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Theoretical substantiation of regular system of horizontal drains (a new approach)

The nonlinear problem of regulating the water-physical conditions of over-drained and over-wetted agricultural lands through a regular system of horizontal drains has been formulated and solved by analytical methods. The dynamics of groundwater reserves are analyzed in a generalized manner, rather than locally as done previously. A dependence has been derived to describe the behavior of the water table averaged over the interdrain space, considering a targeted change in head within the drains. Based on this, a formula has been obtained for calculating the optimal drain spacing in both homogeneous and heterogeneous soils, taking into account the requirements for their water-physical state. An assessment was conducted on the uneven distribution of groundwater caused by the local action of horizontal drains. Examples with typical initial data illustrate the decrease in the average water table over time and demonstrate the potential for rarefaction of reclamation drainage using a new methodological approach. This approach will significantly reduce capital costs for its construction and reconstruction.

Keywords: *drainage system, water table, regulation, calculation, spacing, saturated-unsaturated flow, water loss.*

In relation to practical problems of regulating the water resources of agricultural and special-purpose lands over large areas, regular horizontal subsurface drainage systems have demonstrated high efficiency [1, 2]. Depending on the water-physical state of the upper (biologically active) soil layer and the requirements for water conditions, reclamation drainage, with appropriate technical support, can serve two functions: drainage (during early spring or after heavy precipitation) or wetting (during dry periods). The quality of control of the groundwater flow is significantly reduced, even if the design of the drainage system allows you to quickly raise or lower the water table (WT), due to the high hydraulic resistance of natural porous media and, as a consequence, the curvature of the free surface and the uneven distribution of moisture reserves along it. Thus, water-physical conditions near and away from the drains can differ significantly. It is obvious that an objective idea of the total reserves of water available to crops on the territory of the drainage system is provided by the average, and not the local position of the WT. Therefore, the primary objective of this article, and subsequent theoretical studies, is to enhance the methodology for the

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theoretical substantiation of dual-function drainage based on average characteristics of the regulated water-physical state.

To comprehensively assess the consequences of drainage and potential yield reduction due to uneven groundwater distribution between drains, introducing a specialized dispersion index makes sense. This index would integrally characterize the deviation of optimal water-physical conditions in drained soil from ideal conditions (strictly corresponding to existing standards). However, it is necessary to initially correlate the depth of the WT with the yield of the cultivated crop.

The balance of water in the saturated zone (limited by the free surface and impervious barrier) is maintained through the balance between the processes of interzone water exchange and groundwater flow influenced by nearby drains. In an isotropic uniform (or layered) soil, this balance is described in the hydraulic approximation by the equation

$$k_e \frac{\partial^2}{\partial x^2} W(h) = \frac{\partial U}{\partial t}\,, \tag{1}$$

where h is the piezometric head (or WT), k_e is the (effective) hydraulic conductivity, W, U characterize the groundwater resource and moisture reserves in the aeration zone. In the case of hydrodynamic and structural imperfections of drains, jointly taken into account by means of the hydraulic resistance Φ [3—5], the boundary condition is accepted [6]

$$x = 0, \quad W - 2\Phi \frac{\partial W}{\partial x} = W(m_d)\,, \tag{2}$$

where m_d is the head within the drain. The change in the head inside the drains is not taken into account. For a regular drainage system with a drain spacing $2L$, the condition is also accepted

$$x = L, \quad \frac{\partial W}{\partial x} = 0. \tag{3}$$

In establishing the initial condition, given the typically extreme limitation and even uncertainty of information about the initial position of the water table (WT) and moisture content in the aeration zone, as well as the diminishing influence of groundwater flow dependence over time, it is justified to rely on the primary water reserves in the saturated zone. These reserves in the humid zone of Ukraine are the closest and main source of moisture available to plants. In general, they can be characterized by a constant (average) value h^0, so that

$$t = 0, \;\; h = h^0. \tag{4}$$

To identify the patterns of water exchange between the saturated and unsaturated zones of the soil, two fundamentally different methodological approaches have traditionally been used. Implementing a thorough approach requires labor-intensive experimental studies, involving complex nonlinear mathematical models and numerical methods for their solution. It is evident that such an approach is difficult to implement for specific water reclamation objects.

