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On the role played by anticommutativity in Leibniz algebras

Presented by Academician of the NAS of Ukraine A.M. Samoilenko

Lie algebras are exactly the anticommutative Leibniz algebras. We conduct a brief analysis of the approach to Leibniz algebras which is based on the concept of anticenter (Lie-center) and antinilpotency (Lie nilpotentency).

Keywords: *Leibniz algebra, Lie algebra, center, Lie-center, anticenter, central series, anticentral series, Lie-central series.*

Let L be an algebra over a field F with the binary operations + and [,]. Then L is called a *Leibniz algebra* (more precisely, a *left Leibniz algebra)* if it satisfies the Leibniz identity

$$[a, [b, c]] = [[a, b], c] + [b, [a, c]] \text{ for all } a, b, c \in L.$$

If L is a Lie algebra, then L is a Leibniz algebra. Conversely, if L is a Leibniz algebra such that $[a, a] = 0$ for each element $a \in L$, then L is a Lie algebra. Therefore, Lie algebras can be characterized as the Leibniz algebras in which $[a, a] = 0$ for every element a. In other words, Lie algebras can be described as anticommutative Leibniz algebras.

The following analogy comes up:

{Abelian groups} ⇔ {Lie algebras} and
{non-Abelian groups} ⇔ {Leibniz algebras}.

It is immediately understandable that such an analogy cannot be sufficiently deep, since the properties of commutativity and anticommutativity differ significantly (they coincide in the case of algebras over a field of characteristic 2). In this, we convince ourselves by looking at cyclic subgroups in groups and cyclic subalgebras in Leibniz algebras. Cyclic subgroups in an arbitrary group are commutative, while cyclic subalgebras in Leibniz algebras do not generally possess anticommutativity, as can be seen from their description given in [1].

A Leibniz algebra L has one specific ideal. Denote, by Leib(L), the subspace generated by the elements [a, a], $a \in L$. It is possible to prove that Leib(L) is an ideal of L. Moreover, $L/$Leib(L) is a Lie algebra. Conversely, if H is an ideal of L such that L/H is a Lie algebra, then Leib$(L) \leqslant H$. The ideal Leib(L) is called the *Leibniz kernel* of the algebra L.

We can consider the Leibniz kernel as an analog of the derived subgroup in a group. To this analogy, we will come again later. So, the difference between Lie algebras and Leibniz algebras is that they have a non-zero Leibniz kernel, just as the difference between Abelian groups and non-Abelian groups is in the presence of non-trivial derived subgroups in the latter.

Let us try to continue the analogy with the theory of groups. Along with the derived subgroup in G, there is another characteristic subgroup, namely, its center $\zeta(G)$, that is, the set of all elements z such that $zg = gz$ for each element $g \in G$. Taking into account the fact that the difference between Leibniz algebras and Lie algebras consists in the absence of anticommutativity, we naturally come to the next object in Leibniz algebras.

Let L be a Leibniz algebra. Put

$$\alpha(L) = \{z \in L \mid [a, z] = -[z, a] \text{ for every elements } a \in L\}.$$

This subset is called ***the anticenter*** of a Leibniz algebra L.

Clearly, the anticenter is a subspace of L. It is also a subalgebra of L. Indeed, let $z, y \in \alpha(L)$ and a be an arbitrary element of L. Then

$$[[z, y], a] = [z, [y, a]] - [y, [z, a]] = -[z, [a, y]] + [y, [a, z]] =$$
$$= -[z, [a, y]] - [[a, z], y] = -([[a, z], y] + [z, [a, y]]) = -[a, [z, y]].$$

Moreover, the anticenter is an ideal of L. In fact, let $z \in \alpha(L)$, and let a be an arbitrary element of A. For every element $b \in A$, we have

$$[[z, a], b] = [z, [a, b]] - [a, [z, b] = -[[a, b], z] + [a, [b, z]] =$$
$$-[[a, b], z] + [[a, b], z] + [b, [a, z]] = [b, [a, z]] = -[b, [z, a]],$$

$$[[a, z], b] = [a, [z, b]] - [z, [a, b] = -[a, [b, z]] + [[a, b], z] =$$
$$-[a, [b, z] + [a, [b, z]] - [b, [a, z]] = -[b, [a, z]].$$

Note that, in [2], the Lie-center term for a Leibniz algebra is used. However, the property of anticommutativity is inherent not only to Lie algebras. Therefore, instead of the Lie-center term, it seems preferable to us to use a more general term: anticenter.

