Advances in Astronomy and Space Physics, **7**, 17-22 (2017) doi: 10.17721/2227-1481.7.17-22

Qualitative types of cosmological evolution in hydrodynamic models with barotropic equation of state

S. S. Dylda,* *V. I. Zhdanov*†

Astronomical Observatory, Taras Shevchenko National University of Kyiv, Observatorna str., 3, Kyiv, 04053, Ukraine

We study solutions of the Friedmann equations in case of the homogeneous isotropic Universe filled with a perfect fluid. The main points concern the monotony properties of the solutions, the possibility to extend the solutions on all times and occurrence of singularities. We present a qualitative classification of all possible solutions in case of the general smooth barotropic equation of state of the fluid, provided the speed of sound is finite. The list of possible scenarios includes analogues of the "Big Rip" in the future and/or in the past as well as singularity free solutions and oscillating Universes. Extensions of the results to the multicomponent fluids are discussed.

Key words: cosmology: theory, early Universe, dark energy

INTRODUCTION

Modern ΛCDM cosmological model successfully describes most of the observational data of extragalactic astronomy. Nevertheless, due to the well-known horizon and flatness problems, modifications of the standard cosmological model are widely discussed to ensure the existence of the inflationary stage of the cosmological evolution. In this view a dynamical models of the dark energy (DE) have been introduced, which are different from the unchanging cosmological constant. Various DE models involve cosmological fields, extra dimensions, modified gravity etc. (see [1, 9, 11] for a review). Hydrodynamic approach is often sufficient to analyse these issues; in this approach either all the matter in the Universe or the dynamic DE is modelled by means of a relativistic fluid with some equation of state (EoS). On this way a number of analytical solutions have been found (e. g. [2, 8, 10] and references therein).

In these studies, a considerable attention is paid to the qualitative properties of solutions, such as monotonicity, intervals of existence and limiting properties of the solutions. Recently, interest in the solutions like "Big Rip" [5] and in some other types of singular behaviour [3, 5, 8, 10] has grown. Typical questions are as follows. Does a solution of the Friedmann equations exist for all $t \to \infty$? Otherwise, does the cosmological scale factor and/or $e(t)$ blow up at some singularity point? Is the energy density $e(t)$ bounded?

In paper [6], such a qualitative behaviour of solutions has been studied for a special form of EoS subjected to some restrictions. In the present paper we relax these restrictions. We consider the homogeneous isotropic Universe with a general barotropic EoS $p = p(e)$, that relates the pressure p to the invariant energy density $e > 0$. The only conditions imposed are the smoothness of the function $p(e)$ and the existence of an upper bound for dp/de, i. e. the speed of sound is supposed to be bounded. We describe possible scenarios of the cosmological evolution with a focus on roots of specific enthalpy $h(e) = e + p(e)$. The smoothness of $p(e)$ and $h(e)$ is rather a strict condition; for example, it prohibits crossings of the "phantom line" ($e + p = 0$). We present below a complete list of all possible scenarios with various qualitative behaviours.

BASIC EQUATIONS

The homogeneous isotropic cosmology is described by the Friedmann-Lemaitre-Robertson-Walker (FLRW) metric:

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\left[\mathrm{d}\chi^2 + F^2(\chi)\mathrm{d}O^2\right],$$

where $F(x) = \sin(x)$, $\sinh(x)$ or x respectively, for the closed, open and spatially-flat Universe. This corresponds to the following values of the parameter $k = 1, -1, 0$ in the Friedmann equations:

$$\frac{\mathrm{d}^2 a}{\mathrm{d}t^2} = -\frac{4\pi}{3}a(e+3p), \qquad (1)$$

$$\left(\frac{1}{a}\frac{\mathrm{d}a}{\mathrm{d}t}\right)^2 = \frac{8\pi}{3}e - \frac{k}{a^2}, \qquad (2)$$

