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# MHD waves in the plasma system with dipole magnetic field configuration

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The eigenmode spectrum of the ultra low frequency (ULF) waves in the Earth magnetosphere is discrete and consists of Alfvén and slow magnetosonic modes. Their interaction depends on ionospheric conductivity and the magnetic field curvature. We present the physical conditions of resonant ULF waves realization obtained for different wave polarization types. ULF waves with poloidal polarization are strongly coupled to slow magnetohydrodynamic (MHD) waves. The magnetic field pressure oscillates with 180° phase shift with respect to the plasma pressure. Thus, for such coupled waves the partial pressure compensation is observed. The crucial influence of the background magnetic field shear on the poloidal modes is shown. The toroidal field line resonant ULF waves do not have magnetic pressure and plasma pressure perturbations. Results presented in this paper are common for the ULF waves in the Earth's magnetosphere and include different scale disturbations. The verification of the obtained conditions with parameters of waves collected in the Earth's magnetosphere ULF is carried out by use of AMPTE/CCE and Equator-S data. The good agreement is obtained.

**Key words:** MHD waves and instabilities, plasma waves and instabilities, kinetic and MHD theory

## INTRODUCTION

In the geomagnetic system periodic perturbations of the geomagnetic field are observed since 1864. Low-frequency geomagnetic pulsations are usually processed in terms of the magnetosphere MHD eigenmodes [2, 4, 5, 9]. Ultra low frequency (ULF, waves with periods greater than 1 second) periodic perturbations of the geomagnetic field are one of the main channels of energy transport from the solar wind into the Earth's magnetosphere. The nature of these disturbances, according to the modern view, is associated with a resonant build-up of standing MHD waves on the lines of force of the magnetic field – field line resonance (FLR).

Results obtained in the analitical investigations (see e. g. [2, 3, 4, 5, 9, 10, 12] and references therein) predict existence of characteristic toroidal Alfvén and poloidal surges in the Earth's magnetosphere, being a hybrid of poloidal Alfvén and slow magnetosonic modes. We considered the possibility of modes coupling dependending on the magnetic field topology.

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## POLARIZATION PROPERTIES OF THE RESONANT ULF PULSATIONS IN THE EARTH'S MAGNETOSPHERE

To obtain the equations of small oscillations in the dipole magnetic field we used the equations derived in [6]:

$$\begin{aligned}&\rho\frac{\omega^2}{\left|\vec{\nabla}a\right|^2}\xi+\vec{B}\cdot\nabla\left(\vec{B}\cdot\vec{\nabla}\xi\left|\vec{\nabla}a\right|^{-2}\right)+\\&+\frac{s}{\alpha_s}\left(\gamma_s-s\right)\xi+2\left(\delta p_1+p'\xi+\gamma p\vec{\nabla}\vec{\xi}\right)\times\\&\times\frac{\vec{\chi}\cdot\vec{\nabla}a}{\left|\vec{\nabla}a\right|^2}+\frac{(s-\gamma_s)}{\alpha_s}\vec{B}\cdot\vec{\nabla}\eta=\frac{\vec{\nabla}a\cdot\vec{\nabla}\delta p_1}{\left|\vec{\nabla}a\right|^2},\end{aligned}\tag{1}$$

$$\begin{aligned}&\rho\frac{\omega^2}{\alpha_s}\eta+\vec{B}\cdot\nabla\left(\frac{1}{\alpha_s}+\vec{B}\cdot\vec{\nabla}\eta\right)+2\left(\delta p_1+p'\xi+\right.\\&\left.+\gamma p\vec{\nabla}\vec{\xi}\right)\times\frac{\vec{\chi}\cdot\left[\vec{B}\times\vec{\nabla}a\right]}{\left|\bar{B}\right|^2}=\vec{B}\cdot\vec{\nabla}\left(\frac{s}{\alpha_s}\xi\right)-\\&-\frac{\gamma_s}{\alpha_s}\vec{B}\cdot\vec{\nabla}\xi+\left[\vec{B}\times\vec{\nabla}a\right]\cdot\vec{\nabla}\delta p_1\left|\bar{B}\right|^{-2},\end{aligned}\tag{2}$$

$$\rho\omega^2\tau + \gamma p\vec{B}\cdot\vec{\nabla}\vec{\nabla}\vec{\xi} = 0. \qquad (3)$$

