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Further examination of the 1/*N* portfolio rule: a comparison against Sharpe-optimal portfolios under varying constraints

Abstract. Practical trading constraints (such as asset bounds and transaction costs) are known to affect the efficient frontier of an investment portfolio. In this study, we investigate out-of-sample trading performances of tangency portfolios against the naïve 1/*N* policy under varying constraints. Our aim is to deliberate if such constraints are influential to the relative (individual) performances between (of) the two competing strategies. Using FTSE Bursa Malaysia KLCI constituent stocks of 30 companies listed, we form several portfolios with different *N*s and constraint specifications. Sample period spans 2006 through 2015, in order to alleviate possible confounding affects on risk/return dynamics caused by the 1MDB financial scandal and the U.S. Federal Reserve increasing its key interest rate starting from December 2015. Performance metrics exhibit sensitivity of portfolios to the degree (variability) of constraints, specifically floor, ceiling and the consideration of trading cost.
Among other valuable findings, it has been found out that in all the cases researched the simple 1/*N* portfolio selection rule offers superior outcome as compared to the tangency portfolios. Generally stated, the naïve policy outperforms the more sophisticated portfolio optimization model in terms of the Sharpe criterion, information ratio and maximum drawdown during the period under investigation. Relative performances remain consistent regardless of the number of stocks included in the portfolio.
Keywords: Portfolio Optimization; Sharpe Ratio; Information Ratio; Maximum Drawdown; Naïve Diversification; Practical Constraints.
JEL Classifications: G11; C60
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Подальший розгляд правила 1/*N* портфоліо: порівняння з оптимальними за Шарпом портфелями за різних обмежень
Анотація. Відомо, що практичні торгові обмеження впливають на ефективну межу інвестиційного портфеля. У цій роботі ми досліджуємо позабіржові торговельні показники дотичних портфелів проти наївної політики 1/*N* за різних обмежень. Наша мета - виявити, чи впливають такі обмеження на дії між конкуруючими стратегіями. Використовуючи акції 30 компаній на фондовій біржі FTSE Bursa Malaysia KLCI, ми формуємо кілька портфелів із різними специфікаціями *N* та обмеженнями. Вибірка охоплює період з 2006 по 2015 рік, коли в кінці року усувається можливе змішування впливу на динаміку ризику/прибутковості, викликане фінансовим скандалом 1 МБР, та початком підвищення ключової процентної ставки Федеральною резервною системою США починаючи з грудня 2015 року. Показники ефективності демонструють чутливість портфелів до ступеня (мінливості) обмежень, зокрема, нижньої межі, верхньої межі й урахування вартості торгівлі.
Ключові слова: оптимізація портфеля; коефіцієнт Шарпа; коефіцієнт інформації; максимальна просадка; наївна диверсифікація; практичні обмеження.

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Дальнейшее рассмотрение правила портфеля 1/*N*: сравнение с оптимальными по Шарпу портфелями при различных ограничениях
Аннотация. Известно, что практические торговые ограничения влияют на эффективную границу инвестиционного портфеля. В этой работе мы исследуем внебиржевые торговые показатели касательных портфелей против наивной политики 1/*N* при различных ограничениях. Наша цель - выявить, влияют ли такие ограничения на действия между конкурирующими стратегиями. Используя акции 30 компаний на фондовой бирже FTSE Bursa Malaysia KLCI, мы формируем несколько портфелей с различными спецификациями *N* и ограничениями. Выборка охватывает период с 2006 по 2015 год, чтобы устранить возможное смешение влияния на динамику риска/доходности, вызванное финансовым скандалом 1 МБР, и началом увеличения ключевой процентной ставки Федеральной резервной системой США начиная с декабря 2015 года. Показатели эффективности демонстрируют чувствительность портфелей к степени (изменчивости) ограничений, в частности, нижней границы, верхней границы и учета стоимости торговли.
Ключевые слова: оптимизация портфеля; коэффициент Шарпа; коэффициент информации; максимальная просадка; наивная диверсификация; практические ограничения.