*tunerzinc@gmail.com
†valeryzhdanov@gmail.com

here $G = c = 1$. The only non-trivial hydrodynamics equation is

$$\frac{\mathrm{d}e}{\mathrm{d}t} + \frac{3h}{a}\frac{\mathrm{d}a}{\mathrm{d}t} = 0 \quad (3)$$

Eq. 1-3 are not independent, so we use further Eq. 2, 3. Eq. 3 can be rewritten as a first-order autonomous equation

$$\frac{\mathrm{d}e}{\mathrm{d}X} = -3h, \quad X = \ln a. \quad (4)$$

Next we introduce notations as follows:

f: $a_1 \uparrow a_2$ means that function $f(x)$ is monotonically increasing from a_1 to $a_2 > a_1$ when x belongs to the function domain. Analogously $a_1 \downarrow a_2$ in case of the decreasing function.

f: $a_1 \uparrow a_2 \downarrow a_1$ means that the function f is monotonically increasing from a_1 to $a_2 > a_1$ and then, after reaching the turning point a_2, it is monotonically decreasing to a_1.

We denote decreasing unbounded solutions of Eq. 4 by the symbol $\mathbf{U}\downarrow$, and increasing unbounded solutions by $\mathbf{U}\uparrow$. Analogously, for increasing bounded solutions and decreasing bounded ones we write correspondingly $\mathbf{B}\uparrow$ and $\mathbf{B}\downarrow$.

SOLUTIONS OF THE EQ. 4

We supposed that h is a smooth function. The condition that $\mathrm{d}p/\mathrm{d}e$ is bounded means that $\exists C_0 : 0 < C_0^2 < \infty$ such that:

$$|\mathrm{d}p/\mathrm{d}e| \le C_0^2, \quad (5)$$

and we have $|\mathrm{d}h/\mathrm{d}e| \le 1 + C_0^2$. The right-hand side of Eq. 4 is Lipschitz continuous and $\forall e \in (-\infty, \infty)$ there exists the finite Lipschitz constant $3(1 + C_0^2)$. Then in virtue of the Cauchy-Lipschitz theorem, the Eq. 4 with initial data $e(t_0) = e_0$ has a unique smooth solution $e(X)$ for all $X \in (-\infty, \infty)$.

Suppose we have e_1: $h(e_1) = 0$, then $e(X) \equiv e_1$ is a solution of Eq. 4. In virtue of the uniqueness, any other solution $e(X)$ of this equation cannot intersect the line $e = e_1$. This enables a simple classification of the qualitative behaviour of cosmological scenarios.

Further we impose condition $h(0) = 0$ so as to avoid situations when solutions can be extended to negative values of e in the solutions of Eq. 4. As we pointed out above, the regular solution of Eq. 4 exists $\forall X \in (-\infty, \infty)$.

Let $e_m \ge 0$ be a maximal of all roots of $h(e)$, $h(e_m) = 0$. If, e. g., $h(e) > 0$ for $\forall e > e_m$, then any solution $e(X)$ passing through the point X_0, $e > e_m$ can be extended to all X-axis, it is monotonically decreasing, unbounded and it has the range (e_m, ∞). The bounded solutions in this case are impossible. Indeed, if we suppose that $e(X)$ is bounded then, according to Weierstrass theorem, there exists some finite value e_*, such that $e(X) \to e_* > e_m$ for $X \to -\infty$, whence $\mathrm{d}e/\mathrm{d}X \to 0$ and $h(e_*) = 0$ contradicting to the condition that $h(e) > 0$ for $e > e_m$.

The most simple example of this case: $p = we$, $w > -1$.

Analogously, if $h(e) < 0$ for $e > e_m \ge 0$, then any solution $e(X)$ passing through the point $X_0, e > e_m$ can be extended to all X-axis, it is monotonically decreasing, unbounded (for large negative X) and it has the range (e_m, ∞).

Thus $\mathbf{U}\downarrow$ and $\mathbf{U}\uparrow$ are the only possible types of solutions in the domain $e > e_m$.

