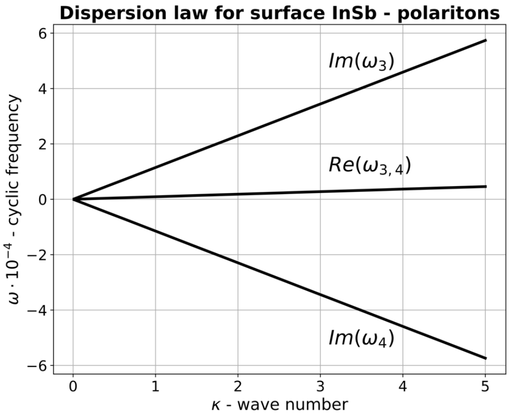

Fig. 14. Dispersion branches of additional hybrid surface plasmon-polaritons in InSb calculated for the dimensionless variables $\bar{\omega}$, $a = 0.089$, $b = 0.769 \times 10^{-3}$, $u_0 = 1.3 \times 10^{-4}$

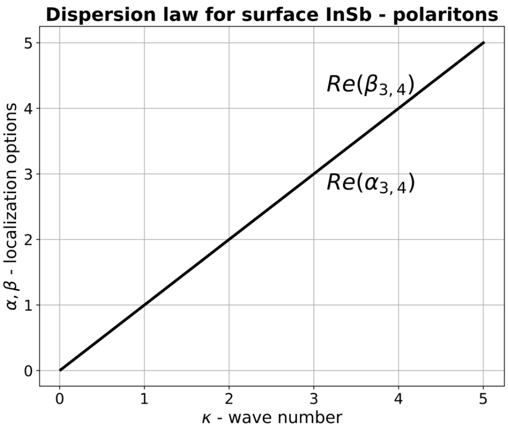

Fig. 15. Localization parameters of additional hybrid surface plasmon-polaritons in InSb calculated for the dimensionless variables $\bar{\omega}$, $a = 0.089$, $b = 0.769 \times 10^{-3}$, $u_0 = 1.3 \times 10^{-4}$

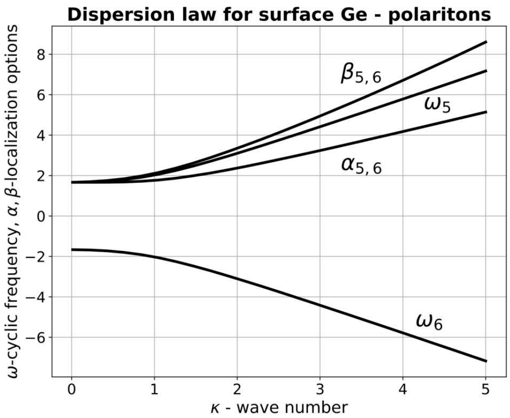

Fig. 16. Dispersion branches of "exotic" plasmon-polaritons in Ge calculated for the dimensionless variables $\bar{\omega}$, $a = 0.866$, $b = 0.487$, $u_0 = 1.3 \times 10^{-3}$

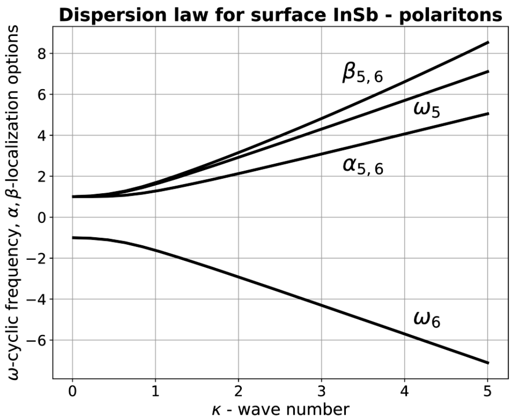

Fig. 17. Dispersion branches of "exotic" plasmon-polaritons in InSb calculated for the dimensionless variables $\bar{\omega}$, $a = 0.089$, $b = 0.769 \times 10^{-3}$, $u_0 = 1.3 \times 10^{-4}$

numerically calculating solutions of Eq. (35), with Ge and InSb semiconductors taken as examples, we confine the consideration to a practically significant long-wave interval of wave numbers κ. In Figs. 10 and 11, the dispersion branches $\omega_{1,2} = \omega_{1,2}(\boldsymbol{\kappa})$ of the main surface plasmon-polaritons and the dependences of their localization parameters $\alpha_{1,2} = \alpha_{1,2}(\boldsymbol{\kappa})$ and $\beta_{1,2} = \beta_{1,2}(\boldsymbol{\kappa})$ on the wave number κ are depicted. At $\mathbf{u}_{0(e,h)} = 0$, these plasmon-polaritons are reduced to the known surface plasmon-polaritons [1, 17, 18].