Note also that if char(F) = 2, then the anticenter of Leibniz algebra coincides with the set

$$\{z \in L \mid [a, z] = [z, a] \text{ for every elements } a \in L\}.$$

This set, in general, is not an ideal. Therefore, it is worthwhile to use the considerations related to the anticenter over the field F such that char$(F) \neq 2$. Here, we assume that char$(F) \neq 2$.

In a Leibniz algebra L, the concept of a center is introduced as follows: The center $\zeta(L)$ is the set of all elements z such that $[z, x] = [x, z] = 0$ for an arbitrary element $x \in L$. Clearly, the center is an ideal of L. The following concept is naturally connected with the center.

Let L be a Leibniz algebra over a field F, M be non-empty subset of L, and H be a subalgebra of L. Put

$$\mathrm{Ann}_{\mathrm{H}}(M) = \{a \in H \mid [a, M] = \langle 0 \rangle = [M, a]\}.$$

The subset $\mathrm{Ann}_H(M)$ is called the *annihilator* or the *centralizer* of M in the subalgebra H. It is not hard to see that the subset $\mathrm{Ann}_H(M)$ is a subalgebra of H. If M is an ideal of L, then $\mathrm{Ann}_{\mathrm{H}}(M)$ is also an ideal of L. The center of L is the intersection of the annihilators of all elements of L. This leads us to the following concept.

Let L be a Leibniz algebra over a field F, M be a non-empty subset of L, and H be a subalgebra of L. Put

$$\mathbf{AC}_H(M) = \{a \in H \mid [a, u] = -[u, a] \text{ for all } x \in M\}.$$

The subset $\mathbf{AC}_H(M)$ is called the *anticentralizer* of M in the subalgebra H. It is clear that the anticenter of L is the intersection of the anticentralizers of all elements of L. But, on this, all the good ends. Unlike an annihilator, the anticentralizer of a subset is not always a subalgebra, so an anticentralizer can no longer be such a good technical tool as a centralizer. This can be seen from the following example.

Example 1. Let F be an arbitrary field, and let L be a vector space over F having a basis $\{a, b, c , d\}$. Define the operation [,] on the elements of the basis by the rule

$$[d, a] = -c, [d, b] = b + c, [d, c] = -b, [a, d] = b + c, [b, d] = [c, d] = 0, [d, d] = b,$$
$$[a, a] = b, [a, b] = c, [a, c] = -b - c, [c, a] = [b, a] = [b, c] = [c, b] = 0$$

and expand it in a natural way on all elements of L. It is possible to prove that L becomes a Leibniz algebra over F. If $x = -2d + a$, $\rho = 0$, $\beta = 1$, then the elements $x = -2d + a$ and $y = d + b + c$ belong to $\mathbf{AC}_L(a)$, but $[x, y] = -2b - c \notin \mathbf{AC}_L(a)$.

