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V.L. Poliakov,
S.M. Kurganska

Institute of Hydromechanics of the NAS of Ukraine, Kyiv, Ukraine
E-mail: v.poliakov.ihm@gmail.com

Modeling of aqueous suspension filtration when combining downward and upward flows

Presented by Corresponding Member of the NAS of Ukraine A.Ya. Oleynik

The physical domain is divided into two subareas of motion, and a nonlinear mathematical problem of water suspension filtration with linear kinetics of interphase detachment mass transfer is formulated with respect to each of them. The structure of gel-like deposit, the dependence of hydraulic conductivity on its concentration, and the relationship of mass transfer coefficients (attachment and detachment) with the filtration rate are taken into account. The corresponding mathematical models contain interconnected clarification and filtration flow compartments. After the introduction of dimensionless variables and parameters, as well as the application of the operational method, rigorous solutions of both problems are obtained. As a result, the most important dependencies and equations were derived for engineering calculations of key characteristics of filtration — concentration of dispersed impurity in filtrate and head losses in distinguished subareas and common in the whole operating layer. The mentioned formalisms were used to determine the main technological times, which limited the time of continuous filter operation due to excessive deterioration of the filtrate quality and mechanical energy consumption for filtration through the clogged medium. As a consequence, the permissible time of its continuous operation (filter run) based on the criteria of effective filtration was established. The similar technological approach to the estimation of the filter performance was applied in parallel to the high-rate filter at the traditional single-flow suspension feeding and the two-flow feeding investigated above. Comparative analysis was performed on test examples with typical initial data for practice of clarification of aqueous suspensions. As a result, it was obtained that the division of the initial flow of suspension into two components coming through the upper and lower bed surfaces can contribute to a significant intensification of the technological process. At the same time, it is realistic to increase the duration of filter runs by 50 % and more, which leads to a tangible decrease in the cost of filtrate. Thus, application of well sorbing filtering materials becomes justified.

Keywords: *filtration, suspension, concentration, dual-flow, filter run, exact solution, head losses.*

Introduction. The cost of the purification of contaminated water significantly depends on the costs of its filtration. Therefore, it is natural that throughout the history of water treatment the intensification of the technological process of clarification on rapid filters remained relevant. Since the total cost of the process consists of capital and operating costs, two ways were realized to reduce the cost

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of separating suspensions of different origin, namely, the improvement of filter design and clarification technology. In the first case, mainly the design was improved, in particular, layered structures were used, they were given a special curvilinear shape, the size of filtering material grains was selected, etc. By supplying contaminated water prepared in a special way, the effective operation of the filters was prolonged, thus saving on operating costs. Obviously, the second way deserves special attention with regard to operating filters. Various techniques and methods ensuring its implementation in practice have been the subject of theoretical and experimental studies, the results of which are set forth in extensive literature, and we can mention, for example, the following works [1—5].

One of indicative examples of the technological improvement of direct-flow filtration on rapid filters can be non-traditional methods of suspension feeding on them. They assume splitting of the initial suspension flow (hydraulic load) into two or more components with their subsequent localization at separate sections of the packed bed [6]. Usually, the suspension enters the filter bed only through its upper (downward filtration) or lower (upward) surfaces [7]. The filtrate is taken from the opposite side. However, it is not technically difficult to feed the suspension simultaneously from both sides. At the same time, it is structurally feasible to implement the suspension inflow into the operating layer from the inside. In the first case it is possible to increase the efficiency of the technological process by varying the ratio between V_1 and V_2, as well as the position of the drainage device (L_d). In the second case, the increase of the operating period is achieved by selecting the position of the internal feeder (L_s) and also the above-mentioned ratio. In both cases, minimal design changes are required. It is important that it is possible to achieve a significant clarifying effect by selecting V_1, V_2, L_d, L_s, which is reflected in a noticeable extension of the continuous operation of the filter. From a physical point of view, this result can be explained by a more uniform distribution of deposit in the bed. A natural consequence of the sharp reduction of the maximum clogging of a filter media is a significant reduction of the total head losses in it.

Just the purpose of the article was to quantify the clarifying effect due to the separate feeding of aqueous suspension through the upper and lower surfaces of the packed bed of a water treatment filter.