Equations (1)-(3) are independent on the choice of coordinate system, because they are obtained by use of general properties of differential operators. For this reason they are accurate and represent arbitrary MHD perturbations in ideal plasma and do not impose any restrictions on the pressure, current and electromagnetic fields. These equations were previously obtained in [5] in a different form:

$$\vec{\xi} = \xi\frac{\vec{\nabla}a}{\left|\vec{\nabla}a\right|^2} + \eta\frac{\left[\vec{B}\times\vec{\nabla}a\right]}{\left|\vec{B}\right|^2} + \frac{\tau\vec{B}}{\left|\vec{B}\right|^2}, \quad \vec{\tau} = \frac{\vec{B}}{\left|\vec{B}\right|},$$
$$\vec{\chi} = \left(\vec{\tau}\cdot\vec{\nabla}\right)\vec{\tau}, \quad \alpha_s = \frac{\left|\vec{B}\right|^2}{\left|\vec{\nabla}a\right|^2}, \quad \gamma_s = \frac{\vec{j}\cdot\vec{B}}{\left|\vec{\nabla}a\right|^2}, \qquad (4)$$
$$s = \frac{\left[\vec{B}\times\vec{\nabla}a\right]}{\left|\vec{B}\right|^2}\cdot\frac{\left[\vec{\nabla}\times\left[\vec{B}\times\vec{\nabla}a\right]\right]}{\left|\vec{\nabla}a\right|^2},$$
$$\delta p_1 = -\gamma p\vec{\nabla}\vec{\xi} - \left|\vec{B}\right|^2\left(\vec{\nabla}\vec{\xi}_\perp + 2\vec{\chi}\cdot\vec{\xi}_\perp\right).$$

In (1)-(4): $\rho$ is the density, $p$ is the plasma pressure, $\gamma$ is the adiabatic index, $\vec{j}$ is the current density, $\vec{B}$ is the magnetic field, $\vec{\xi}$ is the displacement vector of a unit volume of plasma, $\vec{\chi}$ is the curvature vector, $s$ is the magnetic field shear, $\delta p_1$ is the total pressure of perturbed plasma, $\omega$ is the perturbations frequency, and $a$ is the label of the magnetic surfaces and it satisfies the following relations:

$$\vec{B}\cdot\vec{\nabla}a = 0, \;\; \vec{j}\cdot\vec{\nabla}a = 0.$$

With the magnetic surfaces [11] we mean the surfaces containing lines of force of magnetic field generated by electric current. At a given magnetic surface, the value of $a$ is constant. In (1)-(3) it was taken into account that directions of $\vec{\nabla}a$, $\left[\vec{\tau}\times\vec{\nabla}a\right]$ and $\vec{\tau}$ are orthogonal. For simplicity, we used the following scaling for variables $\vec{B}$ and $\vec{j}$:

$$\frac{\vec{B}}{\sqrt{4\pi}} \to \vec{B}, \quad \frac{\sqrt{4\pi}}{c}\vec{j} \to \vec{j}.$$

