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Project of program package for exploring of cosmological fractals

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The value of fractal dimension and inhomogeneity scale of the spatial distribution of galaxies is a widely discussed problem in the modern cosmology. To solve it one should carefully test the reliability of methods of determination of the values of these physical quantities. The project of a program package to explore fractal properties of isolated points (galaxies) distribution is presented. The package includes tools to load catalogues of galaxies as well as to simulate fractal and uniform distributions of points, which are used for testing applicability limits of correlation methods.

Key words: galaxies: statistics, large-scale structure of Universe; methods: numerical

INTRODUCTION

Recent huge catalogues of galaxy redshifts, such as 2dF [4] and SDSS [20], allow to study galaxy spatial distribution up to several hundreds of Mpc [17]. The correlation methods of the large scale structure (LSS) analysis, and the results of its applications are described in [1, 6, 11, 14, 16, 18]. According to modern data [17] the power-law exponent of the correlation function (CF) is $\gamma \approx 0.9$ at scale 0.5–30 Mpc/h and become $\gamma \approx 0.2$ at scales 30–100 Mpc/h, also they detected very large structures like "walls" of size about 400 Mpc/h (Sloan Great Wall). However some papers (e.g. [14]) based on calculations of the reduced two-point correlation function (R2CF) method claimed that the observed R2CF has the power exponent $\gamma \approx 1.7$ and the inhomogeneity scale about $3r_0 \approx 15$ Mpc/h. This demonstrates the necessity of a careful analysis of methods for determination of the fractal dimension and homogeneity scale, including their borders of applicability.

DEFINITIONS

Defining the fractal model we follow the approach described in [6]. For a set of N isolated points (r_0, r_1, … r_N) one can define the microscopic density $\rho(\mathbf{r})$ ($\rho(\mathbf{r}) = \sum_{i=1}^{N} \delta(\mathbf{r} - \mathbf{r}_i)$) and the average space density:

$$\rho_0 = \lim_{R\to\infty} \|C(R;\mathbf{x}_0)\|^{-1} \int_{C(R;x_0)} \rho(\mathbf{r})\mathrm{d}^3\mathbf{r},$$

where $\|C(R;\mathbf{x}_0)\| \equiv 4/3\pi R^3$ is the volume of the sphere $C(R;\mathbf{x}_0)$ of the radius R and the centre $\mathbf{x}_0$. If the average density is strictly positive and independent of arbitrary point $\mathbf{x}_0$ the homogeneity scale is defined as λ_0:

$$\forall R > \lambda_0 \forall \mathbf{x}_0 \left| \|C(R;\mathbf{x}_0)\|^{-1} \times \right.$$
$$\left. \times \int_{C(R;x_0)} \rho(\mathbf{r})\mathrm{d}^3\mathbf{r} - \rho_0 \right| < \rho_0, \quad (1)$$

i.e. the density fluctuations do not exceed the value of the average density and we can talk about homogeneity.

For fractals such average density $\rho_0 \equiv 0$ and homogeneity scale λ_0 could not be defined. An important characteristic of fractal set is the fractal dimension. It can be defined in different ways. One easy and popular way is the so-called box-counting dimension:

$$D_B = \lim_{\epsilon\to 0}\big(\log N(\epsilon)/\log(1/\epsilon)\big),$$

where $N(\epsilon)$ is the minimal number of boxes of size ϵ needed to cover the whole set. For a uniform distribution $D_B = 3$ and for the true fractals the value of $D_B < 3$. Note that in the case of natural sets such as LSS, we are exploring a space-limited sample that has a positive average density ρ_0 even if the set is a pure fractal. Natural sets can have different values of fractal dimension at different scales. Also a uni-

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form set on scales $\lambda < \lambda_0$ can have $D_B < 3$, e.g. due to discretness of a sample.

PROJECT OF PROGRAM PACKAGE

We present a project of a program package which includes several methods to explore fractal properties of sets of isolated points (as galaxies are treated in this kind of LSS studies). Currently, we use the correlation function (complete and incomplete one) [6], approximation of radial distribution of galaxies [7, 8, 12, 15] and SL-statistics [18].