It is easier to evaluate the consequences of splitting suspension feeding formally using mathematical modeling methods. For its initial evaluation it is enough to limit ourselves to the theoretical study of dual-flow filtration. Just below we analyze the effectiveness of the above-mentioned technique of the intensification of rapid filtration, namely, if the suspension is fed into the filter bed simultaneously through its upper and lower boundaries ($z=0$ and L) with constant rates, respectively, V_1 and $V_2 = V - V_1$, where V is the hydraulic load on the filter. Filtrate is taken away by drainage device inside the filter media at the depth L_d $(z = L_d)$. Washing of the clogged bed is carried out by the flows of the treated water in reverse directions. Thus, a single domain of motion in the conventional filtration (single-flow) is divided into two unrelated sub-domains.

It is preliminary assumed that the detachment filtration, linear kinetics of mass transfer between liquid and solid phases take place; the hydraulic conductivity of the clogged medium is generalized as a nonlinear function of the volumetric deposit concentration S_S [8—10]

$$k(S_S) = k_0 f_k(S_S) = k_0[1-(S_S/n_0)^{m_{k1}}]^{m_{k2}}, \tag{1}$$

where k_0, n_0 are the hydraulic conductivity and porosity of the clean bed, $m_{k1,k2}$ are the empirical coefficients.

Then the formulation of the corresponding mathematical problem includes for the upper bed section ($0 \le z \le L_d$) the system of equations

$$V_1 \frac{\partial C_1}{\partial z} + \frac{\partial S_1}{\partial t} = 0, \tag{2}$$

$$\frac{\partial S_1}{\partial t} = \alpha_V V_1^l C_1 - \beta_V V_1^q S_1, \tag{3}$$

$$V_1 = -k_0 f_k(S_1) \frac{\partial h_1}{\partial z}, \tag{4}$$

$$f_k(S_i) = \{1 - [\gamma(S_i) S_i]^{m_{k1}}\}^{m_{k2}} \quad (i = 1,\ 2) \tag{5}$$

and the boundary conditions operator

$$t = 0,\quad S_1 = 0; \quad z = 0,\quad C_1 = C_0; \quad z = L_d \quad h_1 = H_d. \tag{6}$$

Hereinafter it was assumed: C_i is the volumetric concentration of the suspended solids within $i-th$ section $(i = 1, 2)$; S_i and h_i are the volumetric concentration of the deposited particles and piezometric head there; H_d is the constant piezometric head at the filter outlet $(z = L_d)$. The following relationship between the concentrations of the deposit and deposited particles was used in the expression for f_k (5) [11]

$$S_S = \gamma(S_i) S_i, \tag{7}$$

where γ is the functional bulk factor. In addition, rate coefficients of the suspension particles attachment and detachment are related to the filtration rate V as follows [12-13]

$$\alpha = \alpha_V V^l, \qquad \beta = \beta_V V^q, \tag{8}$$

where α_V, β_V are the reduced rate coefficients of the suspension particles attachment and detachment, which do not depend on the flow characteristics. Similarly for the bottom section ($L_d \le z \le L$) will be

$$V_2 \frac{\partial C_2}{\partial z} - \frac{\partial S_2}{\partial t} = 0, \tag{9}$$

$$\frac{\partial S_2}{\partial t} = \alpha_V V_2^l C_2 - \beta_V V_2^q S_2, \tag{10}$$

$$V_2 = k_0 f_k(S_2) \frac{\partial h_2}{\partial z}, \tag{11}$$

$$t = 0,\quad S_2 = 0; \quad z = L_d,\quad h_2 = H_d. \tag{12}$$

and also (5) at $i=2$. After introducing dimensionless variables and parameters in the standard way: $\bar{C}_i = \dfrac{C_i}{C_0}$, $\bar{S}_i = \dfrac{S_i}{n_0 C_0}$, $\bar{z} = \dfrac{z}{L}$, $\bar{t} = \dfrac{Vt}{n_0 L}$, $\tilde{h}_i = \dfrac{k_0}{VL}(h_i - H_d)$, $\bar{V}_i = \dfrac{V_i}{V}$, $\bar{L}_d = \dfrac{L_d}{L}$, $\bar{\alpha}_V = \alpha_V V^{l-1}$, $\bar{\beta}_V = \beta_V V^{q-1}$, $V = V_1 + V_2$, $\bar{\gamma} = \gamma C_0$ problem (2)—(6), (9)—(12) is transformed to the following form, specifically,
for the upper section ($0 \le \bar{z} \le \bar{L}_d$)

