

Application of multichannel singular spectrum analysis to geophysical fields and astronomical images

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The results of application of the multichannel singular spectrum analysis (MSSA) to the Earth monitoring data from Jason, GRACE and Aqua satellites are presented. The method helps to divide periodic components, separate trend, filter out noises and distinguish patterns of similar spatiotemporal behaviour. Increase of CO₂ concentration in troposphere by ~ 2 ppm/yr, sea level by ~ 3 mm/year, melting of ice sheets and glaciers are clearly seen, that proofs we are living in the epoch of global warming. We also applied MSSA to the series of astronomical images from MASTER robotic telescope and found it as a powerful method for denoising, finding low luminosity stars and detecting variable objects.

Key words: geodesy; climatology; instruments and techniques

INTRODUCTION

Satellite methods of monitoring of the Earth and the space play more and more important role in modern astronomy and geophysics. Large amount of data requires contemporary methods of processing and careful interpretation. We present the results of the multichannel singular spectrum analysis (MSSA), also known as extended empirical orthogonal functions decomposition, for the first time applied to altimetry, gravity and tropospheric CO₂ maps obtained from Jason, GRACE and Aqua satellites and to optical telescope observations. The equally-sampled sequence of maps or images was considered as a time-variable two-dimensional signal (field), whose one component (channel) represents the time series for one pixel. Method allows to separate and localize time-variable patterns with correlated behaviour in the respective data series. In application to Earth observing satellites, it simplifies study of global warming trends, ice sheets melting, hydrological cycle, greenhouse effect. For astronomical images the method significantly improves detectability of stars through noise and atmospheric effects filtering. The main purpose of this work is to show MSSA advantages.

THE METHOD

MSSA was developed in 1980-th as a generalization of the Singular Spectrum Analysis (SSA) over

the multidimensional time series [1]. It consists of four main steps: a) trajectory matrix construction, b) its Singular Value Decomposition (SVD), c) singular numbers (SNs) grouping, d) principal components (PCs) reconstruction through Hankelization. The main parameter of the algorithm is lag L , it defines the dimension of the embedding space, where we try to find correlations or finite-dimensional dynamics of the time series. Since MSSA can not be explained in this small paper in details, we refer to [1, 2, 4].

MSSA is quite time-consuming: for M -dimensional time series given in N time moments the trajectory matrix X of size $L \cdot M \times (N - L + 1)$, formed as result of sequential selection of L -dimensional vectors, would require big computer memory for storage and computational power for stages b) and d) of the algorithm. Particularly SVD, when the trajectory matrix is decomposed into three matrices

$$X = USV^T, \quad (1)$$

is better to compute in parallel mode on the super-computer cluster. In (1) matrix S elements are all equal to zero, except the diagonal of the left upper square block, where SNs are arranged in decreasing order. Each SN, generally, corresponds to the PC. The first largest SNs represent the main part of time series variability. Sum of all PCs is equal to the original time series. So, the method decomposes initial data into the components, which, in case of correct choice of L and grouping, represent meaningful

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modes of the time series behaviour, such as trend and oscillations of different periods, possibly with amplitude modulation. Selection of proper L is an heuristic procedure, based on the theory of asymptotic separability. According to recommendations [2] we selected L as the multiplier of the period (annual) expected in the processed time series. In the next section we present the results of MSSA application to different types of data. Animated results are available in the Internet¹.

EXAMPLES OF MSSA APPLICATION

European Internet-archive AVISO² contains data on the ocean altimetry. We used Merged Maps of Sea Level Anomalies & Geostrophic Velocity Anomalies (MSLA), obtained by merging Topex/Poseidon, Jason 1,2, EOS 1,2, and Envisat measurements covering latitudes from -66° to $+66^\circ$, gridded at $1^\circ \times 1^\circ$ latitude. Influence of tides, atmospheric currents, pressure are excluded during the preliminary processing stage at the data centre. Weekly files are available since X/1992. We used each fourth file (28 day step, 13 files per year). MSSA parameter $L = 26$ (~ 2 years) was selected. After the processing long periodic changes moved to PC 1, annual variability of the sea level – to PC 2, noises and high frequency components – to PCs with large proper numbers. Fig. 1 represents secular sea level change (trend), obtained by subtraction of PC 1 maps for X/1992 from XI/2010. Global averaging over the grid leads to the sea level rise rate estimate 2.8 mm/yr (without glacial isostatic adjustment GIA correction of 0.3 mm/yr [3]) [4].