Let $h(e_1) = 0$, $h(e_2) = 0$, $e_1 < e_2$. Then in the domain $e \in (e_1, e_2)$ we have either $\mathbf{B}\uparrow$ or $\mathbf{B}\downarrow$ type depending on the sign of $h(e)$.

COSMOLOGICAL SCENARIOS FOR $k = 0, -1$

We are interested in the domain, range and monotonicity of the functions $a(t)$, $e(t) > 0$ satisfying Eq. 2, 3. From Eq. 2 we have:

$$\frac{\mathrm{d}X}{\mathrm{d}t} = s\sqrt{\frac{8\pi}{3}e - k\exp(-2X)}, \quad (6)$$

where $s = 1$ for the cosmological expansion ($\dot{a} > 0$) and $s = -1$ for the contraction ($\dot{a} < 0$). For $k = 0, -1$ (open or spatially-flat Universe), the right-hand side of Eq. 2 is always non-vanishing, and the sign s does not change.

The condition, which allows us to extend solution $a(t), e(t)$ on all values of $|t|$, is the divergence of the integral:

$$I(X_1, X_2) = \int_{X_1}^{X_2} \mathrm{d}X \left[\frac{8\pi}{3}e(X) - k\exp(-2X)\right]^{-1/2}, \quad (7)$$

both for $X_2 \to \infty$ and for $X_1 \to -\infty$, as $e(X)$ is extended to corresponding values of the argument. If one of the conditions is not satisfied, the solution meets a singularity and exists respectively only for $t < t_* < \infty$ or for $t > t_0 > -\infty$.

For $k = -1$, $s = 1$ all possible types of solutions in regions with a different signs of $h(e)$ are described in Table 1 (see Fig. 1). For $k = -1$ the evolution always starts from finite time $t = t_0 > -\infty$, since

$$I(X_1, X_2) \le \int_{X_1}^{X_2} \mathrm{d}X \exp(X) = \exp(X_2) - \exp(X_1), \quad (8)$$

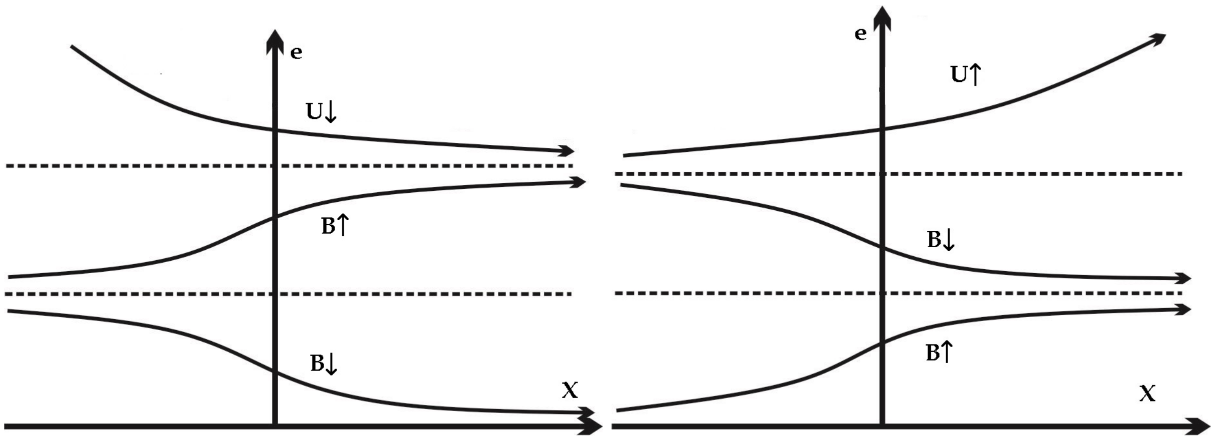

Fig. 1: Examples of the qualitative behaviour of solutions of Eq. 4 for $k = 0$ and $k = -1$. The arrows show the direction of the evolution for $s = +1$. Solutions for monotonic decreasing (left) and increasing(right) functions.

