

ГЕОДЕЗІЯ

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REGIONAL QUASIGEOID SOLUTIONS FOR THE UKRAINE AREA

The goal. The UQG2012 regional quasigeoid solution of an accuracy better than 4 cm with respect to the GPS-levelling data of the 1st and 2nd order was constructed by means of the least squares collocation method. In the first iteration the gravimetry-only quasigeoid UQG2011 was developed from the gravity anomalies for the subsequent detection of gross errors in GPS-leveling data. All terrain reductions were based on the 3"x3" digital terrain model SRTM3. **Scientific significance.** Thus, the final UQG2012 solution consists of gravity anomalies and quasigeoid heights at the points of a 2'x3' grid evaluated by means of the collocation method applied to the set of 4070 GPS-leveling quasigeoid heights plus the above mentioned gravimetry data. After first iteration, the comparison of the UQG2012 solution with all independent GPS-leveling data (1st – 4th order networks given in the Baltic height system) shows a good agreement with rms < 4 cm. This noise level corresponds to an estimated accuracy of the quasigeoid UQG2012 for the Ukraine and Moldova area higher than 4–5 cm with respect to GPS-leveling points of different orders. The evaluation of the UQG2012 solution with independent GPS-leveling data of the 1st and 2nd orders gives a significantly better agreement with rms of about 1.5 cm. Finally, the comparison with the European quasigeoid EGG08 leads to differences of about 20-50 cm (with rms level about 10 cm) in certain areas and to the total mean shift of 25 cm caused by the different height systems used.

Key words. Ukrainian regional quasigeoid – Gravimetry – Altimetry – GNSS/leveling – Least squares collocation – Regularization – EGM08 global geopotential model – Remove-restore technique

1. Introduction

Since 1997 the gravimetric quasigeoid EGG97 [Denker and Torge, 1998] became the most representative solution for the Ukraine and Moldova area. MOLDGEO2004 and MOLDGEO2005 quasigeoids with an accuracy better than 10 cm were based on the fit of EGG97 to the Moldavian dense set of GPS/leveling (preferably of the 4th order) data given in the Baltic 1977 height system [Marchenko and Monin, 2004]. In this case the EGG97 solution has been preliminarily transformed to the Baltic height system. A similar approach in the Ukraine area led to pessimistic results due to a low accuracy of the EGG97 quasigeoid in some of the Ukrainian regions (such as Crimea), which is probably caused by the digital terrain model (DTM) of ca. 5'x5' resolution applied in [Denker and Torge, 1998] for this area as well as a poor land gravity data sets which were used in the Ukrainian area.

Thus, one can say that the first high-resolution gravimetric quasigeoid EGG97 (1'x1.5' grid), including the East European and Ukraine areas, was constructed in 1997 [Denker and Torge, 1998] and related to the NAP (Normaal Amsterdam Piel) or Normal Amsterdam Level. This solution was essentially improved as the EGG07 (quasi)geoid [Denker et al., 2007] and subsequently (2008) as the EGG08 (quasi)geoid within the European Gravity and Geoid Project (EGGP). An independent gravimetric

(UQG2006 – Ukrainian QuasiGeoid 2006) and combined (UQG2007 – from ca. 160000 gravity anomalies, ca. 640000 altimetry measurements and ca. 3000 GPS/leveling points) solution were derived (for a 2'x3' grid) in 2006 and 2007, respectively, see [Marchenko, Kucher, Renkevych, 2007] and [Marchenko and Kucher, 2008]. The quasigeoid models EGG07, EGG08, UQG2006 and UQG2007 have a variable accuracy of 5 – 10 cm in different areas estimated from the comparison with independent GPS/leveling control points.

This paper focuses on the new quasigeoid solutions with accuracy of about 1.5 cm with respect to GPS/leveling points of the 1st and 2nd order. The UQG2012 model for the Ukraine and Moldova area as well as previous mentioned solutions was constructed on the basis of the least squares collocation method with regularization [Neyman, 1979; Moritz, 1980]. Since kernel functions corresponding to radial multipoles were applied [Marchenko, 1998; Marchenko and Lelgemann 1998], only singular point harmonic functions were used for regional quasigeoid solutions on the basis of gravimetry, satellite altimetry, and GPS/leveling data. Terrain reductions rest on the 3"x3" digital terrain model SRTM3 (Jarvis et al., 2006) having (in comparison with leveling data) over one order better accuracy than DTM GEBCO and ETOPO1. The solution UQG2012 was constructed by the least squares collocation method applied within two

iterations. In the first iteration the gravimetry-only quasigeoid UQG2011 was obtained from the digitized gravity anomalies (continental area) and gravity anomalies in the Black Sea and Azov Sea areas derived from altimetry data of ERS-1, ERS-2, TOPEX/POSEIDON, JASON-1, ENVISAT, GFO missions (1992 – 2005), additionally to the national and BGI marine gravimetry data. The UQG2011 quasigeoid was used especially for an effective detection of gross errors in GPS-leveling data.

