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## THE OPTIMIZATION MODEL OF WORKING TIME FUND ALLOCATION IN THE THEORY OF MARGINAL UTILITY

Today Ukrainian economy is in the state of severe crisis. The hopes of the 1990s for the thriving market economy failed. The attempts to put the state economy on market platform, to privatize state-owned enterprises, to liberalize prices were all vain. Different political forces have always tried to promote their successful

economic policy when proclaiming rapid growth of state economy in their time. But the long-run comparative study leads to deplorable results. Comparative sampling statistics on selective economic sectors over the period of 1970-2010 [1,2] bear witness to the economic hardship of Ukrainian (see Table 1).

Table 1

Selective commodity production output in Ukraine over the period of 1970-2010

Commodity list	Unit of measurement	Years				
		1970	1980	1990	2000	2010
Electrical power generation	billion kWh	137,6	236,0	298,5	171,4	188,8
Iron ore and concentrates output	million tons	111,0	125,0	105,0	55,9	78,5
Flour production	thousand tons	7526	7485	7671	3076	2632
Iron production	million tons	41,4	46,5	44,9	25,7	27,4
Steel production	million tons	46,2	49,9	48,5	25,8	17,5
Agricultural sector tractors production	thousand units	148	136	106	4,0	5,2

The data analysis in Table 1 shows that the economic collapse in Ukraine in the late 1990s had disastrous consequences. Those losses have not been covered yet. Despite the positive dynamic of the production output in Ukraine over the past ten years, Ukrainian economy has not reached the level of maturity of 1970-80s yet.

The given situation provokes scientists-economists into analyzing the causes of the economic emergency state along with the ways to overcome this crisis. The following causes are given: major blunders in home and foreign policies (including economic policy) made by the government and the head of the state, the lack of investment, economic and political instability, ineffective legislative and regulatory framework etc. The given conclusion has reasonable grounds. The results of the analysis are to logically lead to the search of realistic well-grounded ways out of the situation. In this regard the scientists and politicians are not unanimous as for the solution.

One of the causes of unsatisfactory economic state is due to the ignorance about low workforce productiv-

ity and the absence of the relative incentive mechanism. In spite of the extensive scientific research into this problem, the comprehensive management of labour concept has not been developed to perfectly fit into Ukrainian market environment. The existing foreign experience does not demonstrate the single approach to solve the problem albeit the developed countries achieved much more results in tackling the questions of well-grounded standards of wages and social security. This statement is supported by the data of Table 2. The countries under analysis were chosen according to the compatibility of the number of their population (except for the Russian Federation). The low remuneration of labour rate in Ukraine to a great extent comes from the low level of workforce productivity and the high level of energy and material consumption in the industries.

Despite the above said selective analysis of the relative figures in Ukraine and developed countries (Table 3) shows that developed countries outperform Ukraine by remuneration of labour rate.

**Comparative figures in Ukraine and developed countries over 2010**

Countries	Population, million people	Unemployment rate %	GDP per person, US dollar	Salary, US dollar
Ukraine	45,7	8,1	6055	282,2
The United Kingdom	62,2	7,8	32459	3930,4 <sup>1</sup>
Spain	46,1	20,1	26967	3059,6 <sup>1</sup>
Italy	60,5	8,4	27081	3200,2 <sup>1</sup>
Canada	34,1	8,0	35445	3590,4
Poland	38,2	9,6	17342	1069,9
France	64,9	9,7	29598	4001,2 <sup>1</sup>
The Russian Federation	141,8	7,5	14166	698,5

<sup>1</sup> – according to 2009 [2].