$$\bar{V}_1 \frac{\partial \bar{C}_1}{\partial \bar{z}} + \frac{\partial \bar{S}_1}{\partial \bar{t}} = 0, \tag{13}$$

$$\frac{\partial \bar{S}_1}{\partial \bar{t}} = \bar{\alpha}_V \bar{V}_1^l \bar{C}_1 - \bar{\beta}_V \bar{V}_1^q \bar{S}_1, \tag{14}$$

$$\bar{V}_1 = -f_k(\bar{S}_1) \frac{\partial \tilde{h}_1}{\partial \bar{z}}, \tag{15}$$

$$f_k(\bar{S}_i) = \{1 - [\bar{\gamma}(\bar{S}_i)\bar{S}_i]^{m_{k1}}\}^{m_{k2}}; \tag{16}$$

$$\bar{t} = 0, \quad \bar{S}_1 = 0; \quad \bar{z} = 0, \quad \bar{C}_1 = 1; \quad \bar{z} = \bar{L}_d, \quad \tilde{h}_1 = 0; \tag{17}$$

for the lower section ($\bar{L}_d \le \bar{z} \le 1$)

$$\frac{\partial \bar{C}_2}{\partial \bar{z}} - \frac{\partial \bar{S}_2}{\partial \bar{t}} = 0, \tag{18}$$

$$\frac{\partial \bar{S}_2}{\partial \bar{t}} = \bar{\alpha}_V \bar{C}_2 - \bar{\beta}_V \bar{S}_2, \tag{19}$$

$$\bar{V}_2 = f_k(\bar{S}_2) \frac{\partial \tilde{h}_2}{\partial \bar{z}}; \tag{20}$$

$$\bar{t} = 0, \quad \bar{S}_2 = 0; \quad \bar{z} = 1, \bar{C}_2 = 1; \quad \bar{z} = \bar{L}_d, \tilde{h}_2 = 0. \tag{21}$$

The rigorous solution of the problem (13) — (21) was obtained by the operational method. The course of the solution of a similar problem is given, for example, in [10]. Therefore, the following are only the main calculation formulae that are necessary to gain a full understanding of the results and consequences of the suspension clarification in a dual-flow filter with different flow directions. So, they will be for the upper section

$$\bar{C}_{1e}(\bar{t}) = e^{-\bar{\alpha}_V \bar{V}_1^{l-1} \bar{L}_1} \Bigg[e^{-\bar{\beta}_V \bar{V}_1^q \bar{t}} I_0\left(2\sqrt{\bar{\alpha}_V \bar{\beta}_V \bar{V}_1^{l+q-1} \bar{L}_d \bar{t}}\right) + \\ + \bar{\beta}_V \bar{V}_1^q \int_0^{\bar{t}} e^{-\bar{\beta}_V \bar{V}_1^q \xi} I_0\left(2\sqrt{\bar{\alpha}_V \bar{\beta}_V \bar{V}_1^{l+q-1} \bar{L}_d \xi}\right) d\xi \Bigg], \tag{22}$$

$$\bar{S}_1(\bar{z},\bar{t})=\bar{\alpha}_V\bar{V}_1^l e^{-\bar{\alpha}_V\bar{V}_1^{l-1}\bar{z}}\int_0^{\bar{t}} e^{-\bar{\beta}_V\bar{V}_1^q\xi} I_0(2\sqrt{\bar{\alpha}_V\bar{\beta}_V\bar{V}_1^{l+q-1}\bar{z}\xi})\,d\xi, \tag{23}$$

Index "e" means that this characteristic refers to the outlet from the filter.