As a result, the final UQG2012 model consists of gravity anomalies and quasigeoid heights at the points of a 2'x3' grid which were evaluated by means of the least squares collocation method applied to the heterogeneous data set: 4070 GPS-leveling quasigeoid heights plus previous gravimetry data. Comparisons of the UQG2012 solution with GPS-leveling data show a good agreement with rms of about 1.5 cm at the GPS-points of the 1st and 2nd orders. This noise level corresponds to estimated geoid accuracy better than 5 cm for the Ukraine area and to the adopted requirements for the combined adjustment of the Ukrainian geodetic network based on the terrestrial and satellite GNSS data.

2. Initial data. Preprocessing

All the initial free air gravity anomalies Δg were used as two individual sets of land and marine (Black

Sea region) Δg . Let's start with a brief description of the marine Δg basically derived from the following altimetry and gravimetry data in the Black Sea area:

Subset 1 represents 643128 TOPEX/POSEIDON, ERS-1, ERS-2, JASON-1, ENVISAT and GFO Sea Surface Heights (SSH) taken for the period 1992-2005 and corrected by CSL AVISO for different geophysical phenomena and instrumental effects (declared accuracy ~ 2–5 cm)..

Because of data gaps in the corrected SSH (Black Sea region) the following additional set of point gravimetry data is used to support the SSH-only solution for Δg :

Subset 2 represents 20207 values of BGI and national marine point gravimetry measurements in addition to land gravimetry surrounding the Black Sea area not closer than 12 miles to the coast (accuracy ~ 5 mGal).

Fig. 1 illustrates AVISO corrected SSH from six satellite missions in the Black Sea area, which were filtered for residual temporal effects (and here also referred to the GRS80 ellipsoid). It has to be pointed out, that the surface in Fig. 1 differs from the geoid due to small deviations of Sea Surface Topography (SST) and various residual effects averaged in time. Fig. 2 demonstrates the result of processing the above two subsets by regularization method into a 2'x2' grid of the gravity anomalies. They were obtained by Marchenko and Yarema [Marchenko and Yarema, 2006] and in the following we call them as the Set 1 (marine Δg).

