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On the distribution of a rotationally invariant α-stable process at the hitting time of a given hyperplane

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We find out an explicit formula for the distribution of a rotationally invariant α-stable process at that moment of time, when it hits a given hyperplane for the first time. The case of $1 < \alpha \leqslant 2$ is considered.

Keywords: *α-stable process, hitting time, resolvent kernel, local time.*

Introduction. Let $(x(t), M_t, P_x)$ be a standard Markov process in a d-dimensional Euclidean space R^d, whose transition probability density g relative to the Lebesgue measure in R^d is given by the integral

$$g(t,x,y) = (2\pi)^{-d} \int\limits_{R^d} \exp\{i(x-y,\xi) - ct\,|\,\xi\,|^{\alpha}\}\, d\xi, \quad t>0, \quad x \in R^d, \quad y \in R^d, \tag{1}$$

where $c>0$ and $\alpha \in (0,2]$ are given parameters (we use Dynkin's notation from [1]; Theorem 3.14 there guarantees the existence of *a standard* Markov process with its transition probability density given by (1)). This process is called a rotationally invariant α-stable process. In the case of $\alpha = 2$ (and $c = 1/2$), it is nothing else but a standard Brownian motion in R^d. We will suppose throughout this paper that $d \geqslant 2$.

Let $\nu \in R^d$ be a fixed unit vector and S be a hyperplane in R^d orthogonal to ν: $S = \{x \in R^d : (x,\nu) = 0\}$. Denote, by τ, the hitting time of S for the process $(x(t))_{t \geqslant 0}$ (this is a short notation for our process), that is, $\tau = \inf\{t \geqslant 0 : x(t) \in S\}$ (as usual, we put $\tau = +\infty$ in the case of $\{t \geqslant 0 : x(t) \in S\} = \varnothing$).

Assuming that $1 < \alpha \leqslant 2$, we show that, for all $\lambda > 0$ and $x \in R^d$, the formula

$$E_x e^{-\lambda\tau} = \frac{\kappa}{\pi} \int\limits_0^{\infty} \frac{\cos(\rho\lambda^{1/\alpha}(x,\nu))}{1+c\rho^{\alpha}}\, d\rho \tag{2}$$

holds true, where $\kappa = \left(\frac{1}{\pi} \int_0^\infty \frac{d\rho}{1+c\rho^\alpha} \right)^{-1} = \alpha c^{1/\alpha} \sin(\pi/\alpha)$. This formula implies the identity $P_x(\{\tau < +\infty\}) \equiv 1$. Hence, the random vector $x(\tau)$ in S is well-defined. We show that the distribution of $x(\tau)$ with respect to the probability measure P_x for $x \in R^d \setminus S$ is absolutely continuous relative to the Lebesgue measure in S, and the corresponding density denoted by $\pi_0(x, y)$ for $x \in R^d \setminus S$ and $y \in S$ is given by the equality

$$\pi_0(x, y) = \frac{\Gamma((d+\alpha)/2-1)}{\pi^{(d-1)/2}\Gamma((\alpha-1)/2)} \frac{|(x, \nu)|^{\alpha-1}}{\left[(x, \nu)^2 + |y - \tilde{x}|^2\right]^{(d+\alpha)/2-1}}, \tag{3}$$

where $\tilde{x} = x - \nu(x, \nu)$ is the orthogonal projection of x on S.

Note that the expression in (3) does not depend on $c > 0$, that can be easily foreseen.

In the case of $\alpha = 2$, the right-hand side of (3) is the density of a $(d-1)$-dimensional Cauchy distribution; this result for the Brownian motion $(x(t))_{t \geqslant 0}$ can be easily derived from the fact that the processes $(\tilde{x}(t) - \tilde{x}(0))_{t \ldots 0}$ and $((x(t), \nu) - (x(0), \nu))_{t \geqslant 0}$ are then independent with respect to the probability measure P_x, $x \in R^d$. So, in what follows, we will suppose that $1 < \alpha < 2$.