Besides, the head losses $\Delta\bar{h}_1$ are accurately calculated by the formula

$$\Delta\bar{h}_1(\bar{t})=\bar{V}_1\int_0^{\bar{L}_d}\frac{d\bar{z}}{f_k(\bar{S}_1(\bar{z},\bar{t}))}. \tag{24}$$

The following dependencies are recommended for similar calculations for the lower section

$$\bar{C}_{2e}(\bar{t})=e^{-\bar{\alpha}_V(1-\bar{V}_1)^{l-1}(1-\bar{L}_d)}\Big[e^{-\bar{\beta}_V(1-\bar{V}_1)^q\bar{t}} I_0(2\sqrt{\bar{\alpha}_V\bar{\beta}_V(1-\bar{V}_1)^{l+q-1}(1-\bar{L}_d)\bar{t}})+ \\ +\bar{\beta}_V(1-\bar{V}_1)^q\int_0^{\bar{t}} e^{-\bar{\beta}_V(1-\bar{V}_1)^q} I_0(2\sqrt{\bar{\alpha}_V\bar{\beta}_V(1-\bar{V}_1)^{l+q-1}(1-\bar{L}_d)\xi})\,d\xi\Big], \tag{25}$$

$$\bar{S}_2(\bar{z},\bar{t})=\bar{\alpha}_V(1-\bar{V}_1)^l e^{-\bar{\alpha}_V(1-\bar{V}_1)^{l-1}(1-\bar{z})}\int_0^{\bar{t}} e^{-\bar{\beta}_V(1-\bar{V}_1)^q\xi} I_0(2\sqrt{\bar{\alpha}_V\bar{\beta}_V(1-\bar{V}_1)^{l+q-1}(1-\bar{z})\xi}), \tag{26}$$

$$\Delta\bar{h}_2(\bar{t})=(1-\bar{V}_1)\int_{\bar{L}_d}^{1}\frac{d\bar{z}}{f_k(\bar{S}_2(\bar{z},\bar{t}))}. \tag{27}$$

The relative impurity concentration in the filtrate and the total head losses in the bed are calculated as the following sums

$$\bar{C}_e(\bar{t})=\bar{V}_1\bar{C}_{1e}(\bar{t})+(1-\bar{V}_1)\bar{C}_{2e}(\bar{t}), \tag{28}$$

$$\Delta\bar{h}(\bar{t})=\Delta\bar{h}_1(\bar{t})+\Delta\bar{h}_2(\bar{t}). \tag{29}$$

Finally, the clogging of the lower bed surface is proposed to be calculated based on the formula

$$\bar{S}_{2e}(\bar{t})=\bar{S}_2(1,\bar{t})=\frac{\bar{\alpha}_V}{\bar{\beta}_V}(1-\bar{V}_1)^{l-q}(1-e^{-\bar{\beta}_V(1-\bar{V}_1)^q}). \tag{30}$$

The clarifying efficiency of the variety of the dual-flow filtration under consideration was evaluated on a number of test cases with the following fixed data: $\bar{\beta}_V=0.01$, $q=1$, $\bar{C}_*=0.1$, $\Delta\bar{h}_*=8$, $\bar{\gamma}(\bar{S})=2.5\cdot10^{-3}-2\cdot10^{-6}\bar{S}$, $f_k(\bar{S})=[1-\bar{\gamma}(\bar{S})\bar{S}]^3$. In addition, two values of l $(1/3, 1)$, characteristic for Brownian ($d_p\le10^{-6}$) and non-Brownian ($d_p\ge10^{-6}$) suspension particles, respectively, were chosen. Finally, either their typical values or wide ranges of possible values are adopted for $\bar{V}_1$, $\bar{L}_d$, $\bar{\alpha}_V$. The main subject of the calculations was the relative duration of the filter run $\bar{t}_f$ as a key technological parameter. For its establishment, the relative technological times $\bar{t}_p$ and $\bar{t}_h$ were previously found with the involvement of the criterion equations [14]

$$\bar{C}_e(\bar{t}_p)=\bar{C}_{e1}(\bar{t}_p)+\bar{C}_{e2}(\bar{t}_p)=\bar{C}_*, \tag{31}$$

$$\Delta\bar{h}_1(\bar{t}_h) + \Delta\bar{h}_2(\bar{t}_h) = \Delta\bar{h}_., \tag{32}$$

where t_h is the time of reaching maximum allowable head losses in filter bed, t_p is the time of protective effect of filter bed, $\bar{C}_.$, $\Delta\bar{h}_.$ are the relative maximum permissible values of impurity concentration in the filtrate and head losses in the filter bed.