In order to determine the value U strictly, it is necessary to solve, in addition to the groundwater flow problem, the complex nonlinear problem of (vertical) moisture transfer. Previously, we utilized substantially nonlinear soil hydraulic functions (unsaturated conductivity, soil-water retention) for all types of fine-grained mineral soils based on the modern classification, conducting numerous numerical calculations. As a result, it was found that the intensity of water exchange between saturated and unsaturated zones depends primarily on the position of the WT and to a lesser extent on the speed of its movement [7, 8]. This implies that it is legitimate to use the second approach in applications, requiring less initial data, simpler experimental techniques, and allowing the use of analytical methods. Therefore, in engineering developments to regulate the water-physical conditions of lands in the humid zone, it is recommended to use a simplified approach, which is based on the representation

$$\frac{\partial}{\partial t}U(h) \approx \mu_W(h)\frac{\partial h}{\partial t}, \tag{5}$$

where $\mu_W(h)$ exactly under conditions of quasi-stationary moisture transfer and approximately under non-stationary conditions equals $\partial U/\partial h$ and, depending on the functional purpose of the drainage, characterizes either soil saturation (differential water loss) or wetting (lack of saturation). Significant efforts were made to create an appropriate information base, with special attention given to differential water loss or lack of saturation and their averaged analogues [9, 10].

It is crucial to note that errors in the modern techniques arising from the formal simplification of differential water loss techniques were previously theoretically and experimentally assessed, typically falling within the accuracy of experimental methods. However, the feasibility of further developing the applied theory of reclamation drainage in a methodological sense seems evident, considering the scale of land water reclamation and the usual limitation and unreliability of initial information.

The choice of the objective function is fundamentally important when modeling the regulation of the water-physical state against the background of drainage. In view of the close connection between the WT and humidity conditions on the waterlogged lands, it is sufficient to take into account the water-physical conditions and water reserves within the interdrain space, operating, along with the head h, also with two spatially average values (parameters) h_a and U_a, namely,

$$h_a(t) = \frac{1}{L}\int_0^L h(x,t)\,dx, \qquad U_a(t) = \frac{1}{L}\int_0^L U(x,t)\,dx. \tag{6}$$

Moreover, it is proposed to ultimately focus on the characteristics h_a, which allows limiting oneself to a generalized understanding of the main water resource without delving into details at a distance from drains or in their proximity. Then the solution to problem (1)—(3) is represented in the following form

$$W(h) = W(m_d) + \frac{x^2 - 2Lx - 4\Phi L}{2k_e}\frac{dU_a}{dt}. \tag{7}$$

Applying the operator W^{-1} to (7), we obtain

$$h(x,t) = W^{-1}\left[W(m_d) + \frac{x^2 - 2Lx - 4\Phi L}{2k_e}\frac{dU_a}{dt}\right]. \tag{8}$$

Both sides of equality (8) are averaged over x

$$h_a(t)=\frac{1}{L}\int_0^L W^{-1}\left[W(m_d)+\frac{x^2-2Lx-4\Phi L}{2k_e}\frac{dU_a}{dt}\right]dx \tag{9}$$

and then the operator U is applied. The result will be

$$U(h)=U\left\{W^{-1}\left[W(m_d)+\frac{x^2-2Lx-4\Phi L}{2k_e}\frac{dU_a}{dt}\right]\right\}. \tag{10}$$

Averaging expression (10) also over the interdrain space gives

$$U_a=\frac{1}{L}\int_0^L U\left\{W^{-1}\left[W(m_d)+\frac{x^2-2Lx-4\Phi L}{2k_e}\frac{dU_a}{dt}\right]\right\}dx\,. \tag{11}$$

Next, it is formally presented

$$p=\frac{dU_a}{dt}$$

and both sides of equation (11) are differentiated with respect to t. Thus, the following problem is formulated regarding p

$$p=\frac{d}{dp}\Psi(p)\frac{dp}{dt}, \tag{12}$$

$$t=0,\quad p=p_0, \tag{13}$$

where $\Psi(p)=\frac{1}{L}\int_0^L U\left\{W^{-1}\left[W(m_d)+\frac{x^2-2Lx-4\Phi L}{2k_e}\frac{dU_a}{dt}\right]\right\}dx$,

p_0 is determined by selection from equation (9), namely,

$$\frac{1}{L}\int_0^L W^{-1}\left[W(m_d)+\frac{x^2-2Lx-4\Phi L}{2k_e}p_0\right]dx=h^0. \tag{14}$$

Finally

$$t(p)=\int_{p_0}^{p}\frac{d}{d\zeta}\Psi(\zeta)\frac{d\zeta}{\zeta}\;. \tag{15}$$

