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THE USE OF DIGITAL IMAGES FOR DETERMINING THE COORDINATES OF OBJECTS IN THE REAL PLANE WITH A FLAT FOUR-POINT TRANSFORMATION

Digital imaging is used to identify objects on the surface as well as in the ground. These objects exist in three dimensions, but after making picture are two-dimensional. The unit of length is changing from meter or centimeter to pixel. There are methods for transforming coordinates from one dimension to another such as Direct Linear Transformation method (DLT) and a flat four-point transformation method. By reading the coordinates of the objects in the pictures, in pixels, you can calculate length (i.e. in meters) using a transformation matrix.

Keywords: parameters DLT-method; four-point transformation method; transformation matrix; linear transformation coefficients.

Introduction

Direct linear transformation (DLT) is a method of determining the three dimensional location of an object (or points on an object) in space using two views of the object. First, let's consider a few different ways of obtaining multiple views of an object: two cameras, one camera and one prism, one camera and an arrangement of mirrors, one camera, two separate images.

DLT method uses a real-space perspective projecttion on the image plane. It allows to calculate the parameters, which directly describe the spatial orientation of photos without the knowledge of approximations of objects.

There are eleven DLT parameters. Determination of the coordinates in the space of 3-D requires the use of all. The 2-D space needs only nine. DLT parameters are obtained from solving the system of linear equations using matrix transformation. The calculations related to the calibration of camera used 6 points and receives 12 linear equations.

The image coordinates of the DLT method, 3-D is calculated from the fallowing formula (1)

$$u = \frac{L_1 x + L_2 y + L_3 z + L_4}{L_9 x + L_{10} y + L_{11} z + 1}$$

$$u = \frac{L_5 x + L_6 y + L_7 z + L_8}{L_9 x + L_{10} y + L_{11} z + 1} \qquad (1)$$

while in 2-D method from the formula below (2)

$$u = \frac{L_1 x + L_2 y + L_3}{L_7 x + L_8 y + 1}, \; u = \frac{L_4 x + L_5 y + L_6}{L_7 x + L_8 y + 1} \qquad (2)$$

Calculations in four-point transformation are using simple mathematical operations and needs the minimum points of assumptions about the quality of the equipment and the position of the optical measurements. Four-point calibration method leads to linear equations for the unknown coefficients of transformation and does not require orthogonality and equal scales the coordinate axes. Transformation is a flat representation that sets points on one plane to another. The first plane is identified with the camera-monitoring plane Π and the second plane – real plane.

The transformation can be represented by the following formula

$$T(P_П) = P_Г. \qquad (3)$$

T-transformation;

$P_П$- the point on the image plane;

$P_Г$ - the point on the real plane.

We found that:

• method does not require the orthogonal coordinate system or equal scales;

• the optical axis of the lens does not have to be perpendicular to the plane of the photosensitive;

• do not have know your digital camera parameters.

The exact relationship between the points belonging to both the plane defined by the formula (4):

$$T(P_П) = P_Г \begin{bmatrix} X_Г \\ y_Г \end{bmatrix} = \begin{bmatrix} \dfrac{a_1 x_П + b_1 y_П + c_1}{\alpha x_П + \beta y_П + 1} \\ \dfrac{a_1 x_П + b_1 y_П + c_2}{\alpha x_П + \beta y_П + 1} \end{bmatrix} \qquad (4)$$

In the formula, there are eight coefficients S:

$$S^T = [a_1, b_1, c_1, a_2, b_2, c_2, \alpha, \beta]. \qquad (5)$$

The coefficients S uniquely define the transformation. Describe the relationship between the real plane and the plane of the monitoring, including representing both the position of the planes to each other as well as the parameters and the optical properties of the camera.

To carry out the identification used to know the position of the monitored points in the plane, and the corresponding points on the real plane. You must know the coordinates of the four pairs of points, no three of which cannot be aligned. This allows using the formula (3), to construct a system of eight linear equations with eight unknowns, which are the coefficients of the transformation (7). Subject to the assumption of a lack of alignment points, the system has exactly one solution (9) where:

S-matrix of the coefficients (5);

C- matrix of transformation;

H-matrix of transformating parameters.