The package includes tools to load a real catalogue of galaxies and to simulate a fractal or uniform distribution of points. Each simulated point has an absolute magnitude distributed according to the Schechter's law (galaxies absolute magnitude distributed according to Schetcher Function $S(M) = 0.92\phi_0 \exp\left(-0.92(\alpha+1)(M-M^*) - e^{-0.92(M-M^*)}\right)$ between $-\infty$ and M_{max}. Form and parameters for simulations taken from [21]), thus, we can simulate magnitude-limited samples and volume-limited samples. Any sample (simulated or real galaxy catalogue) could be cut in space by limiting distance (or redshift) and angles (α, δ). Fractal model represents observed power-law of correlation function, and this "toy model" is useful to determine reliability and borders of applicability of the used methods. Currently, we use randomized 3D Cantor set with known fractal dimension, which is the main parameter of the model. To generate random numbers, we use the MersenneTwister generator [13]. It produces pseudo-random numbers of a huge period and very low correlation. Numbers distributed according to the Schechter's law are produced using Von Neumann technique.

Program is designed to save results to a hard drive for further reference both if they are needed during current or future executions. A unique file name containing short text information on type of results (for user) and unique hex-string generated from used parameters (like coordinates of sample cutting or parameters to generate random magnitude) is assigned to each result. If such file does not exist the result is calculated and saved to a new file or loaded from a disk otherwise.

In addition, to generate a 100000-point volume-limited simulated fractal sample one should start from over 10^8 point set (requires over 2 Gb of disk space), which cannot be fitted into memory. However, only around 100000-point sample and smaller would be used in a calculation that requires random access (for example, complete correlation function would be calculated in 10–20 minutes and incomplete one in 3–4 hours with 3.0 GHz AMD Athlon processor), but the process like generating and cutting samples requires only sequential access.

Each file (result or sample) is followed by human-readable information saying what was calculated or generated and which parameters were applied. The main results of program execution are plotted and analysed. Several features are included into the package to map set samples in various projections and plot calculated function estimators. Plots are created using `GNUPlot` package, which is called directly from the program to run program-generated scripts and produce .EPS picture. By little manual editing of that program-generated scripts one can get good plots in the format suitable for LaTeX.

Program is written in C++ using the object-oriented approach and contains over 6500 lines. The code is multi-platform and can be compiled with the `GNU C++` compiler both on `Linux` or `Windows` (using `MinGW` port). However, saved binary data is not portable from one platform to another. Currently, the package does not contain any user interface and could be used as a library only. We are working on a script interpreter to simulate, to load and to analyse catalogues, to produce plots from a user-made scripts and to have `GUI` to help writing of scripts in further plans. Furthermore, an `API` might be created to make the program expandable by user's methods of generating and exploring sets of isolated points. This program package is not limited by cosmological application only.

RESULTS

Conditional density as an estimator of the complete two-point correlation function is widely-used to determine the fractal dimension. It is defined as $n(r) = \langle N_P(r)\rangle_P \,/\, \left(4/3\pi r^3\right)$, where $\langle N(r)\rangle_P$ is the number of points inside a sphere of radius r around the point P averaged through such points P that do not lay close to the spatial border of the set, i.e. closer than considered distance r. It is known that $n(r) \propto r^{D-3}$, i.e. conditional density should show linear law in logarithmic coordinates for fractal and uniform sets. Uniformity means that $n(r) = \text{const}$. Full definition and description of the correlation functions could be found in [6]. However, analysis of behaviour of conditional density for pure fractal set shows that for r comparable to the size of a sample it is unpredictable and cannot be used to determine the fractal dimension. In fact, one can only get reliable results if r is about 10 times less than the size of the biggest sphere that can be wholly placed into spatial borders of sample (i.e. about 10% of the maximal scale r where conditional density could be calculated). This is caused by the fact that for larger r only fewer points can be used to average the value of $N(r)$. From Figure 1 one can compare the behaviour of pure fractal set and the volume-limited sample of 2dF Galaxy Redshift Survey [4]. Study of the 2dF catalogue shows that fractal dimension is equal to 2.25 ± 0.2 at scales from 5 to 20 Mpc/h and cannot be determined reliably on larger scales.