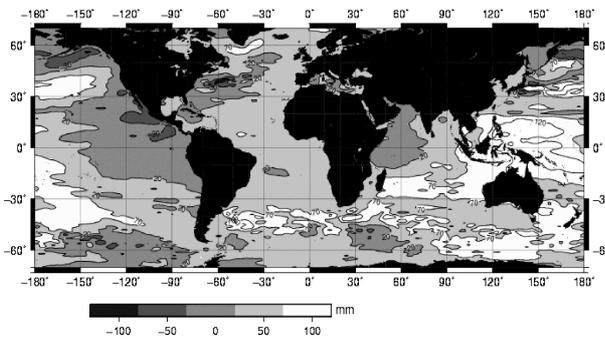


Fig. 1: Change of the sea level since X/1992 till V/2011, obtained as a result of MSSA altimetry data processing.

Atmospheric Infrared Sounder AIRS on board Aqua satellite determines the atmosphere temperature and moistures along the vertical profiles. It also supplies data upon the greenhouse gases, such as concentrations of water vapour, CH₄, CO₂, CO, O₃.

We used AIRX3C28 data upon the CO₂ concentrations in troposphere (~ 8 km) from NASA Mirador archive³ with 8 days step since IX/2002 till V/2011 (gaps were linearly interpolated). MSSA with the parameter $L = 91$ (~ 2 years) was used. Method allows to filter data, to separate trend, annual and semi-annual components. Fig. 4 represents initial data and first three PCs averaged around the globe. CO₂ trend is at the level of 1.9 ppm/yr. Maps analysis shows that concentration increases more rapidly in the northern hemisphere, what could be related with higher human population and less ocean surface, then in the southern hemisphere.

GRACE twin-satellite provides monthly solutions for gravitational field of the Earth since IV/2002. Data files were obtained from NASA PO.DAAC archive⁴. CSR Level 2 solution was used in form of Stokes coefficients up to the degree and order 60. After averaging of the field and Paulson 2007 GIA model [3] subtraction MSSA with parameter $L = 24$ (~ 2 years) was applied. Noises in form of meridional stripes which aggravate GRACE data were successfully filtered out. They passed to PCs with high proper numbers. Annual hydrological cycle went to PC 1. Secular long-periodic changes – to PC 2. Fig. 2 represents trend in the gravity field obtained by subtraction of PC 2 map for II/2003 from II/2011. Regions with ice sheets and cups melting are clearly seen on this map. Comparison with Principal Component Analysis ($L = 1$) [1] has shown advantages of MSSA, which provides better separability and filtering quality [5].

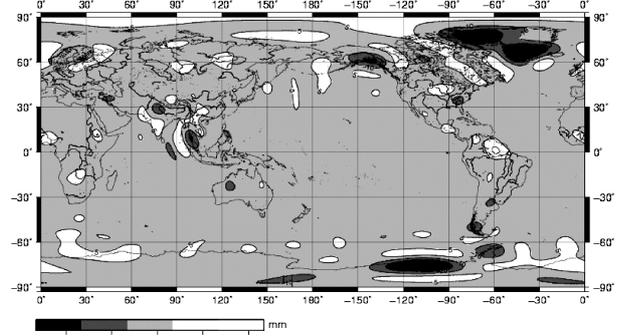


Fig. 2: Gravity field anomaly from GRACE trend since II/2003 till II/2011, represented in equivalent water level.