At already known relative values of the filter run duration $\bar{t}_f$, the relative excesses (or reductions under unfavorable conditions) $\Delta\bar{t}_f$ corresponding to them were determined due to the splitting of a hydraulic load. Thus, $\Delta\bar{t}_f = \bar{t}_f / \bar{t}_{f*}$, where $\bar{t}_{f*}$ is the duration of the filter run at only downward filtration under similar conditions.

The dependences for the relative concentrations $\bar{C}_{ie}$, $\bar{S}$ and head losses $\Delta\bar{h}_i$ were involved: (24)—(26) for the first section ($0 \le \bar{z} \le \bar{L}_d$) and (27)—(29) for the second section ($\bar{L}_d < \bar{z} \le 1$). In order to establish the conditions that contribute to the productive operation of the dual-flow filter and to evaluate the gain therein due to the distribution of the hydraulic load between the upper and lower bed surfaces, the relative duration $\bar{t}_f$ was calculated depending on the position of the filtrate runoff (L_d), as well as on the ratio between the upper (V_1) and lower $(V - V_1)$ flow rates and, finally, on the absorption capability of the filter bed $(\bar{\alpha}_V)$. The mass exchange coefficients were assumed to be independent of the filtration flow direction. First of all, the sensitivity of the relative duration $\bar{t}_f$ to the position of the drainage device was analyzed at equal upper and lower flow rates $(\bar{V}_1 = \bar{V}_2 = 0.5)$. It was found out that the technological process is limited in time only by the protective capability of a bed for coarse impurities, and it is reasonable to place the drainage device in the middle of the filter media $(\bar{L}_d = 0.5)$. Thus, it is possible to increase $\bar{t}_f$ by 32.7 % in comparison with the traditional single-flow filtration. In case of finely dispersed impurity, firstly, the effect of splitting hydraulic load is even greater and the maximum value of $\Delta\bar{t}_f$ (1.643) is reached at $\bar{L}_d = \bar{L}_* = 0.256$, and the filter operation has to be stopped at $\bar{L}_d < \bar{L}_*$, because of the excessive deterioration of the filtrate quality, and due to the excessive head losses at $\bar{L}_d > \bar{L}_*$. However, the calculated value of $\Delta\bar{t}_f$ is only slightly less than the maximum value (1.632) even at $\bar{L}_d = 0.5$. Therefore, the value $\bar{L}_d = 0.5$ is fixed in subsequent calculations. At the same time, it is inadmissible to set $\bar{L}_d$ too low, which may cause complete non-functionality of the filter. Indeed, it is not difficult to specify such a critical value L_{cr}, that the first portion of a suspension will be insufficiently clarified at $L_d \le L_{cr}$. Proceeding from dependences (22), (25) the following equation concerning $\bar{L}_{cr}$ is derived

$$\bar{V}_1 e^{-\bar{\alpha}_V[\bar{V}_1^{l-1} + (1-\bar{V}_1)^{l-1}]\bar{L}_{cr}} - \bar{C}_* e^{-\bar{\alpha}_V(1-\bar{V}_1)^{l-1}\bar{L}_{cr}} + (1-\bar{V}_1)e^{-\bar{\alpha}_V(1-\bar{V}_1)^{l-1}} = 0\,. \tag{33}$$

Equation (33) is easily solved in the general case by the fitting procedure, and the simple formula follows from it in the special case $\bar{V}_1 = \bar{V}_2 = 0.5$

$$\bar{L}_{cr} = -\frac{\ln\left(\bar{C}_* \pm \sqrt{\bar{C}_*^2 - e^{-0.5^{l-1}\bar{\alpha}_V}}\right)}{0.5^{l-1}\bar{\alpha}_V}. \tag{34}$$

Both roots of equation (34) have physical meaning and correspond to two critical positions of the drainage device, which are equidistant from the center of the bed and its nearby inlet. Specifically, in the second series of examples, $\bar{L}_{cr}$ equals to 0.1267 and 0.8733, respectively, at $l = 1/3$