Table 1: Types of qualitative behaviour of solutions for $k = -1$ ($s = 1$) in regions with different signs $h(e)$, $t_* < \infty$, $0 \le e_0 < e_1 < \infty$.

Type	$e(X)$	Domain (t)	$a(t)$	$e(t)$
Region $e > e_0$: $h(e) > 0$; $h(e_0) = 0$				
1.1	$\mathbf{U}\downarrow$	$(0, \infty)$	$0 \uparrow \infty$	$\infty \downarrow e_0$
Region $e > e_0 \ge 0$: $h(e) < 0$; $h(e_0) = 0$				
1.2	$\mathbf{U}\uparrow$	$(0,\ infty)$	$0 \uparrow \infty$	$e_0 \uparrow \infty$
1.3	$\mathbf{U}\uparrow$	$(0, t_*)$	$0 \uparrow \infty$	$e_0 \uparrow \infty$
Region $e \in (e_0, e_1)$: $h(e) > 0$; $h(e_0) = h(e_1) = 0$				
1.4	$\mathbf{B}\uparrow$	$(0, \infty)$	$0 \uparrow \infty$	$e_0 \uparrow e_1$
Region $e \in (e_0, e_1)$: $h(e) < 0$; $h(e_0) = h(e_1) = 0$				
1.5	$\mathbf{B}\downarrow$	$(0, \infty)$	$0 \uparrow \infty$	$e_1 \downarrow e_0$

for all of the cases of behaviour of $e(X) > 0$, i. e. the integral Eq. 7 is convergent on the lower boundary. Because the system involved is autonomous, we put in this case $t_0 = 0$.

In case of $\mathbf{U}\downarrow$ we get an infinite (monotonic) increasing of the scale factor from zero to infinity and monotonic decreasing of the energy density $e(t) \to e_0 \ge 0$ for $t \to \infty$ (type 1.1 of Table 1).

If $h(e) < 0$, then we have increasing $e(t)$ and the solution either can be or cannot be extended to all times in future; in the latter case there must be a singularity of energy density at some finite value a_0 of the scale factor (cf. "Big Rip" [4]). Then for $\mathbf{U}\uparrow$ we have two types: 1.2 – when Eq. 7 is divergent on upper boundary, $a(t) \to \infty$, $e(t) \to \infty$ as $t \to \infty$ and 1.3 – when $X(e)$: $-\infty \uparrow \infty$ and Eq. 7 is convergent on upper limit.

For cases $\mathbf{B}\downarrow$, $\mathbf{B}\uparrow$ cosmological evolution continues from $t = 0$ to infinite times, and energy density is always finite. Note that if $e(t)$ tends to a finite value, then it can be identified with the current value of the dark energy density.

In case of spatially-flat Universe ($k = 0$) we do not have an estimate like Eq. 8; then for certain equations of state there are solutions that can be extended to $t \to -\infty$, since there are cases when $I(X_1, X_2)$ is divergent on the lower limit. All possible cases for $k = 0$ are presented in Table 2. The example of the case 2.1 (Table 2) is given by $p(e) = -e + (e - e_0)\sqrt{e_1/(e + e_1)}$, $e_0 \ge 0$, $e_1 > 0$, and the example of the case 2.2: $p = we$, $w > -1$.

The solutions corresponding to contracting Universe ($s = -1$) for $k = 0, -1$ are obtained from the previous considerations by the change $t \to -t$.

Table 2: Types of qualitative behaviour for $k = 0$ ($s = 1$) in regions with different signs $h(e)$, $t_* < \infty$, $0 \le e_0 < e_1 < \infty$.