Another matter is additional surface plasmon-polaritons. The dispersion laws $\omega_{3,4} = \omega_{3,4}(\boldsymbol{\kappa})$ and the dependences of localization parameters $\alpha_{3,4} = \alpha_{3,4}(\boldsymbol{\kappa})$ and $\beta_{3,4} = \beta_{3,4}(\boldsymbol{\kappa})$ on the wave number κ for them are shown in Figs. 12 to 15. These surface plasmon-polaritons are unstable. In particular, the amplitudes of dynamic variables of surface plasmon-polaritons with $\omega_3 = \omega_3(\boldsymbol{\kappa})$ grow exponentially with the increment of growth $\gamma_3 = \mathrm{Im}(\omega_3)$, and those with $\omega_4 = \omega_4(\boldsymbol{\kappa})$ decrease exponentially with the decrement $\gamma_4 = \mathrm{Im}(\omega_4)$; see Figs. 12 and 14. Note that the localization parameters $\alpha_{3,4} = \alpha_{3,4}(\boldsymbol{\kappa})$ and $\beta_{3,4} = \beta_{3,4}(\boldsymbol{\kappa})$, are, generally speaking, complex quantities. However, their imaginary parts are many orders of magnitude smaller than the real ones. In particular, $|\mathrm{Im}(\alpha_{3,4})| \sim \mathrm{Re}(\alpha_{3,4}) \times 10^{-7}$ and $\mathrm{Im}(\beta_{3,4})| \sim \mathrm{Re}(\beta_{3,4}) \times 10^{-13}$ for Ge, and $|\mathrm{Im}(\alpha_{3,4})| \sim$ $\sim \mathrm{Re}(\alpha)_{3,4} \times 10^{-11}$ and $|\mathrm{Im}(\beta_{3,4})| \sim \mathrm{Re}(\beta_{3,4}) \times 10^{-19}$ for InSb. The imaginary parts of the localization parameters lead to oscillations of the dynamic parameters of surface plasmon-polaritons along the Z-axis, as they move away from the interface $z = 0$ between the two media.

Surface plasmon polaritons are "genetically" related to surface plasmons. But the dispersion equation for surface plasmons (22) has four solutions, whereas the dispersion equation for surface plasmon-polaritons (34) has six solutions. This fact points to the existence, in this case, of "exotic" plasmon-polariton states in the semiconductors, which correspond to the fifth and sixth solutions of dispersion equation (35).

It is essential that the parameters $\alpha_{5,6}$ and $\beta_{5,6}$ of these plasmon-polaritons are purely imaginary, i.e., $\mathrm{Re}(\alpha_{5,6}) = 0$ and $\mathrm{Re}(\beta_{5,6}) = 0$. The signs of the parameters $\alpha_{5,6}$ and $\beta_{5,6}$ are such that they define bulk plasmon-polaritons that propagate from the interface into the depth of contacting media. The dispersion branches $\omega_{5,6} = \omega_{5,6}(\boldsymbol{\kappa})$ and the parameters $\alpha_{5,6}$ and $\beta_{5,6}$ of "exotic" plasmon-polaritons are shown in Figs. 16 and 17.

5. Summary

By analyzing the influence of direct electric current on the dynamics of plasmons and plasmon-polaritons (quasiparticles) in semiconductors, it has been found that the counter motion of macroscopic continua of electrons and holes induced by an external electromotive force gives rise to the appearance, in addition to the main bulk and surface quasiparticles, of unstable bulk and surface AHQPs, which are "genetically" related to both electrons and holes and emerge in pairs. The dynamic variables of one of those AHQPs grow exponentially in time, and the dynamic variables of the other AHQP decrease exponentially. The phenomenon of AHQP instability takes place within a limited interval of quasiparticle wavenumber values.

The effect of quasiparticle hybridization in a semiconductor is substantially affected by the difference between the plasmon frequencies of electrons and holes. As this difference increases, the degree of quasiparticle hybridization decreases.

It is shown that one of six solutions of the dispersion equation for surface AHQPs corresponds to "exotic" bulk plasmon-polaritons, which propagate from the air-semiconductor interface into the depth of the contacting media.

By analyzing the solutions of the dispersion equations (Figs. 1 to 17), it iss found that the AHQP frequencies in semiconductors such as Ge and InSb belong to the terahertz frequency range $\omega = (10^{11} \div 1.5 \times 10^{13})$ Hz.

Attention should be paid to the fact that unstable plasmons and plasmon-polaritons have certain distribution diagrams, which affect their amplitude and frequency characteristics. The latter, in turn, are determined by such factors as the product $(\mathbf{k} \cdot \mathbf{v}_{0(e,h)})$ in formula (14) for the dielectric permittivity of a semiconductor through which a direct electric current flows.

The appearance of unstable AHQPs in a semiconductor through which a direct electric current flows can be used as a basis for a simple excitation method of both bulk and surface AHQPs in the terahertz frequency interval.

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М.М. Чепілко, Ю.В. Бобков, С.О. Пономаренко

ПРО МЕХАНІЗМ ЗБУДЖЕННЯ ГІБРИДНИХ ПЛАЗМОН-ПОЛЯРИТОНІВ У НАПІВПРОВІДНИКАХ

Проаналізовано вплив постійного електричного струму на динаміку плазмон-поляритонів у напівпровідниках. Показано, що зустрічний рух макроскопічних континіумів електронів і дірок, зумовлений зовнішньою електрорушійною силою, приводить до появи нестійких додаткових гібридних об'ємних та поверхневих плазмонів та плазмон-поляритонів, які "генетично" зв'язані як з електронами, так і з дірками. Закон дисперсії та інкремент (декремент) зростання (спадання) амплітуд динамічних змінних додаткових гібридних плазмонів та плазмон-поляритонів суттєво залежать від стаціонарної швидкості руху заряджених частинок, зумовленої постійним електричним струмом. Показано, що одному із розв'язків дисперсійного рівняння поверхневих плазмон-поляритонів відповідають "екзотичні" об'ємні плазмон-поляритони, які розповсюджуються від межі розподілу "повітря–напівпровідник" в глибину контактуючих середовищ. Нестійкість додаткових гібридних плазмонів та плазмон-поляритонів можна покласти в основу простого методу збудження додаткових гібридних плазмонів та плазмон-поляритонів у напівпровідниках.

Ключові слова: електрони, дірки, електричне поле, поляризація, густина електричного струму, плазмонна частота, просторова дисперсія, дисперсійне рівняння, нестійкість, об'ємні плазмони, поверхневі плазмони, об'ємні поляритони, поверхневі поляритони, інкремент зростання, декремент спадання.