Our result goes along with the results of many of the old articles, for example, [2—4], as well as more modern ones, for example, [5, 6]. The authors of those articles dealt with the first hit of the interior/exterior of a given *ball* for an α-stable process in R^d, while our result concerns the first hit of a $(d-1)$-dimensional *surface* in R^d (a hyperplane). The hitting time τ introduced above coincides with the instant of time, when the one-dimensional process $((x(t), \nu))_{t \geqslant 0}$ hits the origin. For this process, the moment τ is the point of continuity.

The main result of this paper is proved in Section 2. Some auxiliary results are expounded in Section 1.

1. Preliminaries.

1.1. One auxiliary result. Note that S is a $(d-1)$-dimensional subspace of R^d, so that $R^d = S \times R^1$. We denote the Lebesgue measure in S by the same symbol as that in R^d. For a given function $(\varphi(x))_{x \in R^d}$, the integral $\int_S \varphi(y)\,dy$ can be considered as a surface integral of φ over S.

Lemma 1. *For $t > 0$, $x \in R^d$, and $\xi \in S$, the equality*

$$\int_S g(t, x, y) e^{i(y, \xi)} dy = e^{i(\tilde{x}, \xi)} \frac{1}{\pi} \int_0^\infty e^{-ct(|\xi|^2 + \rho^2)^{\alpha/2}} \cos(\rho(x, \nu))\, d\rho \tag{4}$$

holds.

Proof. Formula (1) implies that the equality

$$\int_S \int_{R^1} \exp\{i(y, \xi) + i\rho\zeta\}\, g(t, x, y + \zeta\nu)\, dy\, d\zeta = \exp\{i(\tilde{x}, \xi) + i\rho(x, \nu) - ct(|\xi|^2 + \rho^2)^{\alpha/2}\}$$

is valid for all $t > 0$, $x \in R^d$, $\rho \in R^1$, and $\xi \in S$. Integrating both sides of this equality with respect to $\rho \in [-M, M]$, where $M > 0$ is an arbitrary number, we get

$$\int\limits_{R^1}\left[\int\limits_S e^{i(y,\xi)}g(t,x,y+\zeta\nu)dy\right]\frac{\sin(M\zeta)}{\zeta}d\zeta = e^{i(\tilde{x},\xi)}\int\limits_0^M \exp\{-ct(|\xi|^2+\rho^2)^{\alpha/2}\}\cos(\rho(x,\nu))d\rho. \quad (5)$$

Note that, for fixed $t>0$, $x\in R^d$, and $\xi\in S$, the function

$$\left(\int\limits_S e^{i(y,\xi)}g(t,x,y+\zeta\nu)dy\right)_{\zeta\in R^1}$$

is continuous and absolutely integrable over R^1. Therefore, according to the Fourier integral formula (see [7], Ch. II, § 8), we can write down the relation ($t>0$, $x\in R^d$, $\xi\in S$)

$$\lim_{M\to+\infty}\int\limits_{R^1}\left[\int\limits_S e^{i(y,\xi)}g(t,x,y+\zeta\nu)dy\right]\frac{\sin(M\zeta)}{\zeta}d\zeta = \pi\int\limits_S e^{i(y,\xi)}g(t,x,y)dy.$$

Now, passing to the limit in (5), as $M\to+\infty$, we get formula (4). The lemma has been proved.

Remark. A proof of formula (4) different from that given above can be found in our paper [8].

1.2. The resolvent kernel. The following estimate is a simple consequence of Theorem 2.1 in [9]. For all $t>0$, $x\in R^d$, and $y\in R^d$, the inequality

$$g(t,x,y)\leqslant N\frac{t}{(t^{1/\alpha}+|y-x|)^{d+\alpha}} \quad (6)$$

holds true with some constant $N>0$. Similar estimations in much more general situations including the estimates for (fractional) derivatives of g were established in [10], Ch. IV and V.