Type	$e(X)$	Domain (t)	$a(t)$	$e(t)$
		$e > e_0$: $h(e) > 0$; $h(e_0) = 0$		
2.1	$\mathbf{U}\downarrow$	$(-\infty, \infty)$	$0 \uparrow \infty$	$\infty \downarrow e_0$
2.2	$\mathbf{U}\downarrow$	$(0, \infty)$	$0 \uparrow \infty$	$\infty \downarrow e_0$
		$e > e_0 \ge 0$: $h(e) < 0$; $h(e_0) = 0$		
2.3	$\mathbf{U}\uparrow$	$(-\infty, \infty)$	$0 \uparrow \infty$	$e_0 \uparrow \infty$
2.4	$\mathbf{U}\uparrow$	$(-\infty, t_*)$	$0 \uparrow \infty$	$e_0 \uparrow \infty$
		$e \in (e_0, e_1)$: $h(e) > 0$; $h(e_0) = h(e_1) = 0$		
2.5	$\mathbf{B}\uparrow$	$(-\infty, \infty)$	$0 \uparrow \infty$	$e_0 \uparrow e_1$
		$e \in (e_0, e_1)$: $h(e) < 0$; $h(e_0) = h(e_1) = 0$		
2.6	$\mathbf{B}\downarrow$	$(-\infty, \infty)$	$0 \uparrow \infty$	$e_1 \downarrow e_0$

COSMOLOGICAL SCENARIOS FOR $k = 1$

For $k = 1$ the Universe is closed and its evolution depends on zeros of the function $F(X) = (8\pi/3)e(X) - \exp(-2X)$. First we must separate degenerate cases when $F(X_0) = 0$, $F'(X_0) = 0$. In these cases we have $F'(X) = (8\pi/3)e'(X) + 2\exp(-2X) = -(8\pi/3)(e+3p) = 0$ at $X = X_0$. It is easy to see that there is the solution $a(t) \equiv \exp(X_0)$ of the Friedmann equations (1,2); also, there are solutions such that the point X_0 is an attractor or repeller: $X(t) \to X_0$ for $t \to \infty$ or/and $t \to -\infty$.

Further we confine ourselves to the case when zeros of $F(X)$ either do not exist or they are simple: $F(X_0) = 0$, $F'(X_0) \neq 0$; in the latter case these zeros are the turning points for $X(t)$ where change between expansion and contraction occurs (the change of sign of s).

Consider first the case $\mathbf{U}\downarrow$ of the unbounded $e(X)$. Let $h(e) > 0$ for $e > e_0$, $h(e_0) = 0$.

In the region $e > e_0$, if $\forall X : F(X) > 0$, then only the types analogous to 2.1, 2.2 of $k = 0$ (see 3.1, 3.2 of Table 3) are possible. The cases of expansion ($s = 1$) and contraction ($s = -1$) are related by means of the change $t \to -t$. The cases analogous to 2.3, 2.4 are impossible here, because in the region $e > e_0$, $h(e_0) = 0$, in case of $U \uparrow$ there must be a root of $F(X)$.

Let there is a root X_r: $F(X_r) = 0$ and $F(X) > 0$, $X < X_r < \infty$. Then the expansion is followed by contraction ($s = 1 \to s = -1$) at the turning point $a_r = \exp X_r$. The evolution starts with an infinite density and ends similarly. However, the behaviour for $X \to -\infty$ is similar to previous cases 3.1, 3.2: there can be either a solution that is extended for infinite times $t \to \pm\infty$ (type 3.3 of Table 3) or a solution with singularities at $t = \pm t_*$, $|t_*| < \infty$ (type 3.4).

If $F(X) > 0$, $X > X_r > -\infty$ then we have an evolution from $t = -\infty$ to $t = \infty$ with a bounce at $X = X_r$; here we have a change from contraction to expansion and the energy density is always bounded (type 3.5 of Table 3).

For $\mathbf{U}\uparrow$ type, it is easy to see that necessarily there is a root X_r: $F(X_r) = 0$ and $F(X) > 0$, $X > X_r > -\infty$. We have an evolution with the bounce with a different types of behaviour from contraction to infinite expansion depending on the rate of increase of $e(X)$: a type when solutions are defined for all t and a type with the "Big Rip" in the future and in the past (see types 3.6, 3.7).