MSSA was also applied to the astronomical images from MASTER robotic observatory in Kislovodsk. The main purpose of MASTER is to detect optical afterglows of the gamma ray bursts (GRB). Series of observations of the GRB 110328 region were made several tens minutes after the SWIFT alert. GRB 110328 was located in the centre

¹<http://lnfm1.sai.msu.ru/~tempus/science/MSSA>
²<http://www.aviso.oceanobs.com/>
³<http://mirador.gsfc.nasa.gov/>
⁴<http://podaac.jpl.nasa.gov/>

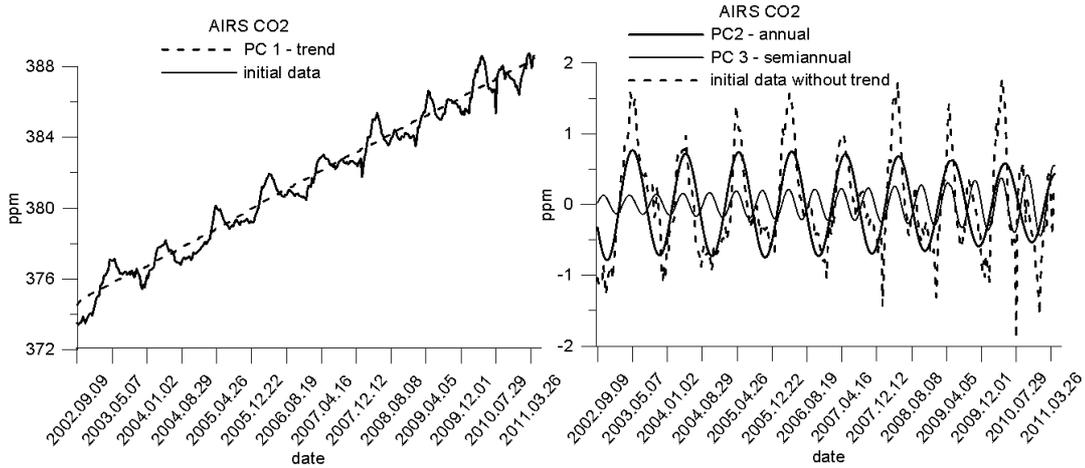


Fig. 4: Tropospheric CO₂ global average and MSSA components.

of galaxy with redshift $z = 0.351$ and had several repetitions. It was interpreted as a separated into parts stellar object falling to the super-massive black hole in the centre of the host galaxy. We processed 90 images with 3 min exposure by MSSA with parameter $L = 20$. Initial and processed (PC 1) images are represented in Fig. 3. While the series of initial images (left) contains noises and changes of point spread function (PSF), PC 1 obtained after MSSA-processing found to be stable. If in the initial image the stars of 19^m are seen, processed image increases the detectability up to 21^m stars. Though we did not detect optical afterglow of GRB (in the image centre), advantages of MSSA-processing are clearly seen.

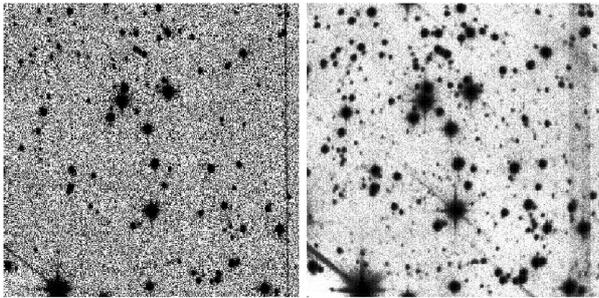


Fig. 3: Comparison of MASTER initial image (left) and MSSA PC 1 for it (right).

CONCLUSIONS

We suggested MSSA as an advanced technique for processing of different types of multidimensional time series in purpose to detect variable correlated behaviour, to separate components and to denoise data. Decomposition into PCs simplify the study

of comprehensive data obtained by the modern observational techniques such as satellites and robotic telescopes. It helps to find hidden effects, to classify and to interpret information. MSSA can be also useful as a stabilizing procedure for inverse problems solving [6], for prediction of the finite-dimensional components [2], and can be extended to non-equally sampled data. We hope the presented examples were illustrative and convincing enough, to encourage MSSA usage.

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