Fig. 1. Dependence $\Delta \overline{t}_f(\overline{V}_1)$: *1* — $l=1/3$, *2* — $l=1$

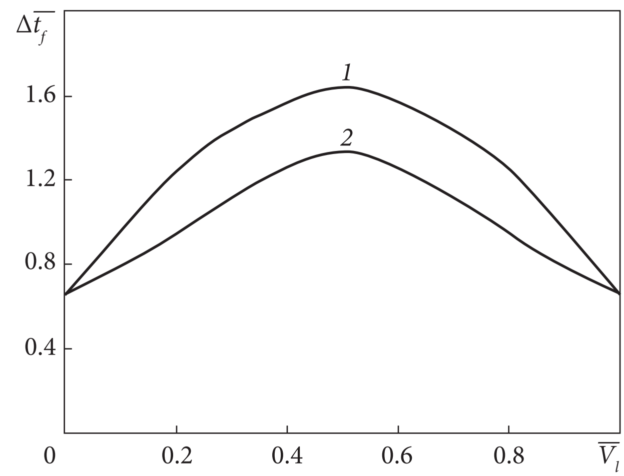
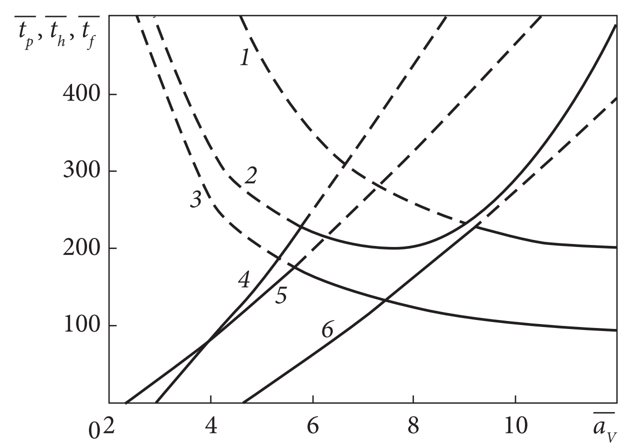


Fig. 2. Dependencies $\overline{t}_p(\overline{\alpha}_V)$, $\overline{t}_h(\overline{\alpha}_V)$, $\overline{t}_f(\overline{\alpha}_V)$: 1—3 — $\overline{t}_h$, 4—6 — $\overline{t}_p$, $\overline{t}_f$ — solid lines; 1, 6 — dual-flow ($l=1$); 2, 4 — dual-flow ($l=1/3$); 3, 5 — single-flow



and already $\overline{L}_{cr}=0.2022$ and 0.7978 at $l=1$. And further attention was emphasized on the relationship between $\overline{t}_f$ and $\overline{V}_1$ or $\overline{\alpha}_V$. Curves describing the excess of $\overline{t}_f$ over the basic value for comparison $\overline{t}_{f*}$ (single flow filtration with parameters $\overline{V}_1=\overline{L}_d=1$) at $\overline{L}_d=0.5$ and changing $\overline{V}_1$ from 0 (upward filtration through a bed with half the height) to 1 (downward filtration through a similar bed) are shown in Fig. 1. Naturally, the reduction of the bed height in both limiting situations $(\overline{V}_1=0$ and $1)$ twice causes a serious (one and a half times) reduction of $\overline{t}_f$ at the same total hydraulic load V. If the suspension is fed through the both surfaces with equal flow rates, the increment of $\overline{t}_f$ is maximum and will be 32.7 % at $l=1/3$, and it doubles (63.2 %) at $l=1$. Also, the technological times $\overline{t}_p$, $\overline{t}_h$, $\overline{t}_f$ were calculated as functions of $\overline{\alpha}_V$ at the fixed values of $\overline{V}_1$ (0.7), $\overline{L}_d$. The two series of plots obtained for the fine and coarse impurities based on (22) — (27) are shown in Fig. 2. Here the solid lines highlight the curves of the dependence $\overline{t}_f(\overline{\alpha}_V)$ the most important for practice, which are continuous and have one fracture each. Comparison of the optimum values of $\overline{t}_f$ in three considered cases of dual- and single-flow filtration is indicative. A comparison of the peak values of $\overline{t}_f$ shows that the corresponding effect can be estimated at 30 %. Also, an increase in the times $\overline{t}_h$, $\overline{t}_f$ is observed here with an increase in $\overline{\alpha}_V$ from the value 7.9. However, the observed even more significant effect should not be overestimated due to the possible inadequacy of linear kinetics of mass exchange during prolonged filtration through a well-absorbing media.