In case of $\mathbf{B}\downarrow$ there is always at least one root X_r: $F(X_r) = 0$. Consider the domain $e \in (e_1, e_2)$, $h(e_1) = h(e_2) = 0$, $e_1 < e(X_r) < e_2$. Then we have the same qualitative behaviour as in the case $\mathbf{U}\downarrow$ (see type 3.5).

In case of $\mathbf{B}\uparrow$ there can be only one root X_r: $F(X_r) = 0$. We have an evolution $\forall t$ with the bounce from contraction to infinite expansion; the energy density is always finite.

At last, let $X_0 < X_1$ be finite roots of the function $F(X)$: $F(X_i) = 0$, $F'(X_i) \neq 0$, $i = 0, 1$, such that $F(X) > 0$ for $X_0 < X < X_1$. This can be only either in case of $\mathbf{U}\downarrow$ or $\mathbf{B}\downarrow$. Here we have an oscillating solution $a(t), e(t)$ of the Friedmann equations.

Possible types of qualitative behaviour are summarised in Table 3 (see Fig. 2).

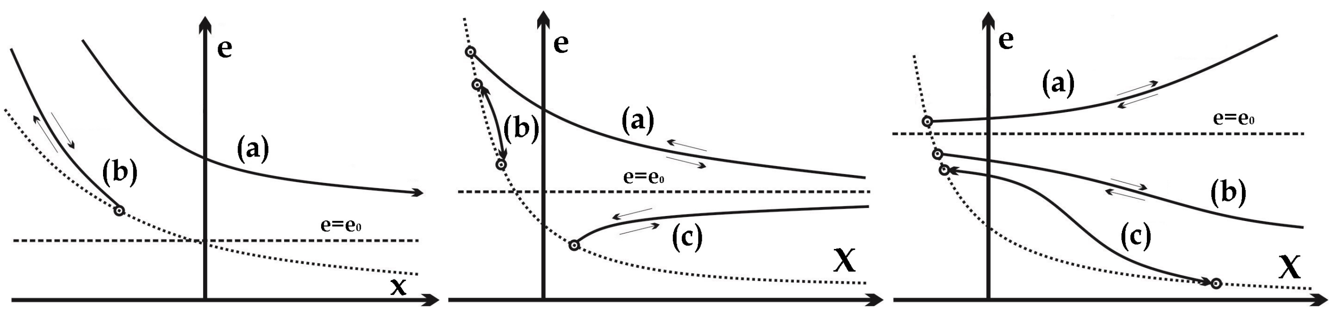

Fig. 2: Examples of the qualitative behaviour of solutions of Eq. 4 for $k = 1$; the arrows show the possible directions of evolution. The small rings indicate the turning points; the occurrence of two such points corresponds to periodic solutions. Left: (a): types 3.1-3.2, (b): types 3.3-3.4; center: (a): type 3.5, (b): type 3.9, (c): type 3.8; right: (a): types 3.6-3.7, (b): type 3.5, (c): type 3.9.

SOME GENERALISATIONS

The consideration of previous sections can be easily generalised to the case of a multi-component fluid under assumption that different DE components do not interact with each other. In this case we have the Eq. 4 for each component e_n, $n = 1, \ldots, N$ separately and the reasoning given in the corresponding section are valid for these components. Instead of Eq. 6 we have

$$\frac{\mathrm{d}X}{\mathrm{d}t} = s\sqrt{\frac{8\pi}{3}e_{\mathrm{tot}} - k\exp(-2X)}, \quad e_{\mathrm{tot}} = \sum_{n=1}^{N} e_n.$$

The main difference from the above discussion is that $e_{\mathrm{tot}}(X)$ can be a non-monotonous function and it is possible that $e_{\mathrm{tot}}(X) \to \infty$ both for $X \to \infty$ and $X \to -\infty$. For example, this can be the case of $e_1(X) \to \infty$ for $X \to \infty$ and $e_2(X) \to \infty$ for $X \to -\infty$. In this case for $k = 0$ and $k = 1$ it is allowed that $a(t)$: $0 \uparrow \infty$ on a finite interval $(0, t_*)$, $t_* < \infty$. This behaviour has been prohibited in case of one component. Note that here $a(t)$ is a monotonous function, in contrast to type 3.4 or 3.7 of Table 3. For the one-component case with $k = -1$ such domain and range of $a(t)$ is also possible (cf. type 1.3 of Table 1), but here the energy density is a monotonous function. For $k = 1$, the multicomponent case yields one more possible scenario with $a(t)$: $0 \uparrow \infty$ on $(-\infty, t_*)$, $t_* < \infty$ in addition to the types of Table 3.

