Telescope inaccuracy model based upon satellite laser ranging data

V. P. Zhaborovskyy[1][*], *V. O. Pap*[1], *M. M. Medvedsky*[1], *V. Ya. Choliy*[1,2]

[1]Main Astronomical Observatory of NAS of Ukraine, 27 Akademika Zabolotnoho St., 03680 Kyiv, Ukraine
[2]Taras Shevchenko National University of Kyiv, Glushkova ave., 4, 03127, Kyiv, Ukraine

In this paper, a new approach to constructing a telescope pointing model is described. Procedures of data collection, data processing, and model construction are presented. Telescope encoder countings, obtained during satellite laser ranging, are used as input data for the construction of the model. The model is presented as a harmonical series with frequencies obtained by the maximum entropy spectral analysis method.

Key words: telescopes, methods: data analysis, site testing

INTRODUCTION

Accurate pointing of the satellite laser ranging telescope is necessary, particularly if the satellite is not visible. A telescope model is constructed to solve this issue. Two-axial telescopes are used for laser ranging, and one such telescope is located at the laser ranging station "Golosiiv-Kyiv" (ILRS # 1824). The telescope is equipped with azimuth (A) and height (h) encoders.

The functions $f(h_c, A_c, e_i)$ and $g(h_c, A, e_i)$, of kind:

$$
\begin{aligned}
A_o &= f(A_c, h_c, e_i), \\
h_o &= g(A_c, h_c, e_i),
\end{aligned} \quad (1)
$$

need to be constructed, where A_c, h_c are calculated from ephemeris coordinates, A_o, h_o are angular encoders countings, and e_i are various parameters, e. g. temperature and, possibly, time. These variables are referred to as functions the telescope inaccuracy model.

The model is considered "good" if $|\Delta A| = |A_{obs} - A_{eph}| <= \varepsilon$ and $|\Delta h| = |h_{obs} - h_{eph}| <= \varepsilon$, where ε is the half-width of laser beam. For the "Golosiiv-Kyiv" station, $\varepsilon = 10 - 15$ arcsec.

INPUT DATA

FOR MODEL CONSTRUCTION

Several models were created at station 1824 in the past ten years, using star and satellite observations [1, 2]. Satellite observations are preferable for the model construction, as they are conducted during the main observation program.

The database from [4] was used in this work. In Fig. 1 through Fig. 4 the Lageos-1 and Lageos-2 observations are shown in black, while all other satel-

lite observations are in grey. The total number of the observations is 95253 and 6817 for Lageos-1 and Lageos-2, respectively.

There were two reasons for creating our model using solely Lageos satellite observations. Firstly, they are high priority targets for observation. Secondly, their observations can be considered representative.

MODEL CONSTRUCTION

From the analysis of Fig. 1 through Fig. 2 it is evident that ΔA and Δh depend periodically upon A. From Fig. 4 — Δh depends linearly on h, and from Fig. 3 — ΔA does not depend on h, and here is why:

$$
f(A) = \sum_{n=1}^{N} (B_n \sin (A\varphi_n) +
$$
$$
+ C_n \cos (A\varphi_n)) + D, \quad (2)
$$

$$
g(A, h) = \sum_{n=1}^{N} (E_n \sin (A\psi_n) +
$$
$$
+ F_n \cos (A\psi_n)) + Gh + H, \quad (3)
$$

where selected as $f(A, h)$ and $g(A, h)$ (see (1)), where $B_n, C_n, E_n, F_n, D, G, H$ are linear parameters, and φ_n and ψ_n are non-linear parameters of the model. The model is to be used for $h > 20°$.

The telescope errors were decomposed into the set of the periodical functions. The D, H explain constant discrepancy between the zeros of the encoders and the ephemeris, G describes the linear dependence of the discrepancies from h, φ_n and ψ_n are data series frequencies.