The representation $W(h)=h_a h$ is considered as an example. Then the expression for h according to (8) will be

$$h(x,t)=m_d+\frac{x^2-2Lx-4L\Phi}{2k_e h_a}\frac{dU_a}{dt}. \tag{16}$$

This implies

$$h_a(t)=\frac{m_d}{2}+\sqrt{\frac{m_d^2}{4}-\frac{L^2+6\Phi L}{3k_e}\frac{dU_a}{dt}}\,. \tag{17}$$

By analogy with (11) we obtained

$$U_a=\frac{1}{L}\int_0^L U\left(m_d+\frac{x^2-2Lx-4L\Phi}{2k_eh_a}\frac{dU_a}{dt}\right)dx. \tag{18}$$

The dependence $t(p)$ has the form (15), where

$$\Psi(p)=U\left(m_d+\frac{x^2-2Lx-4L\Phi}{2k_eh_a(p)}p\right),\quad p_0=-3k_e\frac{h^0(h^0-m_d)}{L^2+6\Phi L}. \tag{19}$$

The calculation scheme is implemented as follows: for a given parameter p, it is sequentially calculated p_0 from (19), t and h_a from (15) and (17), respectively. If necessary, the WT is calculated in accordance with (16).

The main design parameter is determined from the condition [11, 12]

$$t=t_*,\ h_a=M-S_*, \tag{20}$$

where M is the thickness of the soil horizon (from the impervious barrier to the soil surface), S_* is the depth to which it is necessary to lower the WT in time t_*. Great number of works are dedicated to the theoretical substantiation of the drain spacing [13—15]. The indicated values t_*, S_* and the required spacing correspond to the parameter p_*, which is expressed in the following way:

$$p_*(L)=-3k_e\frac{(M-S_*)^2-m_d(M-S_*)}{L^2+6\Phi L}. \tag{21}$$

Expressions (19), (21) are substituted into (15) and, thus, an equation is derived for L

$$\int_{p_0(L)}^{p_*(L)}\frac{d}{d\zeta}\Psi(\zeta,L)\frac{d\zeta}{\zeta}=t_*\,. \tag{22}$$

The fundamental difference in the previously used and new approaches to solving the basic problems of reclamation drainage lies in the interpretation of h_a. While previously, the parameter h_a played an auxiliary role, and its approximate value was established in advance based on drainage conditions and requirements for the favorable water-physical conditions, now this value is initially unknown, and ultimately serves as the main (generalized) indicator of the water-physical state of the soil. Based on the h_a dynamics, the optimal values of the most important design $(2L)$ and technological (head within the drain for wetting) parameters are then justified.

It is possible to significantly simplify the calculation expressions without seriously reducing their accuracy if the water exchange between the saturated and unsaturated zones is approximately described in accordance with (5) as follows

$$\frac{\partial U_a}{\partial t} \approx \mu_W(h_a)\frac{\partial h_a}{\partial t}. \tag{23}$$

Thus, it is assumed that the rate of water exchange mentioned above remains constant throughout the entire interdrain space and only changes due to the average movement of the water table (WT).

The system of equations involving h_a, p, t (15), (17) plays a key role in calculating the effect of regular drainage. In the case of the simplified representation (23), which is traditional for the applied theory of reclamation drainage, the specified system is reduced to a dependence of h_a on t in the form of the following inverse function

$$t = \frac{(L^2 + 4\Phi L)}{3k_e}\int_{h_a}^{h^0} \frac{\mu_W(\zeta)}{\zeta^2 - m_d\zeta}d\zeta. \tag{24}$$

With the known dynamics of the average WT, its actual change between the drains according to (8) is described by the formula

$$h(x,t) = m_d - \frac{3}{2(L^2 + 4\Phi L)}(x^2 - 2Lx - 4L\Phi)[h_a(t) - m_d]. \tag{25}$$

Then, for a given standard regulation period (t_*) and rate (S_*), it is proposed to calculate the optimal spacing L using the formula

$$L = \sqrt{\frac{3k_e t_*}{\int_{M-S_*}^{h^0} \frac{\mu_W(\zeta)}{\zeta^2 - m_d\zeta}} + 9\Phi^2} - 3\Phi. \tag{26}$$

If we operate with a constant value μ_{Wa} (but varying depending on h_a), then expressions (24)—(26) are significantly simplified. Thus, function (24) reduces to this form

$$t = \frac{\mu_{Wa}(h_a)(L^2 + 4\Phi L)}{3k_e m_d}\ln\frac{h_a(h_a^0 - m_d)}{h_a^0(h_a - m_d)}. \tag{27}$$