The other generalisation concerns the smoothness of EoS and $h(e)$. Evidently, this requirement and the inequality of Eq. 5 can be replaced by a single requirement of Lipschitz continuity $|p(e_1) - p(e_2)| < K|e_1 - e_2|$ for all $e \in (\infty, \infty)$, where $0 < K < \infty$.

If this condition is violated then solutions of Eq. 4 can appear with either crossing of the phantom line $e \equiv e_0$ with $h(e_0) = 0$ or, e. g., with $e(X) < e_0$ for $X < X_0$ and $e(X) \equiv e_0$ for $X \geq X_0$. The simple example of the latter EoS is $p(e) = -e + C_1\sqrt{|e - e_0|}$, $C_1 > 0$, with the solutions $e = e_0 + (3C_1/2)^2(X - X_0)|X - X_0|$ and $e \equiv e_0$. In case of singularities of $p(e)$ even more complicated situations are possible, e. g., when the solution $e(X)$ cannot be extended for all X.

DISCUSSION

Thus, in the framework of the hydrodynamic model of homogeneous isotropic universe with a general smooth barotropic equation of state, we presented a classification of qualitative scenarios of cosmological evolution. We listed all possible types of the solutions depending on whether their domains and ranges can be finite or infinite. The classification includes the "traditional" scenario, which starts from $t = 0$ and continues for infinite times. Also, there is a situation when the time, where a solution exists, could be limited. However, there are equations of state that generate scenarios of the eternal Universe, which exists from the infinite times in the past, as well as scenarios for which the energy density $e(t)$ is always finite. This class includes scenarios for a closed universe with a bounce and oscillating solutions. Note that most of the examples discussed above can be found in the other works in the context of specific problems, in particular possible types of singular behaviour have been analysed in [3, 5, 8, 10]. However, our classification covers all possible qualitative types of cosmological evolution from a unified viewpoint.

It is essential that, within our discussion, it is impossible to pass through any zero point of the enthalpy, e. g., from the region of a regular behaviour with $h(e) > 0$, to the region with $h(e) < 0$, where it is possible for singularities of the scale factor to occur in the future. This is due to the uniqueness of the solution of Eq. 4 in case of smoothness of the equation of state (and specific enthalpy respectively). Our clas-

Table 3: Types of qualitative behaviour for $k = 1$, $F(X) > 0$. Here X_r – finite roots of $F(X)$, $a_r = \exp X_r$.