Axial-symmetric dipole magnetic field is defined as:

$$\vec{B} = \left[\vec{\nabla}\psi\times\vec{\nabla}\varphi\right], \qquad (5)$$

where $\psi$ is the poloidal magnetic flux, $\varphi$ is the toroidal angle. It is the simplest curved three-dimensional model of the Earth's magnetic field. Choosing the function $\psi$ as the magnetic surface label, and considering that for such field [7]:

$$\gamma_s = 0, \;\; \vec{\chi}\cdot\left[\vec{B}\times\vec{\nabla}\psi\right] = 0, \qquad (6)$$

from (1)-(3) and (5) we obtained the new system of equations:

$$\rho\frac{\omega^2}{\left|\vec{\nabla}\psi\right|^2}\xi + \vec{B}\cdot\vec{\nabla}\left(\frac{1}{\left|\vec{\nabla}\psi\right|^2}\vec{B}\cdot\vec{\nabla}\xi\right) + 2\left(\delta p_1 + p'\xi + \gamma p\vec{\nabla}\vec{\xi}\right)\frac{\vec{\chi}\cdot\vec{\nabla}\psi}{\left|\vec{\nabla}\psi\right|^2} = \frac{\vec{\nabla}\psi\cdot\vec{\nabla}\delta p_1}{\left|\vec{\nabla}\psi\right|^2}, \qquad (7)$$

$$\rho\frac{\omega^2}{\alpha_s}\eta + \vec{B}\cdot\vec{\nabla}\left(\frac{1}{\alpha_s}\vec{B}\cdot\vec{\nabla}\eta\right) = \frac{\left[\vec{B}\times\vec{\nabla}\psi\right]\cdot\vec{\nabla}\delta p_1}{\left|\vec{B}\right|^2}, \qquad (8)$$

$$\rho\omega^2\tau + \gamma p\vec{B}\cdot\vec{\nabla}\vec{\nabla}\vec{\xi} = 0. \qquad (9)$$

Excluding the fast magnetosonic mode from consideration and substituting $\delta p_1 = 0$ into (7)-(9) we obtained the following equation:

$$\vec{\nabla}\vec{\xi} = \frac{1}{1+\beta}\left\{\vec{B}\cdot\vec{\nabla}\left(\frac{\tau}{\left|\vec{B}\right|^2}\right) - \frac{2\vec{\chi}\cdot\vec{\nabla}\psi}{\left|\vec{\nabla}\psi\right|^2}\xi\right\}, \qquad (10)$$

where $\beta = \gamma p\left|\vec{B}\right|^{-2}$.

As a result the following exact equations for the MHD modes in the dipole magnetic field were obtained:

$$\rho\frac{\omega^2\vec{\xi}}{\left|\vec{\nabla}\psi\right|^2} + \vec{B}\cdot\vec{\nabla}\left(\frac{1}{\left|\vec{\nabla}\psi\right|^2}\vec{B}\cdot\vec{\nabla}\xi\right) + 2\frac{\vec{\chi}\cdot\vec{\nabla}\psi}{\left|\vec{\nabla}\psi\right|^2}\left[p'\xi + \frac{\gamma p}{1+\beta}\left\{\vec{B}\cdot\vec{\nabla}\left(\frac{\tau}{\left|\vec{B}\right|^2}\right) - \frac{2\vec{\chi}\cdot\vec{\nabla}\psi}{\left|\vec{\nabla}\psi\right|^2}\xi\right\}\right] = 0, \qquad (11)$$

$$\rho\frac{\omega^2\tau}{\left|\vec{B}\right|^2} + \beta\vec{B}\cdot\vec{\nabla}\left[\frac{1}{1+\beta}\left\{\vec{B}\cdot\vec{\nabla}\left(\frac{\tau}{\left|\vec{B}\right|^2}\right) - \frac{2\vec{\chi}\cdot\vec{\nabla}\psi}{\left|\vec{\nabla}\psi\right|^2}\xi\right\}\right] = 0, \qquad (12)$$

$$\rho\frac{\omega^2}{\alpha_s}\eta + \vec{B}\cdot\vec{\nabla}\left(\frac{1}{\alpha_s}\vec{B}\cdot\vec{\nabla}\eta\right) = 0, \qquad (13)$$

where $(\cdots)' = \frac{d}{d\psi}(\cdots)$.