Now, we observe that, for any $a>0$, the integral $\int\limits_0^\infty t(t^{1/\alpha}+a)^{-(d+\alpha)}dt$ is finite (remember that $d\geqslant 2$ and $\alpha\in(1,2)$). Hence, according to estimate (6), the function $\tilde{g}(\lambda,x,y)=\int\limits_0^\infty e^{-\lambda t}g(t,x,y)dt$, $\lambda>0$, $x\in R^d$, $y\in R^d$, is well-defined (it has a singularity at the points $x=y$). It is called the resolvent kernel for our process.

As a simple consequence of Lemma 1, we have the formula

$$\int\limits_S e^{i(y,\xi)}\tilde{g}(\lambda,x,y)dy = e^{i(\tilde{x},\xi)}\frac{1}{\pi}\int\limits_0^\infty \frac{\cos(\rho(x,\nu))}{\lambda+c(|\xi|^2+\rho^2)^{\alpha/2}}d\rho \quad (7)$$

valid for all $\lambda>0$, $x\in R^d$, and $\xi\in S$.

1.3. The local time on S. The following relations are simple consequences of (4)

$$\int\limits_S g(t,x,y)dy = \frac{1}{\pi}\int\limits_0^\infty e^{-ct\rho^\alpha}\cos(\rho(x,\nu))d\rho \leqslant \frac{1}{\pi}\int\limits_0^\infty e^{-ct\rho^\alpha}d\rho = \frac{\Gamma(1/\alpha)}{\pi\alpha c^{1/\alpha}}t^{-1/\alpha}. \quad (8)$$

Therefore, $\int\limits_0^t d\theta\int\limits_S g(\theta,x,y)dy\leqslant \text{const}\cdot t^{1-1/\alpha}$. According to Theorem 6.6 from [1], there exists an additive continuous homogeneous functional $(\eta_t)_{t\leqslant 0}$ of the process $(x(t))_{t\geqslant 0}$ such that its

values are non-negative, and the equality $E_x\eta_t = \int_0^t d\theta \int_S g(\theta, x, y)\,dy$ is valid for all $t \geqslant 0$ and $x \in R^d$. This functional is called the local time on S for the process $(x(t))_{t\geqslant 0}$.

Denote by $q_t(x)$ for $t>0$ and $x\in R^d$ the left-hand side of (8). For fixed $h>0$, the function $(q_h(x))_{x\in R^d}$ is continuous and bounded. So, the functional $\eta_t^{(h)} = \int_0^t q_h(x(s))\,ds$, $t \geqslant 0$, of the process $(x(t))_{t\geqslant 0}$ is well-defined. As was shown in [11], the relation $\lim\limits_{h\to 0+} E_x(\eta_t^{(h)} - \eta_t)^2 = 0$ holds true. It is clear that $q_h(x) \to \delta_S(x)$, as $h \to 0+$, where δ_S is a generalized function on R^d, whose action on a test function $(\varphi(x))_{x\in R^d}$ is given by $\langle \delta_S, \varphi\rangle = \int_S \varphi(x)\,dx$.

We have thus arrived at the conclusion that the trajectories of $(\eta_t)_{t\geqslant 0}$ are increasing at those moments t of time, for which $x(t)\in S$, and those trajectories are continuous.

Now, one can easily verify that, for any continuous bounded function $(\varphi(x))_{x\in R^d}$, the Stieltjes integral $\int_0^t \varphi(x(s))\,d\eta_s$ is well-defined for $t \geqslant 0$, and the equality $E_x \int_0^t \varphi(x(s))\,d\eta_s = \int_0^t d\theta \int_S g(\theta, x, y)\varphi(y)\,dy$ holds true for all $t \geqslant 0$ and $x\in R^d$. As a consequence, we have the relation

$$E_x \int_0^\infty e^{-\lambda t}\varphi(x(s))\,d\eta_t = \int_S \tilde{g}(\lambda, x, y)\varphi(y)\,dy \tag{9}$$

valid for all $\lambda > 0$, $x\in R^d$, and any continuous bounded function $(\varphi(x))_{x\in R^d}$.