In Leibniz algebras, the derived ideal $[L, L]$ generated by all elements $[x, y]$, $x, y \in L$ is dual to the center. From our analogy, we can consider $\alpha(L)$ as an analog of the center, while a subspace ◖L, L◗, generated by all elements ◖x, y◗ $= [x, a] + [a, x]$, $x, a \in L$, can be considered as an analog of the derived subgroup. At once, we remark that this subspace is an ideal. Moreover, if $x, y, z \in L$, then the element $[[x, a] + [a, x], y] = 0$ for every element $y \in L$. Indeed,

$$[[x, y] + [y, x], z] = [[x, y], z] + [[y, x], z] = [[x, [y, z]] \ 2{,}5- [y, [x, z]] + [y, [x, z]] - [x, [y, z]] = 0.$$

Further,

$$[z, [x, y] + [y, x]] = [z, [x, y]] + [z, [y, x]] = [[z, x], y] + [x, [z, y]] + [[z, y], x] + [y, [z, x]] =$$
$$= ([[z, x], y] + [y, [z, x]]) + ([x, [z, y]] + [[z, y], x]).$$

On the other hand, $[a, a] + [a, a] = 2[a, a] \in$ ◖L, L◗, and $\mathrm{char}(F) \neq 2$ implies that $[a, a] \in$ ◖L, L◗, so that $\mathrm{Leib}(L) \leqslant$ ◖L, L◗. Since $L/\mathrm{Leib}(L)$ is a Lie algebra, $[x, a] + [a, x] \in \mathrm{Leib}(L)$, so that Leib(L) = ◖L, L◗. Thus, with this approach, the Leibniz kernel is dual to the anticenter. In this connection, it is useful to recall the presence of another important ideal in Leibniz algebras, namely, the left center. If L is a Leibniz algebra, then we put

$$\zeta^{\mathrm{left}}(L) = \{x \in L \mid [x, y] = 0 \text{ for each element } y \in L\}.$$

It is possible to prove that $\zeta^{\text{left}}(L)$ is an ideal of L and $\text{Leib}(L) \leqslant \zeta^{\text{left}}(L)$.

Starting from the anticenter, we define the upper anticentral series

$$\langle 0 \rangle = \alpha_0(L) \leqslant \alpha_1(L) \leqslant \alpha_2(L) \leqslant \ldots \alpha_\lambda(L) \leqslant \alpha_{\lambda+1}(L) \leqslant \ldots \alpha_\gamma(L) = \alpha_u(L)$$

of a Leibniz algebra L by the following rule: $\alpha_1(L) = \alpha(L)$ is the anticenter of L, and, recursively, $\alpha_{\lambda+1}(L)/\alpha_\lambda(L) = \alpha(L/\alpha_\lambda(L))$ for all ordinals λ, and $\alpha_\mu(L) = \cup_{\nu<\mu}\, \alpha_\nu(L)$ for the limit ordinals μ. By definition, each term of this series is an ideal of L. The last term $\alpha_\infty(L)$ of this series is called the *upper hyperanticenter* of L. A Leibniz algebra L is said to be *hyperanticentral* if it coincides with the upper hypercenter. Denote, by $\text{al}(L)$, the length of upper central series of L. If L is hyperanticentral and $\text{al}(L)$ is finite, then L is said to be *antinilpotent*.

Let A, B be the ideals of L such that $B \leqslant A$. The factor A/B is called *anticentral*, if $A/B \leqslant \leqslant \alpha(L/B)$. By definition, the factor A/B is anticentral if and only if $[x, a] + [a, x] \in B$ for each a $\in A$ and each $x \in L$.

If U, V are the ideals of L, then we denote, by ◖U, V◗, a subspace generated by all elements $[u, v] + [v, u]$, $u \in U$, $v \in V$. As we have seen above, $[u, v] + [v, u] \in \zeta^{\text{left}}(L)$. Using the above arguments, can show that ◖U, V◗ is an ideal of L.

You can immediately note that a factor A/B is anticentral if and only if ◖L, A◗ $\leqslant B$.