**Relative GDP figures and salaries in different countries in 2010**

Countries	Relative excess GDP per person	Relative excess of salary of an employee
Ukraine	1,00	1,00
The United Kingdom	5,36	13,93
Spain	4,45	10,8
Italy	4,47	11,34
Canada	5,85	12,72
Poland	2,86	3,79
France	4,89	14,18
The Russian Federation	2,34	2,47

Table 3 makes it clear that the excess of employee's salary significantly outruns the figure of the excess of GDP per person in the majority of the developed countries. In the United Kingdom the GDP rate per person is 5,36 times more than in Ukraine and as far as the salary rate concerns, the excess comes to 13,93. Poland and the Russian Federation have the smallest differences as the post-soviet states undergo similar problems and tendencies when establishing market economy. To a large extent the given problem has the nature of "vicious circle": the low remuneration of labour rate leads to the low workforce productivity and vice versa the low workforce productivity gives no chance to salary growth. Consequently the problem of effective incentive for the staff productivity under the market-based circumstances is critical for Ukrainian economy. The author suggests solving that problem by means of the theory of marginal utility.

The theory of marginal utility appeared at the end of the 19<sup>th</sup> century. This theory contradicted the fundamental principles of the labour theory of value by K. Marx. Scientific research conducted in this direction was characterized by soviet ideologists as "apologetic", "bourgeois", "vulgar" and "pseudoscientific". That attitude undergoes sustainable change, and the works by Carl Menger, Eugen von Böhm Bawerk, Friedrich von Wieser, Joseph Schumpeter, Antuan Auguste

Walras, Alfred Marshall and other scientists become the benchmarks for the research of our scientists-economists. In Ukraine the methodological issues of the theory of marginal utility have been lately in focus of S.V.Glivenko [3], I.D.Pogorelovska [4], N.N.Ivanov [5], I.V.Golovataya [6, 7], Y.V.Petrovskaya [8], A.M.Zaytseva [9], M.I.Kernitskaya [10], M.M.Artus [11,12], M.F.Goncharenko [13], I.V.Pinchuk [14] and others. Still the potential of the theory of marginal utility is not currently used in full swing. First of all it concerns practical issues.

There is no doubt that the theory of marginal utility is not a perfect instrument and has some obvious disadvantages to work on. The theory of marginal utility has yet to tackle the questions that have been previously ignored by Marxist economics. These are the following questions:

1. Classic barter trade on the market platform is voluntary-driven partnership model. If according to the labour theory of value only the commodity of equal value can be the subject to barter, then we ask: "Why does this barter take place?". Karl Marx deviated from that question himself, believing that "The use values of the commodity make the subject to special discipline – commodity science" [15]. The given statement makes the process of use value assessment on market platform almost useless. Having assessed all the labour results,

it is possible to define the “real” (or socially necessary) value of commodity, but we cannot answer the question if the customer going to buy this commodity. On condition of waiting for the “market recognition” of the commodity the assessment of labour value of the commodity loses its significance. At the same time the theory of marginal utility makes it possible to achieve so called “demand curves”.

2. The statement of barter trade on equal value basis does not explain the mechanism of setting monopoly price from the economic perspective. Despite the fact that the theory of pricing under monopoly is studied by economics in detail, one of the questions is still on the table: “Why do some consumers refuse to buy commodity at monopoly price and why is this price acceptable to the others?” Only the theory of marginal utility has the answer to this question.

3. Labour theory of value presupposes that during the employment the capitalist acquires the commodity of special type – workforce. At the same time the question of the intensive use of this commodity (the question of intensive labour) is completely ignored. Rejecting all the contrary points of view in the theory, Marxist economics was yet bound to acknowledge the importance of incentive mechanisms. The assumption that wages are the price of labour but not of the workforce has been interpreted as sacrilege. At the same time the theory of marginal utility has sufficient theoretical basis to prove the relation between the salary rate and the rate of workforce productivity.

The classic economics has not had the unanimous approach in tackling the contradictory question of effective staff motivation. There is no single defined and shaped opinion on this issue in classic economic study as well as among the modern scientists. We can single out several streams in the works of prominent authors:

1. Adam Smith (1723-1790) focuses on the differences between piece wages and time wages in his work «An inquiry into the nature and causes of the wealth of nations» (1776) in chapter 8 of the Book 1, dedicated to the questions of wages and salaries, he describes the situations, when profitable piece wages stimulate workmen so that they exhaust themselves with work and “are very apt to overwork themselves, and to ruin their health and constitution in a few years.” [16, P.75]. That insight brought Smith to the conclusion that time wages are more preferable as the workman gets fixed wages for previously determined task. That Smith’s assumption may yet be regarded as contradictory from the point of view of work overloads liquidation.