2. The distribution of $x(\tau)$.

2.1. The stopping time τ is finite P_x -a.s. For $\lambda>0$, $x\in R^d$, and $\xi\in S$, we put $u(\lambda, x, \xi) = E_x \int_0^\infty e^{-\lambda t} e^{i(x(t),\xi)}\,d\eta_t$. Since $\eta_t = 0$ for $t \leqslant \tau$, one can write down

$$u(\lambda, x, \xi) = E_x \int_\tau^\infty e^{-\lambda t} e^{i(x(t),\xi)}\,d\eta_t = E_x e^{-\lambda\tau} 1_{\{\tau<+\infty\}} \theta_\tau \left(\int_0^\infty e^{-\lambda t + i(x(t),\xi)}\,d\eta_t \right),$$

where $(\theta_t)_{t\geqslant 0}$ is the semigroup of shift operators associated with $(x(t))_{t\geqslant 0}$ (see [1], Ch. 3). Making use of the property of our process to be a strong Markovian one, we get

$$u(\lambda, x, \xi) = E_x e^{-\lambda\tau} 1_{\{\tau<+\infty\}} E_{x(\tau)} \left(\int_0^\infty e^{-\lambda t + i(x(t),\xi)}\,d\eta_t \right) = E_x e^{-\lambda\tau} 1_{\{\tau<+\infty\}} u(\lambda, x(\tau), \xi).$$

We now put $\pi_\lambda(x, \Delta) = E_x(e^{-\lambda\tau} 1_\Delta(x(\tau)) 1_{\{\tau<+\infty\}})$ for $\lambda>0$, $x\in R^d$, and any measurable set $\Delta \subseteq S$. Then the previous relation can be rewritten in the form $u(\lambda, x, \xi) = \int_S u(\lambda, y, \xi) \times \pi_\lambda(x, dy)$, $\lambda>0$, $x\in R^d$, $\xi\in S$.

On the other hand, according to Lemma 1 and formulae (7) and (9), we have the relation $u(\lambda, x, \xi) = \frac{1}{\pi} e^{i(\tilde{x}, \xi)} \int_0^\infty \frac{\cos(\rho(x, \nu))}{\lambda + c(\rho^2 + |\xi|^2)^{\alpha/2}} d\rho$ valid for all $\lambda > 0$, $x \in R^d$, and $\xi \in S$.

We have thus proved the following assertion.

Lemma 2. *For all $\lambda > 0$, $x \in R^d$, and $\xi \in S$, the equality*

$$\int_S e^{i(y, \xi)} \pi_\lambda(x, dy) \int_0^\infty \frac{d\rho}{\lambda + c(\rho^2 + |\xi|^2)^{\alpha/2}} = e^{i(x, \xi)} \int_0^\infty \frac{\cos(\rho(x, \nu))}{\lambda + c(\rho^2 + |\xi|^2)^{\alpha/2}} d\rho \tag{10}$$

holds true.

Now, put $\xi = 0$ in (10). Taking into account that $\int_S \pi_\lambda(x, dy) = E_x e^{-\lambda\tau}$ for $\lambda > 0$ and $x \in R^d$ (we believe that $e^{-\lambda\tau} = 0$ on the set $\{\tau = +\infty\}$), we get $E_x e^{-\lambda\tau} = \kappa \lambda^{1-1/\alpha} \frac{1}{\pi} \int_0^\infty \frac{\cos(\rho(x, \nu))}{\lambda + c\rho^\alpha} d\rho$, where κ is the constant defined in Introduction. Formula (2) is a simple consequence of this equality. Letting $\lambda \to 0+$ in (2), we arrive at the conclusion that $P_x(\{\tau < +\infty\}) \equiv 1$. So, in the definition of π_λ, we can omit the indicator $1_{\{\tau < +\infty\}}$.