Classical identification algorithm involves placing the camera in the calibration standard with marked points, respectively, defining the actual plane. These points are being developed in the camera image (plane monitoring). Then, based on knowledge of the position of points on the two planes, using the formula (7) is identified transformation flat. After completion of the calibration pattern may be removed from the field of the camera. Graphic interpretation of the situation shown in Fig. 1.

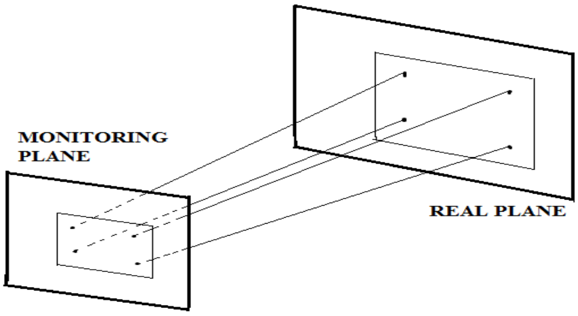

Fig. 1. Interpretation of the removing

In order to determine the four calibration points can be used prepared flat square pattern with marked corners. Coordinates of the corners of the square:

$$P_{\Gamma_1} = [0 \quad 0]^T, \qquad P_{\Gamma_1} = [1 \quad 0]^T$$

$$P_{\Gamma_1} = [0 \quad 1]^T, \qquad P_{\Gamma_1} = [1 \quad 1]^T \qquad (6)$$

Calibrating the transformation can be performed in a flat five stages:

• We put a square pattern on a local level, linking certain points the plane of vectors corners;

• We picture the pattern in the camera setting in which it will be working in the future;

• We recover, through image processing, monitor the position of the corners of the square;

• We do identify factors S (calibration) according to the formula (6).

Found coefficients represent information about the relative position of the camera and the monitoring plane.

Research and calculations

The model of a rectangle measuring 18 cm x26 cm divided into 1 cm squares. A digital image of model was analyzed in the Matlab software environment.

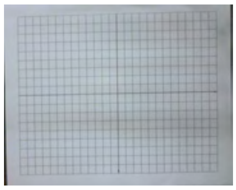

Fig. 2. Digital image of model

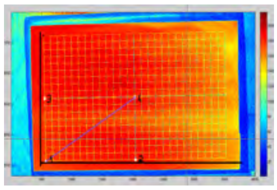

Fig. 3. Picture from Matlab

For four points from the image plane take theirs value of coordinates (in pixels). Insert to the transformation matrix C and to the matrix H (in centymeters) (7).

Receive values of S (parameters of camera calibration). Parameters S with the values of unknown point in pixels, insert into the formula (4).

After the calculations obtained the coordinates in the plane of the real system (in centymeters). The algorithm written in Matlab, automatically performs the calculations.

Errors are calculated using the standard deviation.

Table 1

Example

Coordinates of points in the real plane, cm	Coordinates of points in the image plane, pixels
PГ1=[1;1]	PП 1=[573;2428]
PГ2=[14;10]	PП 2=[2025;1382]
PГ3=[11;1]	PП 3=[1705;2408]
PГ4=[1;10]	PП 4=[553;1417]

S parameters obtained:

a1=0.0090; a2=-0.0002;
b1=-0.0002; b2=-0.0089;
c1=-3.7011; c2=22.8168;
α=0. β=0.

Values of pixels for unknown point:
X=1125 pixels (**x=6 cm**) Y=1860 pixels (**y=6 cm**)

Values of length obtained from the computer:
[x, y] =[6.0519, 6.0378], cm.