2. Frederick Winslow Taylor (1856 –1915) was one of the first management consultants and pioneers of scientific management. In his work “The Principles of Scientific Management”, Taylor cast the new light on the contrary tendency, which he called “soldiering” [17, P.226-236]. That was common practice that took place when the workmen were “deliberately working

slowly so as to avoid doing a full day’s work” [17, C.227]. It is quite transparent in his work that Taylor highly doubts the effective monitoring of the management in order to liquidate the “loafing or soldiering” tendency. Taylor gives the illustrative answer of some master when monitoring the site “Well, I can keep them from sitting down, but the devil can’t make them get a move on while they are at work!” [17, P.231]. This way Taylor admitted that without motivation workers would always slow down the productive capacity of work.

3. Max Weber (1864-1920) in his work “The Protestant Ethic and the Spirit of Capitalism” (1905) shows the differences in the attitudes of separate workers: either to be extremely devoted to their craft and earn a lot or to earn enough to satisfy their traditionally established needs [18, P.80-81]. In Weber’s opinion the attitude of the workman to labour is dictated by inner ethical, cultural and religious factors: the Protestants, being under their religious beliefs, meet the needs of capitalistic society. That is why Protestant descendants are more successful in business. Statistical data supports Weber’s view and gives his position reasonable ground. At the same time Tom Bethel provides numerous facts that disapprove the priority of institute of religion. Bethel gives the institute of property the primary position under the conditions of market economy and he is very persuasive stating that under the equal conditions enterprise and organization, established on the basis of collective property, are less effective than private enterprises [19, P.51-65].

The defined problems were given careful study both by foreign and “home” scientists. Significant results were achieved when tackling the problems that occurred under different conditions. One of the results of the research on this platform was the assumption of the absence of a universal method. It means that the solvation of particular problems and the generation of the relevant practical recommendations can be realized on the scientific basis but have its individual nature.

The basic tasks of the present article are the following:

- the analysis and the grounds of the possibility and rationale to apply the theory of marginal utility in order to give the reason to wise allocation of the resources (time in particular) for achieving maximum utility;
- the analysis of the optimization of the working day time in the context of the theory of marginal utility;
- the development of the model to apply the theory of marginal utility when performing the tasks to optimize time allocation;
- the approbation of the developed model under real conditions.

The basic idea of the research given in the article can be formulated the following way. The process of

the employment can be overlooked as a certain act of the resources (commodity) exchange and each of them has particular value for a person: a person exchanges a piece of his spare time (which has its value) for some material reward (which also has its value). Standing on the ground of the theory of marginal utility it is possible to state that the given exchange is possible till the total utility of spare time exceeds the utility of total money reward. It means that one can define the optimal allocation of total daily time fund of a person where the total utility of the labour reward and the useful time left will be maximal.

The author makes the following assumptions during the research:

1. It is admitted in the classic the theory of marginal utility that the marginal utility lessens with yet another item of benefit. Despite that the statement is not correct enough. For the majority of the material benefits the marginal utility of the benefit initially grows and then goes down. That is why it is more reasonable to apply the quadratic but not the linear dependence for the mathematical description of the total utility of the benefit. The application of the quadratic dependence explains the fact that the unlimited use of the benefits provokes the total result of this use (utility) to fall rather than grow (benefit excesses do harm). The application of the linear dependence makes the mathematical tools of the analyzed models less complicated. In that case one should previously agree on the admissible determination domain i.e. the interval of argument range where utility function can be regarded as linear dependence.