Incomplete correlation function ξ was already mentioned as unreliable for determining the fractal dimension because it relies on the existence of positive average density [6] and confirmed by numeric experiments by Vasyljev N. V. in his diploma work. However, it is referred as a way to determine the scale of homogeneity as a value of correlation scale $\xi = 0$. We used Davis-Peebles estimator [5] of CF defined as $\xi_{DP} = N_r/(N_d - 1) \cdot DD(r)/DR(r)$. We took several samples of simulated fractal sets of different dimension (2.0, 2.3, 2.6) in a whole sphere and angle limited by spherical coordinates α and δ cones ($75^\circ \times 11^\circ$ solid angle) subsample. From each of 6 samples, we cut several subsamples by radius r_{max} from 10 to 100 percent of the original sphere ($R \equiv 1$). Each subsample correlation function is studied for a first root $\xi(r) = 0$. Dependence of value of the first root r on the deepness of the subsample r_{max} is shown in Figure 2 (top). Same dependence for SDSS catalogue [20, 3] 120×35 degrees completed sky part is shown. One can easily see that first root shows a trend: it increases when a subsample expands. It is observed both for simulated pure fractal sets and for SDSS catalogue samples. Thus, one cannot treat the first root of the correlation function as a reliable property of a set, i.e. as a correlation scale (or scale of homogeneity, for example).

CONCLUSION

Correlation function estimators (both complete and incomplete one) shows strong border effects and is not reliable on scales comparable with a size of a sample both to determine the fractal dimension and to find the upper bound of inhomogeneity scale. Lower bound found by conditional density from catalogue 2dF is 20 Mpc. Other methods, radial distribution and SL statistics, described in [9, 10] expands it to 70 Mpc. Growth of the distance to the first zero of the correlation function (Fig. 2) for SDSS VL sample gives the strong lower limit to the baryonic acoustic oscillations. Program package of exploring fractal properties is useful for cosmology research.

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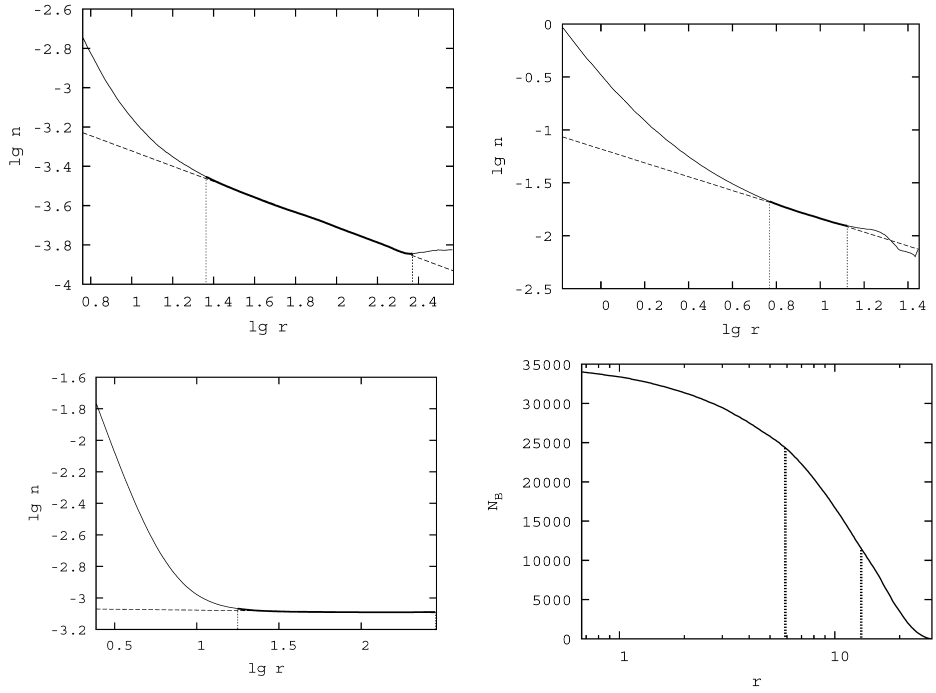