$$\begin{bmatrix} x_{\Gamma_1} \\ y_{\Gamma_1} \\ x_{\Gamma_2} \\ y_{\Gamma_2} \\ x_{\Gamma_3} \\ y_{\Gamma_3} \\ x_{\Gamma_4} \\ y_{\Gamma_4} \end{bmatrix} = \begin{bmatrix} x_{\Pi_1} & y_{\Pi_1} & 1 & 0 & 0 & -x_{\Pi_1} * x_{\Gamma_1} & -y_{\Pi_1} * y_{\Gamma_1} \\ 0 & 0 & 0 & x_{\Pi_1} & y_{\Pi_1} & -x_{\Pi_1} * x_{\Gamma_1} & -y_{\Pi_1} * y_{\Gamma_1} \\ x_{\Pi_2} & y_{\Pi_2} & 1 & 0 & 0 & -x_{\Pi_2} * x_{\Gamma_2} & -y_{\Pi_2} * y_{\Gamma_2} \\ 0 & 0 & 0 & x_{\Pi_1} & y_{\Pi_1} & -x_{\Pi_2} * x_{\Gamma_2} & -y_{\Pi_2} * y_{\Gamma_2} \\ x_{\Pi_3} & y_{\Pi_3} & 1 & 0 & 0 & -x_{\Pi_3} * x_{\Gamma_3} & -y_{\Pi_3} * y_{\Gamma_3} \\ 0 & 0 & 0 & x_{\Pi_1} & y_{\Pi_1} & -x_{\Pi_3} * x_{\Gamma_3} & -y_{\Pi_3} * y_{\Gamma_3} \\ x_{\Pi 4} & y_{\Pi_4} & 1 & 0 & 0 & -x_{\Pi 4} * x_{\Gamma_4} & -y_{\Pi_4} * y_{\Gamma_4} \\ 0 & 0 & 0 & x_{\Pi_1} & y_{\Pi_1} & -x_{\Pi 4} * x_{\Gamma_4} & -y_{\Pi_4} * y_{\Gamma_4} \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \\ c_1 \\ a_2 \\ b_2 \\ c_2 \\ \alpha \\ \beta \end{bmatrix}, \qquad (7)$$

$$H = C * S, \qquad (8)$$

$$S = C^{-1} * H. \qquad (9)$$

Application

1. Four-point transformation is a simple method.
2. Fast way can calculate the coordinates of small objects on the plane and gives distances with little error.
3. No needs to know the parameters of the camera.

Literature

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ВИЗНАЧЕННЯ КООРДИНАТ ОБ’ЄКТІВ У ДІЙСНІЙ ПЛОЩИНІ ЗА ЇХНІМ ЦИФРОВИМ ЗОБРАЖЕНННЯМ З ВИКОРИСТАННЯМ ЧОТИРИТОЧКОВОГО ПЕРЕТВОРЕННЯ

Є. Томашевська

Цифрові зображення використовуються для ідентифікації об’єктів як на поверхні, так і на глибині. Ці об’єкти існують у трьох вимірах, але після створення малюнка є двовимірними. Одиниця довжини змінюється від метрів або сантиметрів до пікселя. Існують методи для перетворення координат від одної розмірності до іншої, такі як пряма лінійна трансформація (DLT) за допомогою плоского чотириточкового методу трансформації. При зчитуванні координат об’єктів на фотографії в пікселях можна обчислити довжину (метрах) за допомогою матриці перетворення.

Ключові слова: параметри DLT-методу, чотириточковий метод трансформації, матриця перетворення, коефіцієнт лінійного перетворення.

ОПРЕДЕЛЕНИЕ КООРДИНАТ ОБЪЕКТОВ В ДЕЙСТВИТЕЛЬНОЙ ПЛОСКОСТИ ПО ИХ ЦИФРОВОМУ ИЗОБРАЖЕНИЮ С ИСПОЛЬЗОВАНИЕМ ЧЕТЫРЕХТОЧЕЧНОГО ПРЕОБРАЗОВАНИЯ

Е. Томашевская

Цифровые изображения используются для идентификации объектов как на поверхности, так и на глубине. Эти объекты существуют в трех измерениях, но после создания рисунка являются двумерными. Единица длины изменяется от метров или сантиметров до пикселя. Существуют методы для преобразования координат от одной размерности к другой, такие как прямая линейная трансформация (DLT) при помощи плоского четрырехточечного метода трансформации. При считывании координат объектов на фотографии в пикселях можно вычислить длину (в метрах) с помощью матрицы преобразования.

Ключевые слова: параметры DLT-метода, четрырехточечный метод трансформации, матрица преобразования, коэффициент линейного преобразования.

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