2. When the process of the exchange of spare time for the labour reward is considered, it is important to keep it in mind that to ensure the minimal level of physiological survival man needs some minimum of primary benefits. He needs time to sleep and money to meet immediate life demands (food, accommodation). In this regard when conducting the research it is necessary to exclude this time from the model for the sake of its accuracy. The given assumption leads to the critical conclusion that the utility of one and the same benefit cannot be overlooked on the whole scale of its use.

3. When considering the "value" of working time it is indispensable to take into account the intensification of a working process that in its turn leads to the change of workforce productivity.

The labour theory of value focuses on two types of labour: simple and complicated. Certain amount of complicated labour can be treated as equal by its value to certain bigger amount of simple labour.

The cost of an hour of a complicated labour grew on account of training and preparation etc. The experience admits the influence of different types of motivation when one and the same person performs his task more or less actively. We should bear in mind the fact that the modern economy has different industrial pro-

cesses that can be divided into two groups according to this quality:

- 1) the processes that accept the intensification of industrial process;

- 2) the processes that do not accept the intensification of workforce labour exploitation.

4. The given article focuses exclusively on the situation of hired labour. This restriction makes it possible for the research to cut any other institutes ( property, cooperation, corporate management etc), that can affect basic factors, influencing the value of the resources under analysis: option of extra benefit, option to invest some amount of money the employee earned, the intensification of the labour.

5. Quadratic form of the dependence of marginal utility is satisfactory only for such types of benefits that can be regarded as primary (or vital). At the same time some types of the secondary benefits that have social nature cannot be described with the help of quadratic equations. The utility of such types of benefits is constantly on the increase and at the same time the speed of utility is constantly on the decrease.

The present article focuses on the optimization of the daily time fund allocation of one person for the following sectors: sleep, rest (spare time), and the work aiming at labour reward. The modeling of the utility function of the benefits under analysis was presented by the following functions:

1. Sleep utility function:

$$U_s = 25 \cdot x - 1,56 \cdot x^2 \quad (1)$$

2. Rest utility function:

$$U_r = 12,5 \cdot x - 0,39 \cdot x^2 \quad (2)$$

3. Wages utility function:

$$U_w = \frac{100}{1 + e^{T-x}} \quad (3)$$

$x$  – time, that a person spends to receive certain type of benefit;

$t$  – active time when an employee earns minimum living wages.

Dependences (1)-(3) are presented below on Fig. 1.

Graph 1 illustrates the dependence that has particular qualities:

1. Sleep utility function and rest utility function are presented in the form of quadratic dependences. For such benefit like sleep, maximum of the benefit utility (100%) is reached at  $t=8h$  and maximum of rest benefit utility is reached at  $t=16h$ . There is no doubt that the given figures for separate people can change, but the model gives average figures.

2. Wages utility function is presented by sigmoid (logistic curve) and  $t$  value was taken equal from 1 to 16 hours in other words the workforce productivity level can vary in significant range.