2.2. An explicit formula for the distribution of $x(\tau)$. Let us return to formula (10). Putting $\pi_0(x, \Delta) = \lim_{\lambda \to 0+} \pi_\lambda(x, \Delta) = P_x(\{x(\tau) \in \Delta\})$ for $x \in R^d$ and any measurable $\Delta \subseteq S$, we obtain the relation

$$\int_S e^{i(\xi, y)} \pi_0(x, dy) = e^{i(\tilde{x}, \xi)} \kappa_1 \int_0^\infty \frac{\cos(\rho |\xi| (x, \nu))}{(1 + \rho^2)^{\alpha/2}} d\rho \tag{11}$$

valid for all $x \in R^d$ and $\xi \in S$, where $\kappa_1 = \left(\int_0^\infty \frac{d\rho}{(1 + \rho^2)^{\alpha/2}} \right)^{-1} = \frac{2}{\sqrt{\pi}} \frac{\Gamma(\alpha/2)}{\Gamma((\alpha - 1)/2)}$.

We are now going to find out an explicit formula for the density of the measure $\pi_0(x, \cdot)$ with respect to the Lebesgue measure in S. Denote this density by $\pi_0(x, y)$ for $x \in R^d \setminus S$ and $y \in S$. Our task now is to derive formula (3) from relation (11).

The integral on the right-hand side of (11) can be written as follows

$$\int_0^\infty \frac{\cos(\rho |\xi| (x, \nu))}{(1 + \rho^2)^{\alpha/2}} d\rho = \frac{\sqrt{\pi} (|\xi| \cdot |(x, \nu)|)^{(\alpha - 1)/2}}{2^{(\alpha - 1)/2} \Gamma(\alpha/2)} K_{\frac{\alpha - 1}{2}}(|\xi| \cdot |(x, \nu)|)$$

in accordance with formula (18) of § 16, Ch. III in [7], where

$$K_{\frac{\alpha - 1}{2}}(\zeta) = \frac{1}{2} \int_{-\infty}^\infty e^{-\zeta \cosh t - t(\alpha - 1)/2} dt \quad (\mathrm{Re}\, \zeta > 0)$$

is a modified Bessel function of the second type and the order $(\alpha - 1)/2$ (see, for example, [12]).

We have thus obtained the equality

$$\int\limits_S e^{i(\xi,y)}\,\pi_0(x,dy)=\frac{2^{(3-\alpha)/2}\,|\xi|^{(\alpha-1)/2}\,|(x,\nu)|^{(\alpha-1)/2}}{\Gamma\big((\alpha-1)/2\big)}\,e^{i(\tilde{x},\xi)}\,K_{\frac{\alpha-1}{2}}(|\xi|\cdot|(x,\nu)|) \tag{12}$$

valid for all $x\in R^d\setminus S$ and $\xi\in S$.

The properties of the function $K_{(\alpha-1)/2}$ (see [12], Ch.7) provide the function on the right-hand side of (12) to be absolutely integrable with respect to ξ over S. Therefore, the density $\pi_0(x,y)$ for $x\in R^d\setminus S$ and $y\in S$ does exist, and it can be written as an inverse Fourier transform

$$\pi_0(x,y)=\frac{1}{(2\pi)^{d-1}}\,\frac{2^{(3-\alpha)/2}\,|(x,\nu)|^{(\alpha-1)/2}}{\Gamma((\alpha-1)/2)}\int\limits_S e^{i(\tilde{x}-y,\xi)}\,|\xi|^{(\alpha-1)/2}\,K_{\frac{\alpha-1}{2}}(|\xi|\cdot|(x,\nu)|)\,d\xi. \tag{13}$$