1. Introduction

Portfolio diversification is a key topic in finance and economics, and has serious implications to both theory and practice. Its principle is applied predominantly by mutual funds, with the global assets under management amounting to nearly USD 45 trillion as of June 2017 (ICI, 2017) [1]. Rudimentary idea for diversifying investment can be traced back to the Talmudic advice of uniform distribution in the 4th century, although mathematical formulation and the resulting efficient frontier emerged relatively recently in Markowitz (1952) [2]. Its main idea is to provide investors with portfolios that give the highest return for a given level of risk along the Pareto optimal front.

The mean variance (MV) portfolio optimization considers both risk and return to compute the optimal weight for each component stock and thus it is supposedly superior to the naïve 1/*N* portfolio allocation policy. This uniform distribution policy simply divides available funds equally to each stock in the portfolio. However, results from the existing literature remain mixed (see for instance DeMiguel et al., 2009; Kritzman et al., 2010; Levy and Duchin 2010; Nor and Islam, 2016) [3-6]. The conflicting observations from prior research have vital theoretical and practical implications, particularly as the economic benefit of optimal diversification is debatable. Further, many investors and mutual and/or pension funds simply divide their capital evenly among *N* assets (Benartzi and Thaler, 2001; Huberman and Jiang, 2006) [7-8].

2. Brief Literature Review

The works of Markowitz (1952) [2] and Sharpe (1963) [9], among others, and later their Nobel Prizes in Economics (officially Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel) reflect the significance of research in the area of investment portfolios. In demonstrating the benefits (or lack thereof) of investment portfolio optimization, recent literature to name a few DeMiguel et al., (2009) [3], Kritzman et al., (2010) [4] and Nor and Islam (2016) [6] extensively explores optimal against uniform portfolios. However, the main limitation of traditional MV is that it ignores practical constraints e.g. floor (lower bound), ceiling (upper bound) and trading costs. Yoshimoto (1996) [10] shows that neglecting costs in the optimization process can result in an inefficient portfolio. As well, disproportionate portfolio weights can lead to excessive management fees, monitoring costs and/or higher exposure of a particular stock. The weight (proportion) bound can also mitigate estimation error (Levy and Duchin, 2010) [5]. Although advancements have been made in this area, including the use of complex constraints (e.g. Mei et al., 2016; Ruiz-Torrubiano and Suárez, 2015; Xue et al., 2006) [11-13], such constraints are often embedded in the MV model and/or its extensions.

3. The purpose of this paper is to construct mathematical portfolios that attempt to maximize risk/return tradeoff of different sized portfolios and compare their out-of-sample performances against the naïve 1/*N* rule. The portfolios are constructed with different *N*s and explored with (and variations of) as well as without several practical constraints. Since emerging markets can provide good portfolio diversification benefit, we employ the data of 30 companies listed in the FTSE Bursa Malaysia KLCI for a 10-year period (from 1 January 2006 to 31 December 2015). Our data ends with 2015 to mitigate significant events such as economic and political factors (which occur during that period) from affecting our findings. Among others, these include the infamous 1MDB financial scandal in Malaysia and also the U.S. Federal Reserve started raising interest rates in December 2015 which would likely channel international investments from emerging markets to them.

The portfolio comprises of firms from diverse industries, including construction, consumer products, finance, properties, trading and services which should allow for proper diversification, in line with Markowitz (1952) [2]. Briefly stated, the main idea of diversification is for the assets to have negative or low positive correlations with other assets in the portfolio in order to reduce risk. With respect to Sharpe-optimal portfolio, the goal is to obtain the best combination of assets (or stocks) that generate the greatest return-per-risk ratio, tangential to the Pareto optimal front.

4. Results

Figure 1 shows the in-sample correlation matrix between 30 FTSE Bursa Malaysia KLCI stocks in graphical form. With an average correlation of only 0.226, it appears that there are opportunities to obtain (out-of-sample) diversification benefit by constructing investment portfolios in the Malaysian stock market.

Levy and Duchin (2010) [5] and Nor and Islam (2016) [6] for instance, show that portfolio size is influential to the relative performance between optimal and naïve portfolio. To see any effects of portfolio size and varying constraints on portfolio performances, we build a total of 16 investment portfolios for analysis. More specifically, a total of four different *N*s (*N* = 30, 25, 20, and 15), and each with four rules, namely Naïve (1/*N*), Unconstrained (UC), Constrained 1 (C1) and Constrained 2 (C2).