Now, we can introduce an analog of the lower central series. Define the lower anticentral series of L,

$$L = \kappa_1(L) \geqslant \kappa_2(L) \geqslant \ldots \kappa_\alpha(L) \geqslant \kappa_{\alpha+1}(L) \geqslant \ldots \kappa_\delta(L),$$

by the following rule: $\kappa_1(L) = L$, $\kappa_2(L) =$ ◖L, L◗, and, recursively, $\kappa_{\lambda+1}(L) =$ ◖L, $\kappa_\lambda(L)$◗ for all ordinals λ and $\kappa_\mu(L) = \cap_{\nu<\mu}\, \kappa_\nu(L)$ for the limit ordinals μ. The last term $\kappa_\delta(L)$ is called the *lower hypoanticenter* of L. We have $\kappa_\delta(L) =$ ◖L, $\kappa_\delta(L)$◗.

As we have seen above, $\kappa_2(L) =$ ◖L, L◗ $= \text{Leib}(L) = \text{K}$. Furthermore, $\kappa_3(L) =$ ◖L, $\kappa_2(L)$◗. If $x \in L$, $a \in K = \kappa_2(L)$, then ◖x, a◗ $= [x, a] + [a, x] = [x, a]$, because $\text{Leib}(L) \leqslant \zeta^{\text{left}}(L)$. It follows that $\kappa_3(L) = [L, \kappa_2(L)] = [L, \text{Leib}(L)]$.

If A is an ideal of L, then we put $\gamma_{\text{L}\,1}(A) = A$, $\gamma_{L\,2}(A) = [L, A]$, and, recursively, $\gamma_{L\,n+1}(A) = [L, \gamma_{\text{L}\,n}(A)]$ for all positive integers n.

Thus, we obtain $\kappa_1(L) = L$, $\kappa_2(L) = \text{Leib}(L)$, $\kappa_3(L) = \gamma_{\text{L}\,2}(\text{Leib}(L))$, $\kappa_{n+1}(\text{L}) = \gamma_{L\,n}(\text{Leib}(L))$ for all positive integers n.

Suppose now that L has a finite series of ideals

$$\langle 0 \rangle = A_0 \leqslant A_1 \leqslant A_2 \leqslant \ldots \leqslant A_n = L.$$

This series is said to be *anticentral*, if every factor A_j/A_{j-1} is anticentral, $1 \leqslant j \leqslant n$.

Proposition 1. *Let L be an Leibniz algebra over a field F and*

$$\langle 0 \rangle = C_0 \leqslant C_1 \leqslant \ldots \leqslant C_n = L$$

be a finite anticentral series of L. Then

(i) $\kappa_j(L) \leqslant C_{n-j+1}$, *so that* $\kappa_{n+1}(L) = \langle 0 \rangle$.

(ii) $C_j \leqslant \alpha_j(L)$, *so that* $\alpha_n(L) = L$.

These statements were proved in [2] for right Leibniz algebras; for left Leibniz algebras, the proof is similar.

Corollary. *Let L be an antinilpotent Leibniz algebra. Then the length of the lower anticentral series coincides with the length of the upper anticentral series. Moreover, the length of these two series is the smallest among the lengths of all anticentral series of L.*

The length of the upper anticentral series (or lower anticentral series) is called *the class of antinilpotency of a Leibniz algebra L* and is denoted by $\mathrm{ancl}(L)$. Note that, in [2], a Lie-nilpotent algebra and the class of Lie-nilpotency are considered. However, the concept of Lie-nilpotency arose much earlier in the theory of associative rings. So, in order to avoid confusion, it is better to use another term. In addition, as we have already noted, the property of anticommutativity is inherent not only in Lie algebras. Therefore, we focus on it.

Note some properties of hyperanticentral Leibniz algebras.

Proposition 2. *Let* $\{ L_\lambda \mid \lambda \in \Lambda \}$ *be a family of Leibniz algebra over a field F.*

(i) If n is a positive integer, then $\alpha_n(\mathrm{Cr}_{\lambda \in \Lambda} L_\lambda) = \mathrm{Cr}_{\lambda \in \Lambda}\, \alpha_n(L_\lambda)$.