Let $d>2$. Denote by $B_r(z)$ a ball in R^{d-1} of radius r and center $z\in R^{d-1}$, and by $\partial B_r(z)$ its boundary. Using the well-known Catalan formula, one can write down the relation $\int\limits_{\partial B_1(0)} e^{i(\tilde{x}-y,\xi)r}\,d\sigma_\xi=\frac{(2\pi)^{(d-1)/2}}{(r\,|\tilde{x}-y|)^{(d-3)/2}}\,J_{\frac{d-3}{2}}(r\,|\tilde{x}-y|)$, $r>0$ (the integral on the left-hand side here is a surface one), where $J_\mu(\zeta)$ is a Bessel function of the order μ: $J_\mu(\zeta)=$ $=\frac{(\zeta/2)^\mu}{\sqrt{\pi}\,\Gamma(\mu+1/2)}\int\limits_{-1}^{1}(1-\theta^2)^{\mu-1/2}\cos(\zeta\theta)\,d\theta$ ($\mathrm{Re}\,\mu>-1/2$). Hence, we get the following relation

$$\pi_0(x,y)=\frac{1}{(2\pi)^{(d-1)/2}}\,\frac{2^{(3-\alpha)/2}\,|(x,\nu)|^{(\alpha-1)/2}}{|\tilde{x}-y|^{(d-3)/2}\,\Gamma((\alpha-1)/2)}\times$$

$$\times\int\limits_0^\infty r^{(d+\alpha)/2-1}K_{\frac{\alpha-1}{2}}(r\,|(x,\nu)|)\,J_{\frac{d-3}{2}}(r\,|\tilde{x}-y|)\,dr.$$

In accordance with formula (39) of 7.14.2 in [12], we have

$$\int\limits_0^\infty r^{(d+\alpha)/2-1}K_{\frac{\alpha-1}{2}}(r\,|(x,\nu)|)\,J_{\frac{d-3}{2}}(r\,|\tilde{x}-y|)\,dr=$$

$$=\frac{(2\,|\tilde{x}-y|)^{(d-3)/2}\,(2\,|(x,\nu)|)^{(\alpha-1)/2}\,\Gamma\left((d+\alpha)/2-1\right)}{(|\tilde{x}-y|^2+(x,\nu)^2)^{(d+\alpha)/2-1}}.$$

It is not a difficult exercise to verify that these formulae are also valid in the case of $d=2$. A very simple calculation allows us to formulate now the following statement.

Theorem. *For all $x\in R^d\setminus S$ and $y\in S$, the formula*

$$\pi_0(x,y)=\frac{\Gamma((d+\alpha)/2-1)}{\pi^{(d-1)/2}\Gamma((\alpha-1)/2)}\,\frac{|(x,\nu)|^{\alpha-1}}{\left[(x,\nu)^2+|y-\tilde{x}|^2\right]^{(d+\alpha)/2-1}}$$

holds true.

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ПРО РОЗПОДІЛ РОТАЦІЙНО ІНВАРІАНТНОГО α-СТІЙКОГО ПРОЦЕСУ В МОМЕНТ ПЕРШИХ ВІДВІДИН НИМ ЗАДАНОЇ ГІПЕРПЛОЩИНИ

Ми знаходимо явну формулу для розподілу ротаційно інваріантного α-стійкого процесу в момент часу, коли він вперше навідується до заданої гіперплощини. Розглядається випадок $1 < \alpha \leqslant 2$.

Ключові слова: *α-стійкий процес, момент досягнення, резольвентне ядро, локальний час.*

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О РАСПРЕДЕЛЕНИИ РОТАЦИОННО ИНВАРИАНТНОГО α-УСТОЙЧИВОГО ПРОЦЕССА В МОМЕНТ ПЕРВОГО ПОПАДАНИЯ ЕГО НА ЗАДАННУЮ ГИПЕРПЛОСКОСТЬ

Находим явную формулу для распределения ротационно инвариантного α-устойчивого процесса в момент времени, когда он впервые посещает заданную гиперплоскость. Рассматривается случай $1 < \alpha \leqslant 2$.

Ключевые слова: *α-устойчивый процесс, момент достижения, резольвентное ядро, локальное время.*