---

*zhskyy@gmail.com

An iterative approach to the construction of the model was used. For example, let's take $g(A, h)$. For every step n there are data series $\tau_i^{(n)}$, and the frequency ψ_n at maximum amplitude is searched for by maximum entropy spectral analysis method [5], according to the procedure from [3], with autoregressive sequences on the order of $4\sqrt{L}$, where L is the data series length. Then, coefficients E_n and F_n of the harmonical functions are determined by the least squares method:

$$\tau_i^{(n+1)} = \tau_i^{(n)} - (E_n \sin{(A\psi_n)} + F_n \cos{(A\psi_n)}). \quad (4)$$

Here, $\Delta h = h_{obs} - h_{eph}$ with mean removed was used as $\tau_i^{(0)}$.
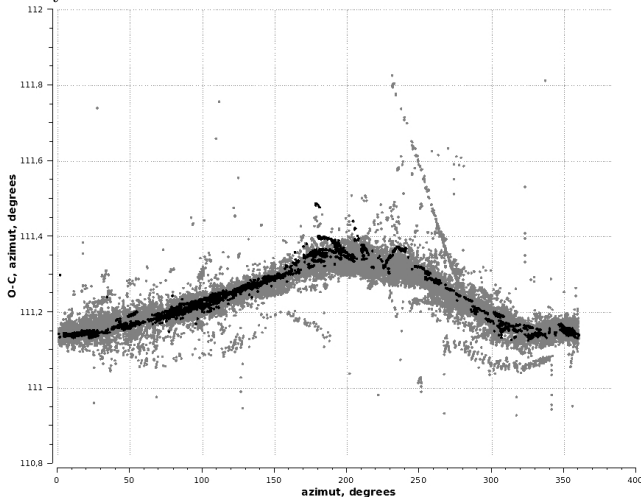

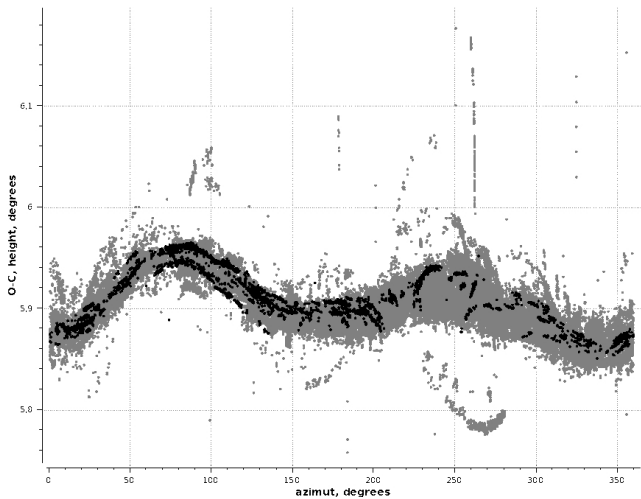Fig. 1: Dependence of $\Delta A = A_{obs} - A_{eph}$ from azimuth.

Fig. 2: Dependence of $\Delta h = h_{obs} - h_{eph}$ from azimuth.

At each step the standard deviation σ is used as a criteria. The iterations run until $\sigma >= \varepsilon$, where $\varepsilon = 10 - 15$ arcsec is half-width of the laser beam for "Golosiiv-Kyiv" station.

The same approach was used for azimuthal model.

RESULTS AND CONCLUSIONS

Numerical values of parameters from equations (2–3) are presented in Table 1 and Table 2. Additionally, $D = 111°.2 \pm 0°.5$, $G = 0.06 \pm 0.02$, $H = 5°.9 \pm 0°.5$.