We divide sample period into two non-overlapping subperiods: (1) In-sample period spans 1 January 2006 to 31 December 2013, (2) the remaining (1 January 2014 to 31 December 2015) is reserved for out-of-sample evaluation. The 80/20 splitting ratio is consonant with Nisbet et al. (2009) and allows for a larger in-sample training data (i.e. parameter optimization). Weekly stock prices are extracted from Yahoo Finance database (http://finance.yahoo.com). We use 3-month Malaysian Treasury bill rate as proxy for the risk-free-rate, sourced from Central Bank of Malaysia website (http://www.bnm.gov.my). Finally, trading costs in the FTSE Bursa Malaysia include brokerage fees, stamp duty and clearing fees, and are computed to be around 1% one-way (i.e. buy or sell).

The classical Markowitz's MV model can be formulated as:

$$\min \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} \tag{1}$$

$$\max \sum_{i=1}^{N} w_i \mu_i \tag{2}$$

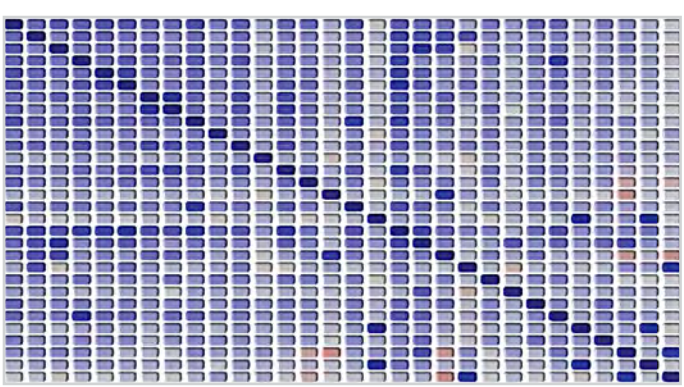
subject to

$$\sum_{i=1}^{N} w_i = 1 \tag{3}$$

$$0 \le w_i \le 1\ , \tag{4}$$

where $i = 1, 2, ..., N$ and N is the number of stocks (or assets) in the portfolio, Equations (1) and (2) represent minimising portfolio risk (variance) and maximising expected returns, respectively subject to constraints that all funds are fully invested and short sell restriction as represented in Equations (3) and (4), accordingly. Equations (1) and (2) represent the MVO as a multi-objective problem. It is also common to present it within a single objective framework as the following quadratic programming problem

$$\max\ (1-\lambda)\left[\sum_{i=1}^{N} w_i \mu_i\right] - \lambda\left[\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij}\right] \tag{5}$$



Note: Blue (peach) cells represent positive (negative) correlations. Darker (lighter) cells indicate higher (lower) correlation. Each diagonal darkest blue cell denotes the value of +1

Fig. 1: **In-sample portfolio correlation matrix between 30 FTSE Bursa Malaysia KLCI stocks**
Source: Computed and elaborated by the authors

subject to the same constraints as (3) and (4), where λ shows the risk aversion parameter, and $0 \le \lambda \le 1$. With $\lambda = 1$, an investor is only concerned with minimising risk (variance) without any consideration for return. Conversely, $\lambda = 0$ shows that the investor attempts to maximise (expected) returns with no concern for risk. Each point on the efficient frontier represents an optimal portfolio that matches the λ level. By default, the single-objective MVO described earlier in Equation (5) ignores trading costs that will incur in order to buy and sell stocks, and rebalance the portfolio. Incorporating trading costs into the equation, the revised optimization objective can be defined as:

$$\max \quad (1-\lambda)\left[\sum_{i=1}^{N} w_i \mu_i - \sum_{i=1}^{N} k_i \left| w_i - w_i^0 \right|\right] - \lambda\left[\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij}\right] \tag{6}$$

subject to

$$\sum_{i=1}^{N} w_i = 1 \quad , \quad l_i \le w_i \le u_i \quad , \tag{7}$$

where l_i and u_i are the lower bound and upper bound respectively, while the total cost is the sum of the differences between the new weight w_i (the optimal weight for stock i) and the original weight w_i^0 (i.e. the weight before rebalancing, which is effectively zero for the initial portfolio), times the cost rate k_i. This total cost, c_i, can be detailed as follows:

$$c_i = \sum_{i=1}^{N} k_i \left| w_i - w_i^0 \right| \quad . \tag{8}$$

Taking into account the preceding equations, our portfolio selection problem is solved by maximizing its risk/return trade-off, i.e. Sharpe criterion. This can be presented as:

$$\max \quad S_p = \frac{\left[\sum_{i=1}^{N} w_i \mu_i - \sum_{i=1}^{N} k_i \left| w_i - w_i^0 \right|\right] - r_f}{\sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij}}} \quad . \tag{9}$$

The same constraint as in Equation (3) applies where investment is based on the available budget. We place lower (floor) and upper (ceiling) bound constraints to reduce estimation error and produce realistic investment outcomes. For $N \ge 25$, we set C1: $l_i = 1\%$, $u_i = 20\%$ and C2: $l_i = 2\%$, $u_i = 10\%$, whereas for $N \le 20$, C1: $l_i = 1\%$, $u_i = 25\%$ and C2: $l_i = 2\%$, $u_i = 15\%$. These figures are used arbitrarily but are sensible for practical applications, and the idea is similar to Levy and Duchin (2010) [5]. The reason for this is rather intuitive; the ceiling for smaller *N*s is set higher because of the lower number of stocks making up these portfolios. Effectively, due to the limitations imposed by the bourse (i.e. Regulated Short Selling), we establish long-only constraint for the portfolios.

As stated earlier, the trading cost per transaction in Malaysia is computed as $k_i = 1\%$. Note that due to online trading facilities and cash up front, brokerage fees can be negotiated and this might lead to a lower overall cost. Nonetheless, the use of 1% is considered reasonable and the results are at best understated. For UC portfolios, no trading cost is considered during the optimization process while lower (upper) bound is restricted to only 0% (100%) to ensure long-only trades (budget to be fully exhausted).

Since optimal portfolio will, by definition, outperform 1/*N* during the in-sample phase, only out-of-sample analysis is relevant for analysis. Nonetheless, some descriptions of in-sample performance are provided for information purposes. By definition, rational investors are mainly concerned with the risk/return tradeoff of an investment. Hence, out-of-sample Sharpe ratio will be used as the primary measure of portfolio performance. Additionally, we explore two different measures, namely maximum drawdown and information ratio. Although our portfolios are not specifically optimized for these two variables, they provide additional information about portfolio performance with respect to the risk of decline and tracking errors.

Maximum drawdown can be defined as the decline (in percent) of the largest peak to valley of the portfolio during the period. Mathematically, it can be presented as:

$$MD\% = \frac{(V - P)}{P} \quad , \tag{10}$$

where $MD\%$ is the maximum drawdown of the largest peak (P) to valley (V) decline. On the other hand, the information ratio (IR) measures the portfolio returns against that of the benchmark FTSE Bursa Malaysia KLCI return over the volatility of those differences in returns (tracking error), and is shown as:

$$IR = \frac{R_p - R_b}{\sigma_{p-b}} \quad , \tag{11}$$

where R_p indicates the portfolio return, R_b is the benchmark return, and σ_{p-b} denotes the tracking error.

We describe some market and portfolio performances during the in-sample period starting from the beginning of 2006 until the end of 2013. Recall that these are provided for information purposes only. Figure 2 exhibits portfolio drawdowns during the first subperiod. Higher percentage indicates greater drawdowns. The chart says that the FTSE Bursa Malaysia KLCI component stocks suffer the greatest peak to valley decline of 32% during the Global Financial Crisis period in 2008. It is not surprising. The rapid drop in the market barometer during this period and its recovery phase in 2009/2010 appear to suggest (to a certain extent) selling pressure among Malaysian investors due to crisis sentiments. This also implies the existence of herding behaviour which leads to overreaction.

Table 1 exhibits the out-of-sample trading performance for each portfolio strategy during the period 1 January 2014 to 31 December 2015. The results tell us several stories.