Fig. 1: Conditional density examples. Top left: fractal set of the dimension 2.6, bottom left: uniform distribution. Top right: 2dF galaxy catalogue, bottom right: number of "valid" spheres the conditional density was averaged to. Solid line shows the value of conditional density, bold denotes the segment where conditional density was approximated by a straight line (dashed line), its borders are shown by vertical dotted lines.

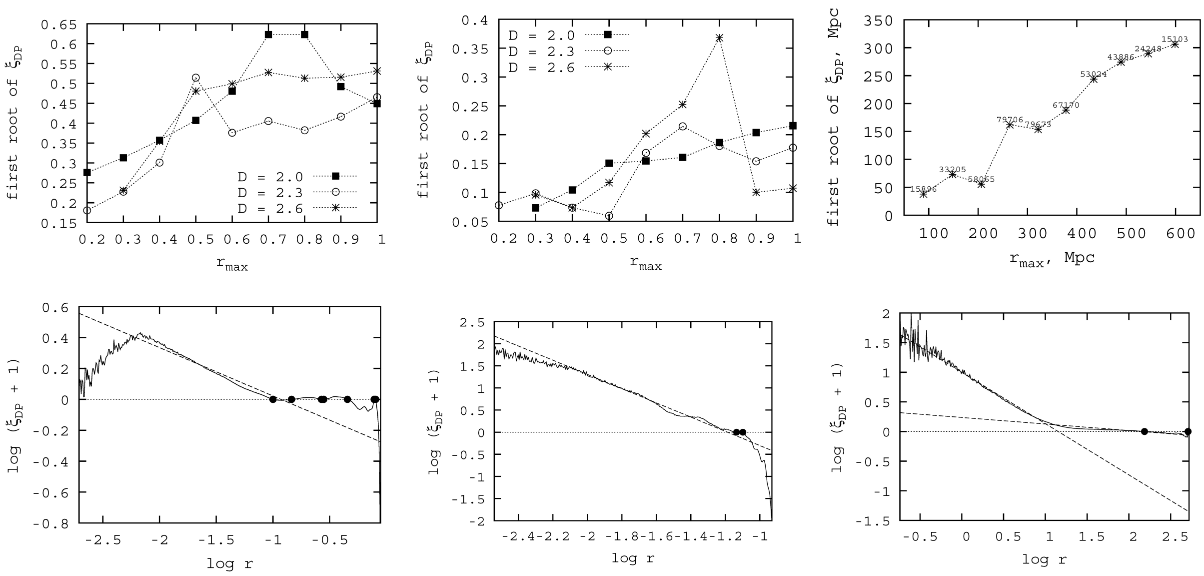

Fig. 2: Correlation function properties. Bottom row: examples of a correlation function for simulated set in a whole sphere (left, dimension 2.6, slope -0.31, 48360 point spherical subsample of radius 0.9 of radius of 66421 points sample, first root 0.1), solid angle (center, dimension 2.0, slope -1.61, 1708 points subsample of radius 0.3 of whole 73977 points sample, first root 0.07) SDSS catalogue (right, volume-limited subsample to $z = 0.11$, ($R = 321.44$ Mpc), 79673 points, slope -0.11 and -0.87, roots at 153.72 and 511.06 Mpc), straight-line approximation, its slope and roots are shown. Top row: dependency of a first root of correlation function for different dimension in whole sphere (left), solid angle (center) from the radius of subsample and radius of volume-limited subsample of SDSS catalogue (right). For SDSS catalogue, each point is supplied with the number of points in corresponding VL subsample.