Equations (11)-(13) describe the longitudinal structure of the perturbations, while (10) describes its transverse structure. In the case of $\xi = 0$ and $\tau = 0$ the equation (13) together with the equation:

$$\left|\vec{B}\right|^{-2}\left[\vec{B}\times\vec{\nabla}\psi\right]\cdot\vec{\nabla}\eta = 0, \qquad (14)$$

describes the toroidal Alfvén modes. If $\eta = 0$, then equations (11), (12) together with the equation:

$$\vec{\nabla}\left(\xi\frac{\vec{\nabla}\psi}{\left|\vec{\nabla}\psi\right|^2}\right) + \frac{2}{1+\beta}\frac{\vec{\chi}\cdot\vec{\nabla}\psi}{\left|\vec{\nabla}\psi\right|^2}\xi + \\ + \frac{\beta}{1+\beta}\vec{B}\cdot\vec{\nabla}\left(\frac{\tau}{|B|^2}\right) = 0, \qquad (15)$$

describe the poloidal Alfvén mode entangled through the radial curvature of the force lines of magnetic field to the slow magnetosonic modes. (14) and (15) are obtained from (10).

Note, that equations (11)-(13) coincide with the equations for a transversely-scale (balloon) perturbations [2, 4, 5], equations (14), (15) in the case of small-scale cross-modes describe their polarization and dependence on the transverse coordinates. Using (11)-(13) in [2, 7, 8] the longitudinal structure of the small-scale (transverse to the background magnetic field) perturbations is discussed in details. Results, obtained here, are more general than in [2, 7, 8] and are valid not only for transverse small-scale perturbations. We refer to the results of these works in the next section, without going into details of obtaining the results themselves.

## POLARIZATION OF RESONANCE ULF PERTURBATIONS FROM SATELLITE MEASUREMENTS

The experimental validation of the obtained results was carried out by statistical analysis of the magnetic field and measurements of the plasma parameters perturbations aboard AMPTE/CCE spacecraft [13]. The Magnetic Field Experiment (MFE) includes a fluxgate magnetometer sensor mounted on a 2 meter boom. The fluxgate sensor was sampled at a rate of 8 vector samples per second. Survey data of the spin-averaged DC magnetic field has ~6 second resolution. The data span the entire AMPTE/CCE mission, from 1984 day 234 (August 21) through 1989 day 009. Dynamics of the magnetic field and the plasma pressure was processed within the field aligned coordinate system.

Next, we considered ULF waves polarization by use of magnetic field measurements aboard AMPTE/CCE spacecraft during 1985-1986. The distribution of the observed ULF events as a function of magnetic local time (MLT) of observation and the polarization of the magnetic field in the wave (shown as a ratio of poloidal $b_{tor}$ and toroidal $b_{POL}$ magnetic field perturbation component) are shown in Fig. 1. Thus, waves with toroidal polarization are concentrated on the bottom of the diagram, and the poloidal are on the top. The circle radii are proportional to the ratio of the longitudinal component of the magnetic field perturbations $b_{||}$ (relative to the local magnetic field) to the transverse component $b_\perp$ ($b_\perp^2 = b_{tor}^2 + b_{pol}^2$). Thus, the larger radius of the circle corresponds to greater perturbation of the magnetic pressure in the wave.

As can be seen from the diagram, the toroidal waves are observed mainly without the perturbation of the magnetic field pressure ($P_M = b_{||}^2/8\pi$), and, thus, without perturbations of the plasma pressure. Most often they are observed on the flanks of the Earth's magnetosphere. The ratio of longitudinal and transverse components of the magnetic field perturbations is usually $b_{||}/b_\perp < 0.5$. For the major part of events with toroidal magnetic field perturbations: $b_{||}/b_\perp < 0.1$. The observed asymmetry of the morning and evening magnetosphere sectors can be explained within the frame of the Kelvin-Helmholtz instability on the flanks of the magnetosphere or waveguide mode coupling with FLR waves [1]. Poloidal waves are characterized by a significant component of the magnetic field pressure and plasma pressure. The ratio $b_{||}/b_\perp$ can reach 4. For poloidal waves this ratio lies within the range $1.5 - 2.5$. This confirms sufficient coupling of the toroidal waves with slow MHD waves obtained analytically (15). As it was obtained for toroidal waves in (14) only pure Alfvén waves are observed without magnetic pressure perturbations. Magnetic field pressure and dynamic plasma pressure ensure 180° phase shift of the FLR wave. It is typical for the slow MHD wave.