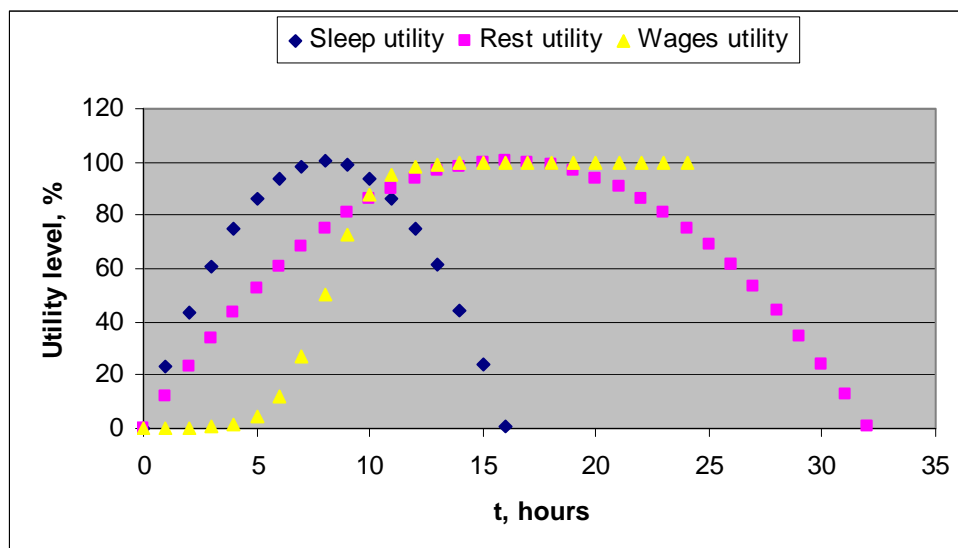


Fig 1. Dependences of the utility level of different types of benefits

The above given model is the result of optimization calculations performed in Excel spreadsheets. Here we solve the task of the maximization of the total integrated utility for a person at common daily time fund restriction of 24 hours. Table 4 and Fig. 2 illustrate the results of the calculations. The following conclusions can be drawn on the basis of Table 4 and Fig. 2:

- at the maximum utility level equal to 300, the achieved utility will be less, that is explained by the 24-hour restriction of daily time fund;
- the optimal working time of one employee always exceed the active time that covers minimum standard of living;

- the optimal sleep time along with the optimal spare time grow progressively as the workforce productivity grows;

- in case of low workforce productivity optimal sleep time and optimal rest time are reallocated in favour of sleep (this conclusion does not contradict the reality as it is).

The research covered in the article resulted in the following conclusions:

1. The research given in the article is grounded on the statistical average data. Actual value of the recommendations will be much higher, if the analogous research is conducted for different categories of employees with the identified wages rate. Analogous approach can be also applied to the environments of the selected sectors and enterprises.

Table 4

**The variants of optimal daily working time fund allocation in relation to the workforce productivity level**

The active time that covers minimum standard of living, hour	The optimal sleep time, hour	The optimal rest time, hour	The optimal working time, hour	The total integrated utility
1	7,1	12,4	4,5	290
2	6,9	11,7	5,4	288
3	6,8	11,1	6,1	284
4	6,6	10,4	7,0	280
5	6,4	9,7	7,9	275
6	6,2	9,0	8,8	270
7	6,1	8,3	9,6	264
8	5,9	7,5	10,6	258
9	5,7	6,8	11,5	251
10	5,5	6,1	12,4	244
11	5,3	5,4	13,3	236
12	5,2	4,6	14,2	227
13	5,0	3,9	15,1	218
14	4,8	3,1	16,1	208
15	4,6	2,4	17,0	198
16	4,4	1,7	17,9	187

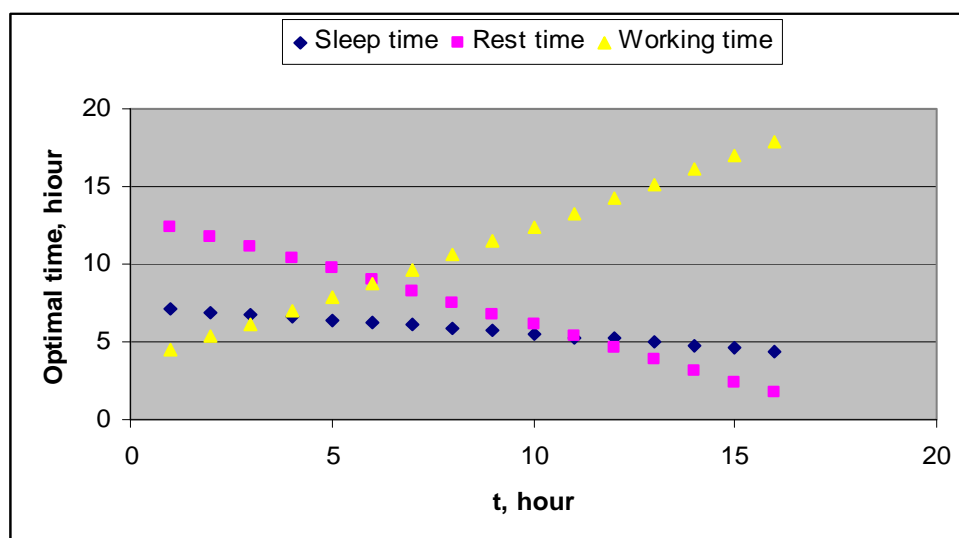


Fig. 2. Optimal daily working time fund allocation

2. Standard of living figure significantly influences the figures in the model. Two critical points should be considered when using it in the model:

- the accurate calculations for this figure in terms of the specific environment;
- the compulsory use of the minimum wages value figure in the model (basically, the wages must exceed the minimum standard of living).