Firstly, ex ante Sharpe-based portfolios (optimized during the in-sample period) and the 1/*N* rule perform very poorly with negative returns to variabilities during the period (with the exception of 1/*N,* where *N* = 30). This suggests that there was a major shift in the returns of the portfolio compositions where in-sample optimal and naïve allocations were no longer profitable during the holdout period. Indeed, Figure 3 shows that with the exception of only three stocks, risks (volatility) are greater than excess returns for the individual stocks in the portfolio.

Secondly, tighter floor-ceiling constraints (smaller gap between the lower and upper bounds) that attempts to mitigate sampling error also seem to result in smaller maximum drawdowns as well as the least negative Sharpe values and information ratios in all sample sizes among the tangency portfolios, with the exception of *N* = 25. The results imply that varying these constraints have some impact on investment performance. We postulate that this is caused by the lower exposure of the portfolio from any particular stock and therefore, to a certain extent, reduces portfolio risk.

Thirdly, excess returns for the UC portfolios are the lowest across all *N*s. These findings appear to corroborate the idea that ignoring transaction costs would be detrimental to portfolio outcome, as documented by Yoshimoto (1996) [10] and Mei et al. (2016) [11]. Accordingly, reward to variability is affected for all *N*s, further supporting the importance of integrating trading costs in formulating the portfolio selection problem during the training phase.

Finally, in all the cases, the simple 1/*N* portfolio selection rule offers superior outcome as compared to the tangency portfolios. Briefly stated, the naïve policy outperforms the more sophisticated portfolio optimization model in terms of the Sharpe criterion, information ratio and maximum drawdown during the period under investigation. Relative performances remain consistent regardless of the number of stocks included in the portfolio. With regard to *N*s, our finding is unlike those of Nor and Islam's (2016) [6] and Levy and Duchin's (2010) [5], among others. This is possibly due to the differences in our objective functions, portfolio constraints as well as the time periods explored.

5. Conclusion

In this paper, we investigate out-of-sample performance of the naïve 1/*N* rule versus the tangency portfolios that maximize the Sharpe ratio. We form our portfolios of different *N*s using FTSE Bursa Malaysia KLCI component stocks, as well as include (and vary) several practical constraints. Overall, we find that Sharpe-based portfolio selection problem performs poorly in itself, and in comparison against the simple equal-weighted scheme.

Our results have important theoretical and practical implications. We find that investment outcomes appear to be sensitive to the degree (variability) of the constraints. Nonetheless, investors who seek to maximize their investment Sharpe ratio by constructing portfolio diversification policies may have to utilize different optimization models, consider other constraints, or even other forms of trading strategies. Negative Sharpe values across all portfolios (apart from 1/*N* policy where *N* = 30) suggest that simple portfolio diversification policy is a challenge for an emerging market like Malaysia.

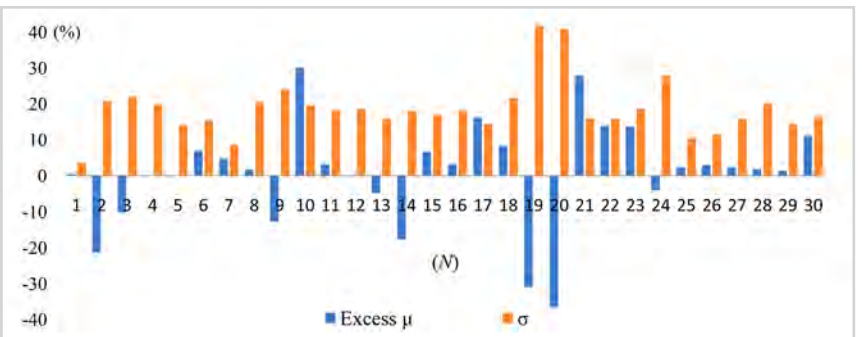
Indeed, the very idea of modern portfolio theory is generally contingent upon the market being information efficient. If active trading strategies cannot offer abnormal returns consistently, investors should properly diversify to obtain the best risk/return tradeoff. Yet recent studies such as by Guidi and Gupta (2013) [14] and Soon et al. (2015) [15] have shown that Bursa Malaysia might not be efficient even at the weak form. In fact, weak form inefficiency has similarly been observed in the developed markets. For example, Nor and Wickremasinghe (2014) [16] show that some forms of technical analysis still have potentials to reap positive returns in Australia, while recently, Shahzad et al. (2017a) [17] find that even in the U.S. market, some sectors have greater degree of inefficiency over the others. The fact that a number of real-world portfolio offered by mutual funds in Malaysia seem to generate poor returns and/or low Sharpe ratios further support our argument (for the list and performances of some mutual funds in the country, refer to http://www.fundsupermart.com.my). Our empirical findings coupled with these mutual fund performances therefore cast