(ii) If ω *is a first infinite ordinal, then* $\alpha_\omega(\mathrm{Cr}_{\lambda \in \Lambda} L_\lambda) \leqslant \mathrm{Cr}_{\lambda \in \Lambda}\, \alpha_\omega(L_\lambda)$.

(iii) If μ *is an arbitrary ordinal, then* $\alpha_\mu(\oplus_{\lambda \in \Lambda} L_\lambda) = \oplus_{\lambda \in \Lambda}\, \alpha_\omega(L_\lambda)$, *in particular, if every algebra* L_λ *is hyperanticentral, then the direct sum* $\oplus_{\lambda \in \Lambda} L_\lambda$ *also is hyperanticentral.*

(iv) If every algebra L_λ *is antinilpotent and there exists a positive integer k such that* $\mathrm{ancl}(L_\lambda) \leqslant k$ *for all* $\lambda \in \Lambda$, *then the Cartesian product* $\mathrm{Cr}_{\lambda \in \Lambda} L_\lambda$ *is also antinilpotent and* $\mathrm{ancl}(\mathrm{Cr}_{\lambda \in \Lambda} L_\lambda) \leqslant k$.

(v) If every algebra L_λ *is antinilpotent and the set* Λ *is finite, then* $\mathrm{Cr}_{\lambda \in \Lambda} L_\lambda = \oplus_{\lambda \in \Lambda} L_\lambda$ *is antinilpotent, moreover,* $\mathrm{ancl}(\oplus_{\lambda \in \Lambda} L_\lambda) \leqslant \max \{\mathrm{ancl}(L_\lambda) \mid \lambda \in \Lambda\}$.

Proposition 3. *Let L be a Leibniz algebra over a field F.*

(i) If L is hyperanticentral, then every subalgebra of L is hyperanticentral and every factor-algebra of L is hyperanticentral.

(ii) If L is antinilpotent, then every subalgebra of L is antinilpotent, and every factor-algebra of L is antinilpotent.

(iii) If A is a non-zero ideal of L such that $A \cap \alpha_\infty(L) \neq \langle 0 \rangle$, *then* $A \cap \alpha\,(L) \neq \langle 0 \rangle$.

(iv) If A, B are antinilpotent ideals of L, then A + B is an antinilpotent ideal of L.

The above properties show a certain analogy between nilpotent and antinilpotent Leibniz algebras. However, this analogy is very shallow. Thus, every principal central factor of a Leibniz algebra L has dimension 1. On the other hand, every principal factor of a Lie algebra is anticentral, but it can have infinite dimension. Further, a finitely generated nilpotent Leibniz algebra has finite dimension [3, Corollary 2.2]. On the other hand, there are finitely generated Lie algebras, which have infinite dimension.

Note the following analog. In work [4, Corollary B1], it was proved that if a center of a Leibniz algebra has finite codimension, then a derived ideal has finite dimension.

Proposition 4. *Let L be a Leibniz algebra over a field F. If the anticenter of L has finite codimension d, then the Leibniz kernel of L has finite dimension at most* d^2.

Proof. Let $A = \alpha(A)$. Then $L = A \oplus \mathrm{B}$ for some subspace B. Choose a basis $\{a_\lambda \mid \lambda \in \Lambda\}$ in A and a basis $\{ b_1, \ldots, b_d \}$ in B. If y is an arbitrary element of L, then $y = a + \Sigma_{1 \leqslant j \leqslant d}\, \beta_\mathrm{j} \mathrm{b}_\mathrm{j}$, where $a \in A$, $\beta_j \in F$, $1 \leqslant j \leqslant d$. We have

$[y, y] = [a + \Sigma_{1 \leqslant j \leqslant d}\, \beta_j b_j, a + \Sigma_{1 \leqslant k \leqslant d}\, \beta_k b_k] =$

$= [a, a] + [a, \Sigma_{1 \leqslant k \leqslant d}\, \beta_k b k_j] + [\Sigma_{1 \leqslant j \leqslant d}\, \beta_j b_j, a] + [\Sigma_{1 \leqslant j \leqslant d}\, \beta_j b_j, \Sigma_{1 \leqslant k \leqslant d}\, \beta_k b_k]$.