Then the drain spacing should be found using the formula

$$L = \sqrt{\frac{3k_e m_d t_*}{\mu_{Wa}(M - S_*)\ln\frac{h_a^0(M - S_* - m_d)}{(h_a^0 - m_d)(M - S_*)}} + 9\Phi^2} - 3\Phi. \tag{28}$$

Due to the localization of the influence of drains, the question naturally arises about the consequences of the inevitable and possibly noticeable difference in the depth of the WT for water consumption of crops. A primary representation can be obtained by calculating the total deviation of the WT from its average (optimal) position between the drains. It is proposed to calculate the corresponding dispersion index ΔS_L in relation to drain system as follows:

$$\Delta S_L(t)=\frac{1}{L}\left\{\int_0^{x_*}[h_a(t)-h(x,t)]dx+\int_{x_*}^{L}[h(x,t)-h_a(t)]dx\right\}. \tag{29}$$

where x_* is from the condition $h(x_*,t)=h_a(t)$. In this case, the coordinate x_* does not depend on time and will be

$$x_*=L\left(1-\frac{1}{\sqrt{3}}\right). \tag{30}$$

Then, taking into account the expression for $h(x,t)$ (25), the pattern of reduction of ΔS_L over time follows from (29)

$$\Delta S_L(t)=0{,}075\frac{L}{L+6\Phi}[h_a(t)-m_d]. \tag{31}$$

If it is possible to link ΔS_L to a decrease in crop productivity due to unfavorable water-physical conditions, it becomes feasible to evaluate the decrease in yield resulting from the peculiarities of groundwater flow against the background of regular horizontal drainage.

To illustrate the calculation dependencies derived above and, crucially, to clarify potential savings in capital costs due to transitioning from a local assessment of reclamation drainage efficiency to a generalized assessment, serial calculations were performed. Their subject was the reduction of the WT on average (h_a) and in the middle between drains (h_L), as well as a key design parameter (L). Initially, the coefficient k_e $(1\ m/day$, the structure of the soil horizon was not detailed) was fixed; differential water loss according to available experimental data (Institute of Water Problems and Land Reclamation) was taken in linear form, namely, $\mu_W(h)=0{,}15(M-h)$, and its averaged analogue $\mu_{Wa}(h)=0{,}075(M-h)$. The thickness of the soil horizon varied discretely (1, 2, 4 m). The depth of the drains, that were constructively perfect, at was given 1 and 1,2 m, respectively. Thus, in the first case, the drainage was hydraulically perfect, and in the second and third cases, it was hydrodynamically imperfect. The corresponding values of Φ were calculated according to the recommendations of [4] and amounted to 1 and 3,5 m.

The dynamics of WT were characterized by its decrease in values h_a and h_L. The corresponding graphs were calculated at $M=4,\ m_d=2{,}8$ m and shown in Fig. 1. At each calculation step, the value h_a (or h_L) was specified, and then the corresponding value of t was determined in four ways. In this case, the appropriate values of Φ and μ_{Wa} were previously calculated. From a formal point of view, the solution obtained by averaging the supply of the groundwater flow over the interdrain space seems more reliable. Widely practiced in theoretical developments on reclamation drainage, the identification of differential water loss (or lack of saturation) $\mu_W(h)$ with subsequent reference to the critical section of the groundwater flow, and in the case under consid-

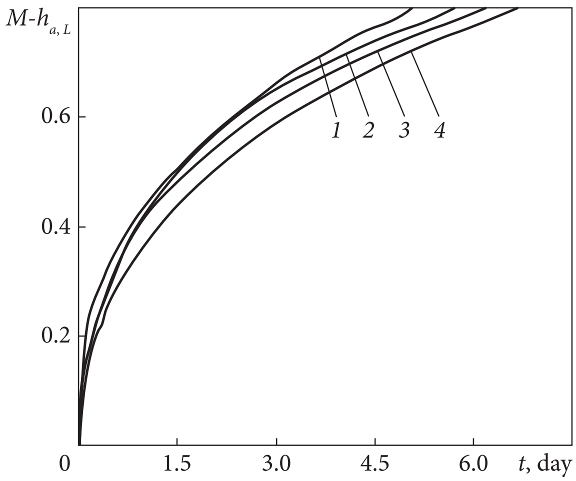

Fig. 1. Decrease in WT on average and in the middle between drains over time: *1—3* — $M-h_a$, *4* — $M-h_L$; *1* — at $\mu_{Wa}(h_a)$ using (27); *2* — using (15), (17); *3* — at $\mu_W(h_a)$ using (24); *4* — at $\mu_{Wa}(h_L)$ according to [6]