Tab. 1: **Out-of-sample investment performance**

	Portfolio	Excess Return	Volatility	Sharpe Ratio	Information Ratio	Maximum Drawdown
N = 30	1/*N*	0.70%*	10.13%	0.07*	0.02*	-14%*
	UC	-12.19%	18.29%	-0.67	-0.76	-34%
	C1	-8.92%	10.01%*	-0.89	-0.67	-27%
	C2	-5.36%	11.61%	-0.46	-0.52	-19%
N = 25	1/*N*	-0.92%*	10.37%*	-0.09*	-0.14*	-15%*
	UC	-18.88%	25.58%	-0.74	-0.86	-46%
	C1	-4.65%	11.71%	-0.40	-0.46	-19%
	C2	-4.96%	11.72%	-0.42	-0.48	-19%
N = 20	1/*N*	-1.95%*	11.03%*	-0.18*	-0.23*	-16%*
	UC	-13.88%	19.31%	-0.72	-0.81	-37%
	C1	-11.39%	16.83%	-0.68	-0.76	-32%
	C2	-7.79%	13.37%	-0.58	-0.65	-24%
N = 15	1/*N*	-0.75%*	11.77%*	-0.06*	-0.12*	-17%*
	UC	-13.13%	15.21%	-0.86	-0.74	-38%
	C1	-9.71%	16.53%	-0.59	-0.67	-31%
	C2	-3.37%	12.98%	-0.26	-0.32	-20%

Note: The table shows out-of-sample (1 January 2014 to 31 December 2015) performance for the naïve 1/*N* policy and the Sharpe-optimal portfolios based on the Unconstrained (UC), Constrained 1 (C1) and Constrained 2 (C2) rules, across four different *N*s: 15, 20, 25 and 30. * indicates the best performance (or least poor) measure among the different-sized portfolio policies.

Source: Computed and elaborated by the authors



Note: The chart exhibits out-of-sample (1 January 2014 to 31 December 2015) return-volatility pairs (as measured by excess μ and σ respectively) for each individual stocks in the portfolio (*N* = 30).

Fig. 3: **Out-of-sample return-volatility pairs**
Source: Computed and elaborated by the authors