Type	$e(X)$	Domain (t)	$a(t)$	$e(t)$
$e(X) > e_0$: $h(e(X)) > 0$, $h(e_0) = 0$; no zeros of $F(X) > 0$				
3.1	$\mathbf{U}\downarrow$	$(-\infty, \infty)$	$0 \uparrow \infty$	$\infty \downarrow e_0$
3.2	$\mathbf{U}\downarrow$	$(0, \infty)$	$0 \uparrow \infty$	$\infty \downarrow e_0$
$e(X) > e(X_r) > e_0$: $h(e(X)) > 0, h(e_0) = 0$; $X < X_r$, $F(X_r) = 0$				
3.3	$\mathbf{U}\downarrow$	$(-\infty, \infty)$	$0 \uparrow a_r \downarrow 0$	$\infty \downarrow e(X_r) \uparrow \infty$
3.4	$\mathbf{U}\downarrow$	$(-t_*, t_*)$, $t_* < \infty$	$0 \uparrow a_r \downarrow 0$	$\infty \downarrow e(X_r) \uparrow \infty$
$e_0 < e(X) < e(X_r)$, $h(e(X)) > 0$, $h(e_0) = 0$; $X > X_r$, $F(X_r) = 0$				
3.5	$\mathbf{U}\downarrow$	$(-\infty, \infty)$	$\infty \downarrow a_r \uparrow \infty$	$e_0 \uparrow e(X_r) \downarrow e_0$
	$\mathbf{B}\downarrow$	$(-\infty, \infty)$	$\infty \downarrow a_r \uparrow \infty$	$e_1 \uparrow e(X_r) \downarrow e_1$
$e_0 < e(X) < e(X_r)$, $h(e) < 0$, $h(e_0) = 0$; $X > X_r$, $F(X_r) = 0$				
3.6	$\mathbf{U}\uparrow$	$(-\infty, \infty)$	$\infty \downarrow a_r \uparrow \infty$	$\infty \downarrow e(X_r) \uparrow \infty$
3.7	$\mathbf{U}\uparrow$	$(-t_*, t_*)$, $0 < t_* < \infty$	$\infty \downarrow a_r \uparrow \infty$	$\infty \downarrow e(X_r) \uparrow \infty$
$e(X_r) < e < e_0$, $h(e) < 0$, $X > X_r$; $F(X_r) = 0$, $h(e_0) = 0$				
3.8	$\mathbf{B}\uparrow$	$(-\infty, \infty)$	$\infty \downarrow a_r \uparrow \infty$	$e_0 \downarrow e_r \uparrow e_0$
Region $X \in (X_{r_1}, X_{r_2})$, $F(X_{r_1}) = F(X_{r_2}) = 0$, $h(e(X)) > 0$				
3.9	$\mathbf{U}\downarrow, \mathbf{B}\downarrow$	$(-\infty, \infty)$	Oscillating	Oscillating

sification does not include non-smooth equations of state, which can lead to solutions intersecting points of zero enthalpy (e. g. [10]). Consideration of such EoS may be of interest because the smoothness condition can be violated in phase transitions. Our qualitative analysis can be generalised to include such options, though when considering the global cosmological behaviour, it would generate too large number of additional types. In the case of smooth equations of state our classification is complete.

ACKNOWLEDGEMENT

This work has been supported in part by the Department of target training of Taras Shevchenko National University of Kyiv under National Academy of Sciences of Ukraine (project 6Φ).

REFERENCES

[1] Alexandrov A. N., Vavilova I. B., Zhdanov V. I. et al. 2015, *'General Relativity Theory: Recognition through Time'*, Naukova dumka, Kyiv

[2] Bamba K. & Odintsov S. D. 2014, [arXiv:1402.5114]

[3] Bouhmadi-López M., González-DíazP̃. F. & Martín-Moruno P. 2008, Phys. Lett. B, 659, 1

[4] Caldwell R. R., Kamionkowski M. & Weinberg N. N. 2003, Phys. Rev. Lett., 91, 071301

[5] Fernández-Jambrina L. 2014, Phys. Rev. D, 90, 064014

[6] Jenkovszky L. L., Zhdanov V. I. & Stukalo E. J. 2014, Phys. Rev. D, 90, 023529

[7] Nojiri S. & Odintsov S. D. 2003, Phys. Rev. D, 68, 123512

[8] Nojiri S., Odintsov S. D. & Tsujikawa S. 2005, Phys. Rev. D, 71, 063004

[9] Novosyadlyi B., Pelykh V., Shtanov Yu. & Zhuk A. 2013, *'Dark energy and dark matter of the universe: in three volumes'*, Akademperiodyka, Kyiv

[10] Parnovsky S. L. 2015, Odessa Astron. Publ., 28, 137

[11] Yatskiv Ya. S., Alexandrov A. N., Vavilova I. B. et al. 2005, *'General Relativity theory: tests through time'*, Akademperiodyka, Kyiv