Table 1: Numerical values of the azimuth model's coefficients.

n	B_n	C_n	φ_n
	arcsec	arcsec	deg
1	19.5 ± 0.8	18.0 ± 0.4	112
2	-86.5 ± 0.5	$114. \pm 4.0$	71
3	16.8 ± 0.6	$-149. \pm 3.9$	62
4	53.7 ± 0.8	-1.53 ± 0.2	46
5	-17.0 ± 1.0	17.0 ± 0.7	41
6	0.84 ± 0.04	-0.84 ± 0.09	22
7	3.5 ± 0.8	1.0 ± 0.1	19
8	-3.2 ± 0.7	2.2 ± 0.2	18
9	-0.33 ± 0.02	-1.2 ± 0.1	16
10	0.09 ± 0.01	-0.09 ± 0.01	15

Table 2: Numerical values of the height model's coefficients.

n	E_n	F_n	ψ_n
	arcsec	arcsec	deg
1	2573 ± 36	-3874 ± 34	292
2	343 ± 6	591 ± 5	137
3	8.30 ± 0.9	69.6 ± 1.8	59
4	-29.4 ± 2.3	-21.1 ± 0.3	51
5	-2.3 ± 0.3	-2.8 ± 0.3	33
6	1.8 ± 0.2	0.4 ± 0.2	31

The input (gray) and modelled (black) data are presented in the Fig. 5 (for azimuth) and Fig. 6 (for height). The residuals of the model are presented in the Fig. 7 and Fig. 8.

The model was implemented in "Golosiiv-Kyiv" station, and now is used during the observations.

REFERENCES

[1] Medvedsky M. & Pap V. 2008, in *16th International Workshop on Laser Ranging*, Proceedings of the conference held 12-17 October, 2008 in Poznan, Poland, 84

[2] Medvedsky M. & Suberlak V. 2002, Artificial Satellites – Journal of Planetary Geodesy, 37, 3

[3] Morgan K. & Somerville C. R. 1976, Canadian Studies in Population, 3, 1

[4] Pap V. O. 2011, Bulletin of The Ukrainian Centre of determination of the Earth Orientation Parameters, 6, 22

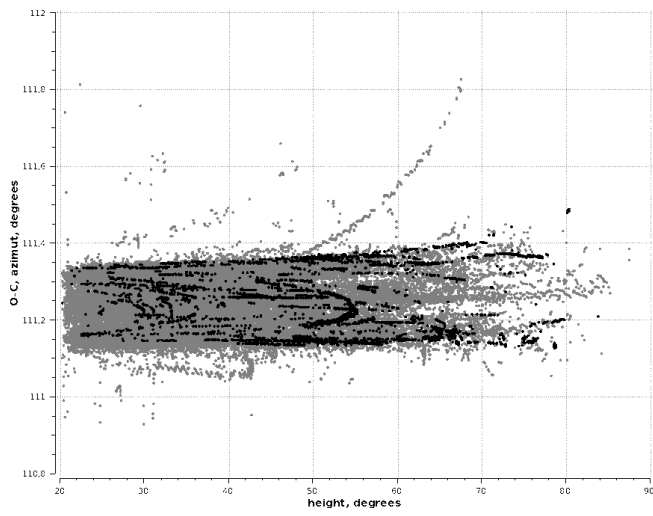[5] Ulrych T. J. 1972, J. Geophys. Res., 77, 1396

Fig. 3: Dependence of $\Delta A = A_{obs} - A_{eph}$ from height.
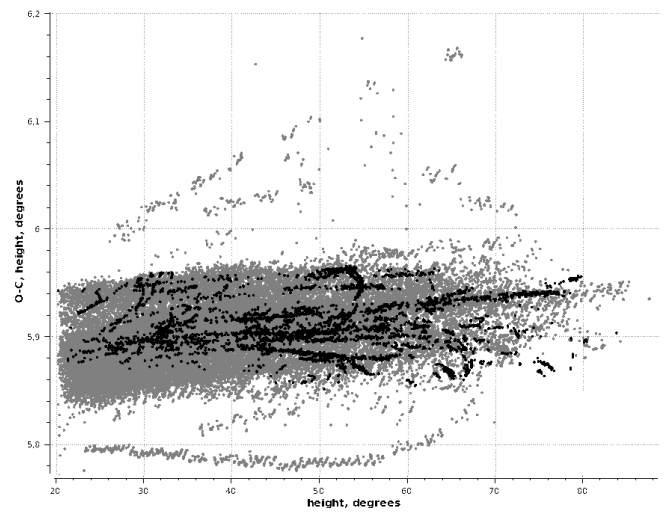