The maximum perturbations of the magnetic pressure component are observed for wave events with circular polarization. The ratio of the longitudinal and transversal amplitudes of the perturbations can reach 5–7. These wave events with similar amplitude toroidal and poloidal components are fast MHD waves, which are not eigenmodes for magnetosphere system, but can transport energy from the magnetopause to the inner magnetosphere and can intensively couple with FLR modes.

## CONCLUSIONS

The eigenmode spectrum of the ULF waves in the Earth magnetosphere is discrete and consists of Alfvén and slow magnetosonic modes. Their inter-

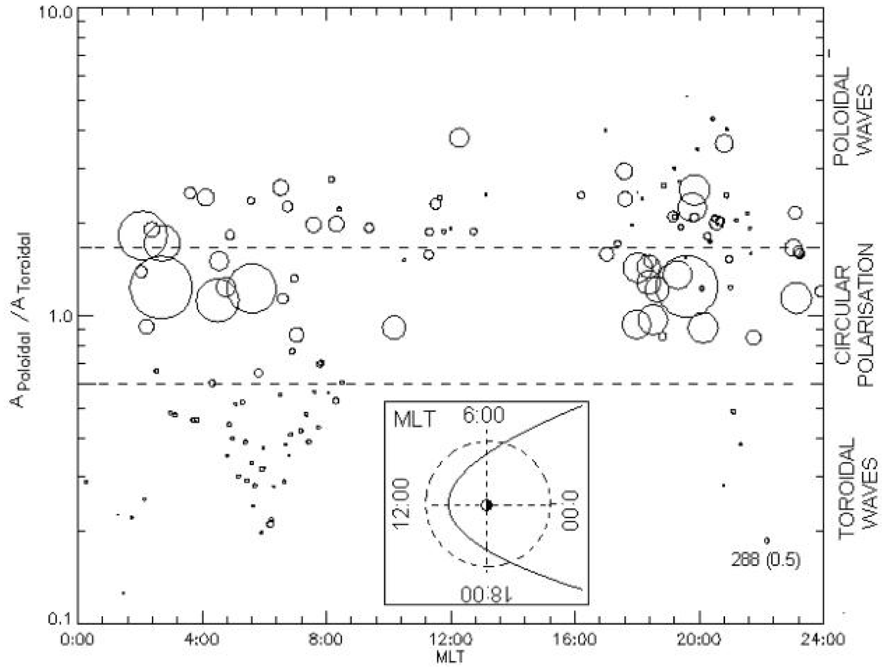


Fig. 1: Polarization characteristics of the ULF waves observed during 1985-1986 aboard AMPTE/CCE spacecraft. All events shown depending on their location (magnetic local time - MLT, shown schematically in the small diagram) and their polarization, shown as a ratio of poloidal and toroidal magnetic field perturbation components. Thus, waves with dominating of poloidal perturbation are concentrated on the top of the diagram. Also the magnetic pressure is indicated by a circle radius for each event.

action depends on ionospheric conductivity and the magnetic field curvature. We present the complex study of the physical conditions for resonant ULF waves realization obtained for different wave polarization type and their generation mechanisms. The poloidal ULF waves are strongly coupled to the slow MHD waves. The magnetic field pressure and plasma pressure anti-correlation oscillation with partial pressure compensation is obtained for such coupled wave. The critical influence of the magnetic shear for the poloidal modes is shown. The toroidal resonant ULF waves have no magnetic pressure and plasma pressure perturbation component. Experimental study of ULF wave activity associated with the resonance modes of the lines of force and verification of the obtained conditions with parameters of the waves collected in the Earth's magnetosphere ULF were carried out.

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