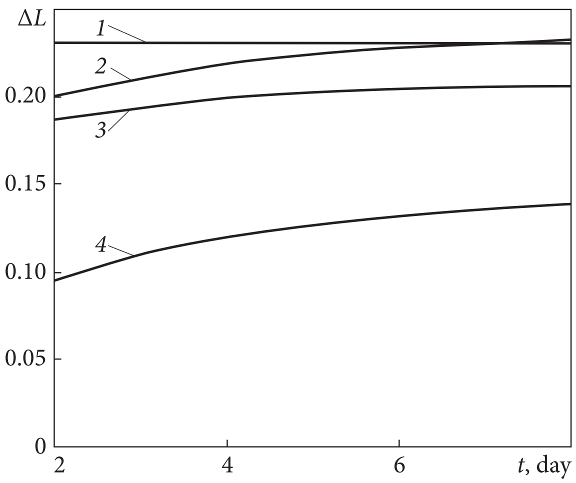

Fig. 2. Dependence $\Delta L(t_*)$: *1*, *2*, *4* — at $\mu_{Wa}(M-S_0)$, L using (28), L_0 according to [6]; *3* — at $\mu_W(M-S_0)$, L using (26), L_0 according to [6]

eration, also with its similar averaging and, finally, additional dynamic averaging (corresponding to $\mu_W(h_L)$, $\mu_W(h_a), \mu_{Wa}(h_a)$) cause an increase in calculating errors. Therefore, the first, two-parameter solution is justified to be considered as the basis for subsequent comparative analysis. The feasibility of such an analysis and the revealing nature of its results follow from methodological considerations. The low position of curve 4 and its slight deviation from the other curves is natural due to the relatively small curvature of the free surface. Such curves are in close proximity to each other and diverge minimally only during prolonged drainage. Particularly noteworthy is the proximity and the intersection of curves *1* and *2* precisely at groundwater depths that provide water-physical conditions favorable for agricultural production in the aeration zone. Therefore, judging by the results of calculations of the drainage effect, it can be recommended to use, along with a two-parameter solution, a significantly simpler solution, which involves dynamic averaging of differential water loss and is expressed by dependencies (23), (24) and formula (25).

In the second series of calculations, the parameter L was calculated as a function of time t_*. In this case, formulas, were based on new (L) and traditional (L_0) approaches, were used to assess assessing the drainage resource of drain system, respectively (22), (24) and from [6]. The results of calculations of this value are presented in Fig. 2 in graphic form. The curves of the dependence $\Delta L(t_*)$, where $\Delta L = (L-L_0)/L_0$, clearly demonstrate the feasibility of correcting the drain spacing determined from the results of the analysis of the water-physical state in the middle between the drains. If we focus exclusively on the total water reserves in both zones and their even distribution between the drains, then a noticeable rarefaction of drainage and, as a consequence, a corresponding reduction in the cost of its design are possible. The indicated savings turn out to be more significant for thin soil horizons and longer drainage time. The value ΔL reached a maximum value of 0,23 in the case of $M=1$ and $t_*=8$ days.

Therefore, the new approach is more conceptual in terms of information since it takes into account the water-physical state, although in general, of the entire of drained (wetting) land.

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ТЕОРЕТИЧНЕ ОБГРУНТУВАННЯ РЕГУЛЯРНОЇ СИСТЕМИ ГОРИЗОНТАЛЬНИХ ДРЕН (НОВИЙ ПІДХІД)

Сформульовано та розв’язано аналітичними методами нелінійну задачу регулювання водного режиму переосушених та перезволожених сільськогосподарських земель регулярною системою горизонтальних дрен. Динаміка запасів ґрунтових вод аналізується узагальнено, а не як раніше локально. Виведено залежність, що описує поведінку середнього за міждренним простором рівня ґрунтових вод при цілеспрямованій зміні напору всередині дрен. На її основі отримано формулу для розрахунку оптимальної відстані між дренами в однорідному і неоднорідному ґрунтах, виходячи з вимог до водного режиму. Виконано оцінку нерівномірності розподілу ґрунтових вод, зумовлену локальною дією горизонтальних дрен. На прикладах із типовими вихідними даними ілюструється зниження осередненого рівня ґрунтових вод з часом, а також показано можливість розрідження меліоративного дренажу завдяки новому методологічному підходу, що дозволить значно економити капітальні витрати на його будівництво та реконструкцію.

Ключові слова: *система дрен, рівень грунтових вод, регулювання, розрахунок, відстань, насичено-ненасичений потік, водовіддача.*