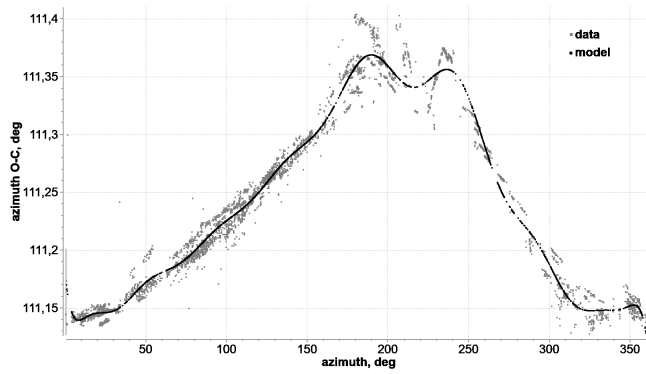Fig. 4: Dependence of $\Delta h = h_{obs} - h_{eph}$ from height.
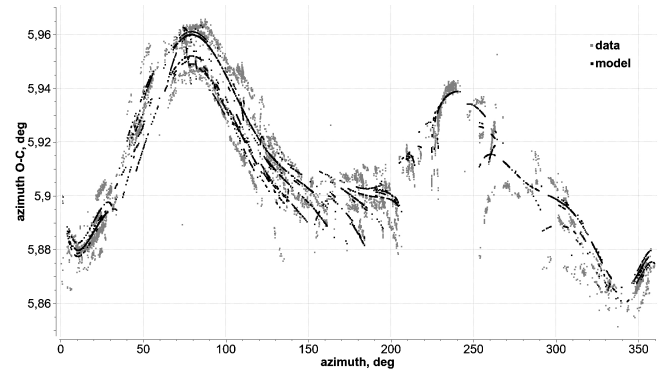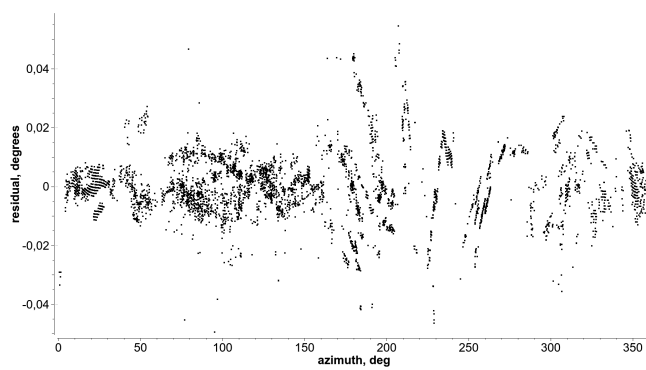

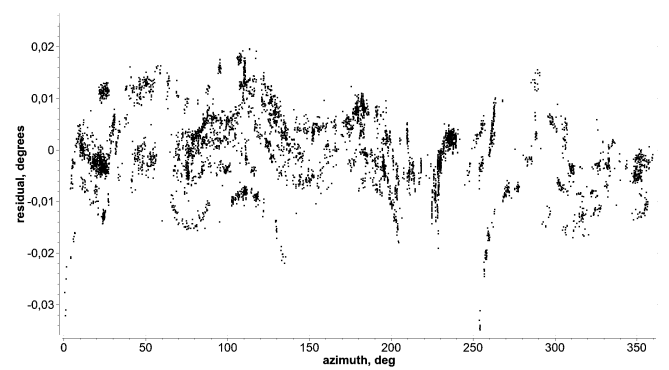Fig. 5: Azimuth model.

Fig. 6: Height model.

Fig. 7: Azimuth's model residuals.

Fig. 8: Height's model residuals.