Thus, due to simultaneous feeding of a suspension into the bed in two places (not only through the upper and lower surfaces), it is possible to significantly intensify the clarification process also in the section of the bed, which practically did not take part in it in the traditional (downward or upward) filtration. Thus, firstly, the absorption resource of the entire filter media is more fully implemented, and secondly, the concentration profiles of the deposit and deposited particles are smoothed out and the mechanical energy expenditure is reduced accordingly. However, the quality of the filtrate deteriorates faster due to the reduction of the filtration rate and the weakening of the absorption $(l > 0)$. Therefore, it is necessary to preselect the technique of feeding a suspension into the filter bed, based on the results of the joint technological analysis of dual-flow and single-flow filtration.

Conclusions. It is established on the basis of strictly solving the mathematical problem of detachment filtration of water suspension at linear kinetics of interphase mass transfer that its simultaneous feeding through the upper and lower surfaces contribute to a significant reduction of mechanical energy consumption for the technological process, which can be expressed in several tens of percent.

Separation of the initial suspension flow, despite the minimal deterioration of water treatment quality, allows to increase the duration of the filter run to the same extent, and in practical terms — to use well sorbing filtering materials.

Due to the large scale of application of rapid filtration in the water industry and in particular in water treatment technologies, it is possible to significantly reduce the cost of filtrate by separate feeding of contaminated water to filters.

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В.Л.Поляков,
С.М. Курганська

Інститут гідромеханіки НАН України, Київ, Україна
E-mail: v.poliakov.ihm@gmail.com

МОДЕЛЮВАННЯ ФІЛЬТРУВАННЯ ВОДНИХ СУСПЕНЗІЙ
У РАЗІ ПОЄДНАННЯ НИЗХІДНОГО ТА ВИСХІДНОГО ПОТОКІВ

Фізичну область розділено на дві підобласті руху, і стосовно кожної з них сформульовано нелінійну математичну задачу фільтрування водної суспензії за лінійної кінетики міжфазного відривного масообміну. При цьому враховано структуру гелеподібного осаду, залежність від його концентрації коефіцієнта фільтрації, зв'язок коефіцієнтів масообміну (прилипання та відриву) зі швидкістю фільтрування. Відповідні математичні моделі містять взаємопов'язані освітлювальний і фільтраційний блоки. Після введення безрозмірних змінних і параметрів, а також застосування операційного методу отримано суворі розв'язки обох задач. У підсумку виведено найважливіші залежності та рівняння, призначені для інженерних розрахунків ключових характеристик фільтрування — концентрації дисперсної домішки у фільтраті та втрат напору у виділених підобластях і загальних у всьому шарі завантаження. Зазначені формалізми використовували для визначення основних технологічних часів, що обмежувало час безперервної роботи фільтра внаслідок надмірного погіршення якості фільтрату та витрат механічної енергії на фільтрацію через засмічене середовище. Як наслідок, встановлювали допустимий виходячи з критеріїв ефективного фільтрування час його безперервної роботи. Подібний технологічний підхід до оцінки працездатності фільтра паралельно застосовували до швидкого фільтра за традиційної однопотокової подачі суспензії та дослідженої вище двопотокової. Порівняльний аналіз виконано на тестових прикладах з типовими для практики освітлення водних суспензій вихідними даними. У результаті отримано, що поділ вихідного потоку суспензії на дві складові, що надходять через верхню та нижню поверхні завантаження, може сприяти значній інтенсифікації технологічного процесу. При цьому реально збільшення тривалості фільтроциклів на 50 % і більше, що призводить до відчутного зниження вартості фільтрату. Таким чином, стає виправданим застосування фільтрувальних матеріалів, що добре сорбують.

Ключові слова: *фільтрування, суспензія, концентрація, двопотоковий, фільтроцикл, точний розв'язок, втрати напору.*