3. The subject of the further scientific research in this direction is about the study of complex influence of both wages rate and minimum standard of living rate (or rather minimal wages rate) on optimal daily working hours of an employee.

On the ground of the conclusions the authors reached they outlines the following suggestions and recommendations along with the further research trends:

1. The problem of the optimal working time fund allocation is quite challenging and its successful solvation needs the real environment of a particular enterprise.

2. The application of the developed model in the particular enterprise environment presupposes the real minimum wage rate identification for particular environment: region, sector, enterprise, occupation etc.

3. The use of the suggested approach helps to define the optimal daily working hours as well as to solve the inverse problem of fixing the appropriate standard wages as a an incentive to daily productive labour. At the same time in the environment of a particular enterprise managers get the chance to efficiently tap into workforce potential in several ways:

- changing the working practices of the staff and the daily working hours (under the existing legislation);
- changing the existing remuneration practices;
- optimizing the wage amount.

4. Advantageous line in further development of the given model is in its expansion on account of the introduction of the extra factor – the aptitude of an employee (economically active person) for saving and investment.

5. The suggested approach would be effective for the workplaces assessment at the enterprise to reveal the workforce potential.

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### **Шевченко В. В., Інякін В. М. Оптимізаційна модель розподілу фонду часу працівника в теорії граничної корисності**

В статті розроблено оптимізаційну модель розподілу добового фонду часу окремого індивіда з

використанням теорії граничної корисності. Використання запропонованого підходу дозволяє не тільки знаходити оптимальну тривалість робочого часу працівника, але і вирішувати протилежне завдання – встановлювати необхідний рівень оплати праці працівника з метою стимулювання його до інтенсивної праці протягом робочої доби. Запропонований підхід доцільно використовувати в процесі атестації робочих місць на підприємстві з метою виявлення можливих резервів підвищення ефективності використання трудових ресурсів.

*Ключові слова:* гранична корисність, модель, оптимізація, фонд часу, продуктивність праці.

### **Шевченко В. В., Інякін В. Н. Оптимізаційна модель розподілу фонду часу працівника в теорії предельної полезности**

В статті розроблена оптимізаційна модель розподілу суточного фонду часу окремого індивіда з використанням теорії предельної полезности. Використання запропонованого підходу дозволяє не тільки визначати оптимальну тривалість робочого дня працівника, але і вирішувати зворотню задачу – встановлювати необхідний рівень оплати праці працівника з метою стимулювання його до інтенсивного виробничого праці в течение робочого дня. Запропонований підхід було б цілком доцільно використовувати в процесі атестації робочих місць на підприємстві з метою виявлення можливих резервів підвищення ефективності використання трудових ресурсів.

*Ключевые слова:* предельная полезность, модель, оптимізація, фонд часу, продуктивність праці.

### **Shevchenko V. V., Inyakin V. M. The Optimization Model of Working Time Fund Allocation in the Theory of Marginal Utility**

The article presents the optimization model of the allocation of daily working time fund per person with the use of the theory of marginal utility. The use of the suggested approach helps to define the optimal daily working hours as well as to solve the inverse problem of fixing the appropriate standard wages as an incentive to daily productive labour. The suggested approach would be effective for the workplaces assessment at the enterprise to reveal the workforce potential.

*Keywords:* marginal utility, the model, optimization, time fund, workforce productivity.

Received by the editors: 10.11.2014  
and final form 23.12.2014