Since $a \in \alpha(A)$, $[a, a] = 0$ and $[a, \Sigma_{1 \leqslant k \leqslant d}\, \beta_k b k_j] + [\Sigma_{1 \leqslant j \leqslant d}\, \beta_j b_j, \mathrm{a}] = 0$. So, we obtain

$[y, y] = \Sigma_{1 \leqslant j \leqslant d,\, 1 \leqslant k \leqslant d}\, \beta_j\, \beta_k\, [b_j, b_k]$.

It follows that Leib(L) generates by the elements $\{[b_j, b_k] \mid 1 \leqslant j \leqslant d,\, 1 \leqslant k \leqslant d\}$. In particular, $\dim_F(\mathrm{Leib}(L)) \leqslant d^2$.

We note at once that the converse is not true. The following example justifies this.

Example 2. Let $F = \mathbf{Q}$ be a field of all rational numbers and let L_n be a vector space with a basis $\{ a_n, c_n \}$, $n \in \mathbf{N}$. We can define an operation [,] on L_n assuming that $[a_n, a_n] = c_n$, $[a_n, c_n] = [c_n, a_n] = [c_n, c_n] = 0$ and expanding this operation, using the property of bilinearity, to all elements of L_n. It is not difficult to verify that such a particular operation makes L_n a Leibniz algebra over $\mathbf{Q}$. Moreover, this algebra is nilpotent and $\mathrm{ncl}(L_n) = 2$. It is possible to show that $\zeta(L) = \mathrm{Leib}(L) = \alpha(L)$. Thus, Leib($L$) has a dimension 1, but $\alpha(L) = \mathrm{Leib}(L)$ has infinite codimension.

In conclusion, we give a result arising from another, more familiar analogy. Above, we already noted one of the results of work [4, Corollary B1], which states that if the center of a Leibniz algebra has finite codimension, then the derived ideal has finite dimension. This result is analogous to the next known group-theoretic result.

If the center of a group G has finite index, then the derived subgroup of G is finite.

This theorem first appeared in the work by B.H. Neumann [5]. Nevertheless, very often it is called the Schur theorem (see [6] on this subject). The inversion of this theorem is false both for groups and for Leibniz algebras. Nevertheless, if the derived subgroup of a group is finite, then the second hypercenter of G has finite index [7]. The same situation holds for the Leibniz algebras, as our following result shows.

Theorem. *Let L be a Leibniz algebra over a field F. Suppose that the derived ideal of L has finite dimension d. Then the second hypercenter of L has finite codimension at most* $2d^2(1 + 2d)$.

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ПРО РОЛЬ АНТИКОМУТАТИВНОСТІ В АЛГЕБРАХ ЛЕЙБНІЦА

Алгебри Лі являють собою антикомутативні алгебри Лейбніца. Розглянуто короткий аналіз підходу до алгебри Лейбніца, який базується на концепції антицентра (Лі-центра) та антинільпотентності (Лі-нільпотентності).

Ключові слова: *алгебра Лейбніца, алгебра Лі, центр, Лі-центр, антицентр, центральні ряди, антицентральні ряди, Лі-центральні ряди.*

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О РОЛИ АНТИКОММУТАТИВНОСТИ В АЛГЕБРАХ ЛЕЙБНИЦА

Алгебры Ли представляют собой антикоммутативные алгебры Лейбница. Рассмотрен краткий анализ подхода к алгебре Лейбница, который базируется на концепции антицентра (Ли-центра) и антинильпотентности (Ли-нильпотентности).

Ключевые слова: *алгебра Лейбница, алгебра Ли, центр, Ли-центр, антицентр, центральные ряды, антицентральные ряды, Ли-центральные ряды.*