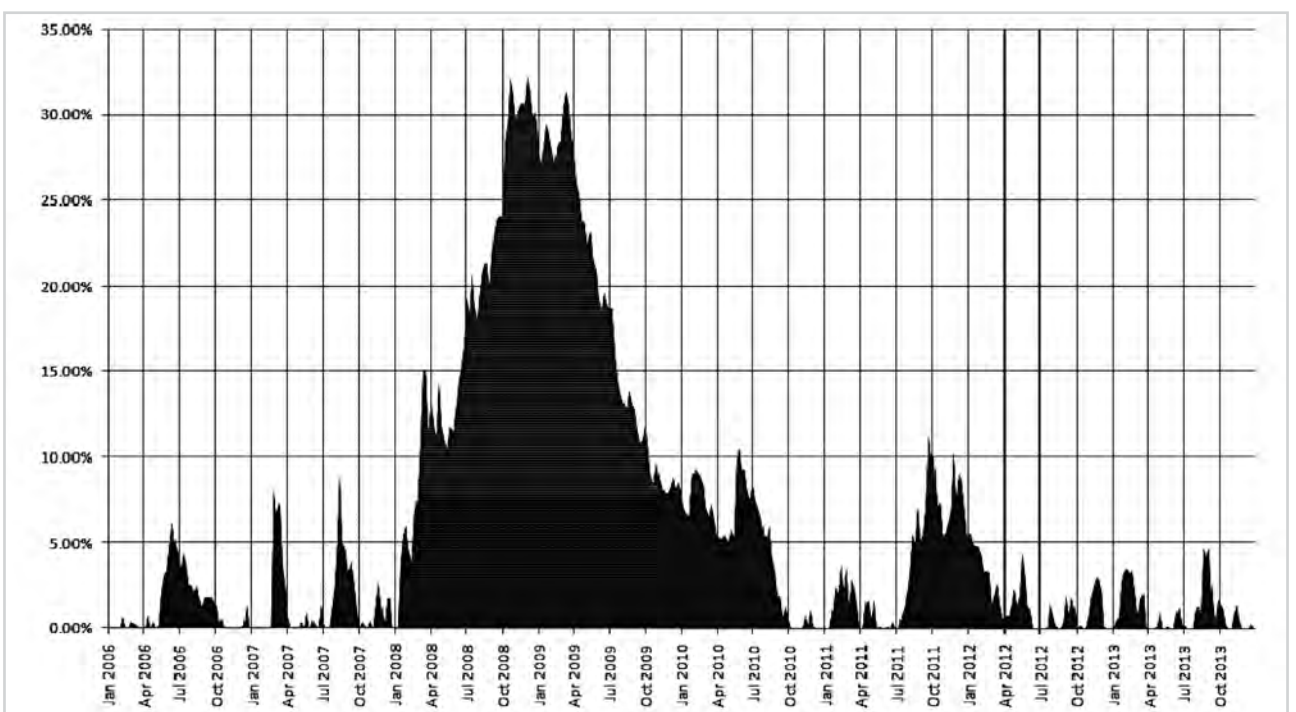


Fig. 2: **In-sample portfolio drawdowns in 2006-2013 of the FTSE Bursa Malaysia KLCI component stocks**
Source: Computed and elaborated by the authors

serious doubts on the value of portfolio optimization within an emerging market context.

Nonetheless, while our Sharpe-based portfolios underperform the simple policy of dividing capital evenly, the results are based on a single out-of-sample period. Since varying constraints seem to affect trading outcomes in some way, future studies can explore these two distinct portfolio rules under the same experimental design using continuous time and walk-forward optimization. As well, this paper lacks any statistical test as this is deemed extraneous, given the underperformance of the portfolios. Such absence is also consistent with existing studies, among others Levy and Duchin (2010) [5] and Nor and Islam (2016) [6]. Relevant tests for the mean returns including Jobson and Korkie's (1981) [18] and Memmel's (2003) [19] test of equality for the Sharpe ratio can thus be utilized.

Portfolio diversification will remain an interesting and important research topic. For instance, dissecting the cause for constraints-and-performance relationship is an appealing avenue. Likewise, other noteworthy issues relating to portfolio diversification within and/or across several asset classes such as asymmetry, causality, contagion and interdependence (see Baur and Löffler, 2016; Cai et al., 2017; Han et al., 2016; Low et al., 2016; Shahzad et al., 2017b, 2017c, 2017d, among others) [20-26] as well as portfolio size, different objective functions and risk measures, international diversification and complex constraints (see Bodnar and Zabolotskyy, 2013; Cesarone et al., 2016; Moosa and Al-Deehani, 2009; Zhang, 2015, 2016) [27-31] can be considered for further analysis.

As noted earlier, our sample ends in December 2015 to alleviate potential problems associated with 1MDB financial scandal and interest rate increment in the U.S., which may result in unprofitable outcomes and/or inefficient portfolios during the extended holdout sample phase. Although the Malaysian market appears more efficient over time (Nor and Wickremasinghe, 2017) [32] and might reinforce the idea of its diversification benefits for the period of 2016 and onwards, structural change in risk/return dynamics imposes misallocation vulnerability within a single optimization procedure. From a practical viewpoint, international investors might reallocate their monies in view of the above scandal and/or opportunity elsewhere. Since the current article focuses on the performances of optimal portfolios against naïve diversification policy under varying constraints, future studies can thus explore walk-forward optimization to reflect continuous change in market prices (at specific intervals) which might incorporate emerging issues such as above, as long as the market is information efficient.

Taking everything into account, the performance of investment portfolios is indisputably influenced by many factors. Given the theoretical and practical implications of portfolio diversification, further investigations are needed to assess its relevance and true potential for emanating adequate risk-adjusted returns.

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