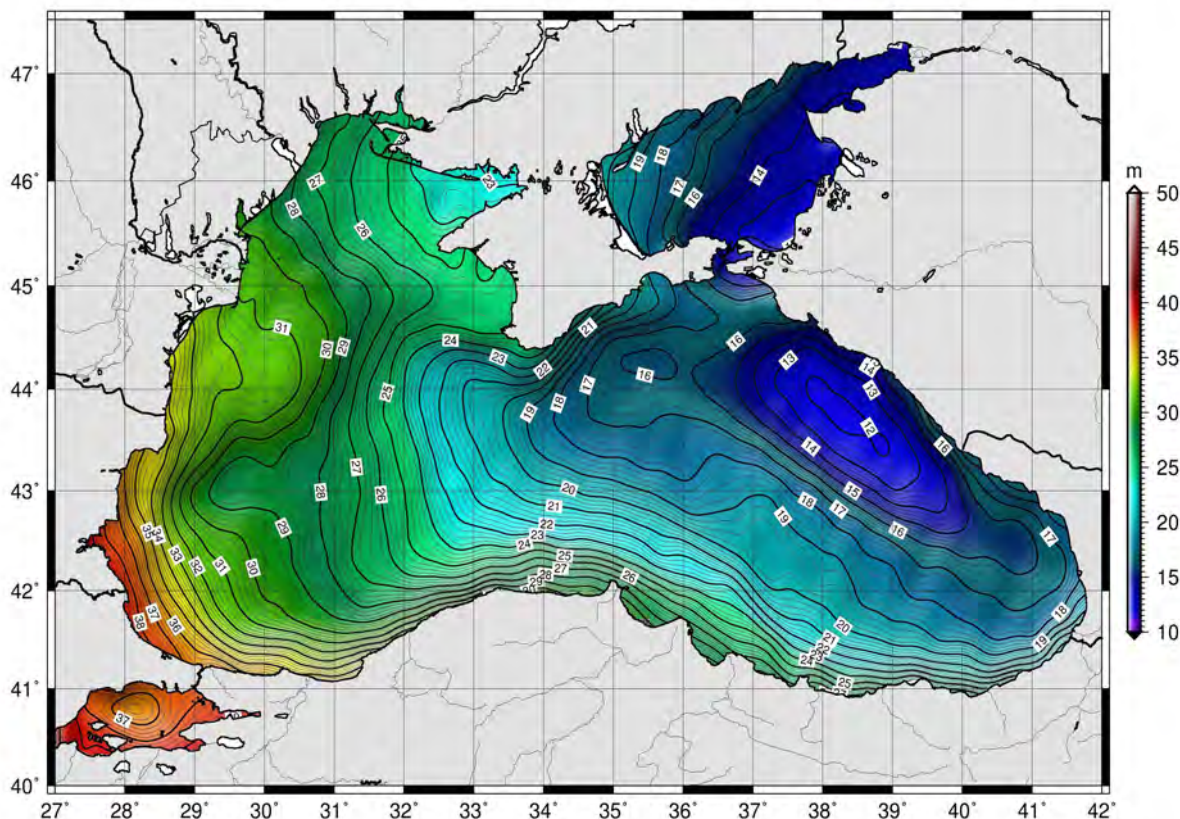


Fig. 1. AVISO corrected SSH [m] from TOPEX/POSEIDON, ERS1, ERS2, JASON-1, ENVISAT and GFO altimetry, filtered for residual temporal effects

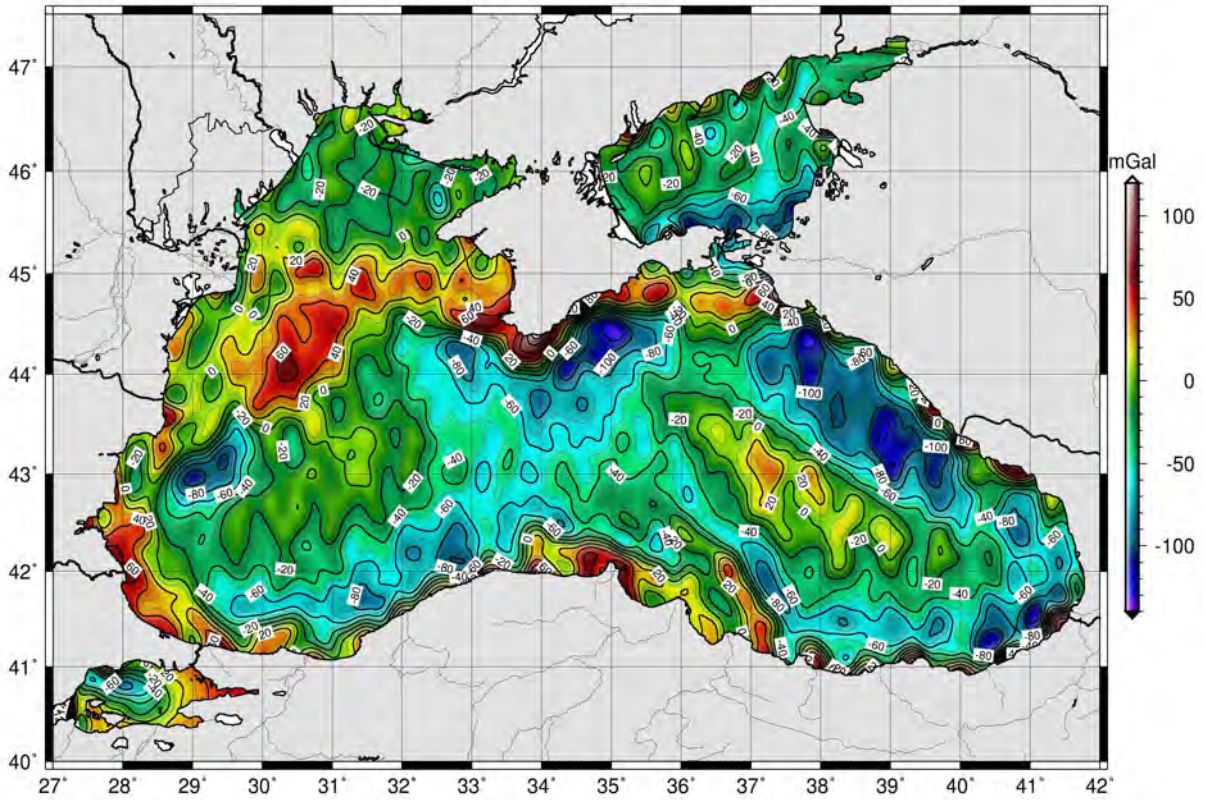


Fig. 2. Gravity anomalies [mGal] from six satellites missions (1992–2005) and BGI gravimetry plus national marine gravimetry measurements in the Black Sea area

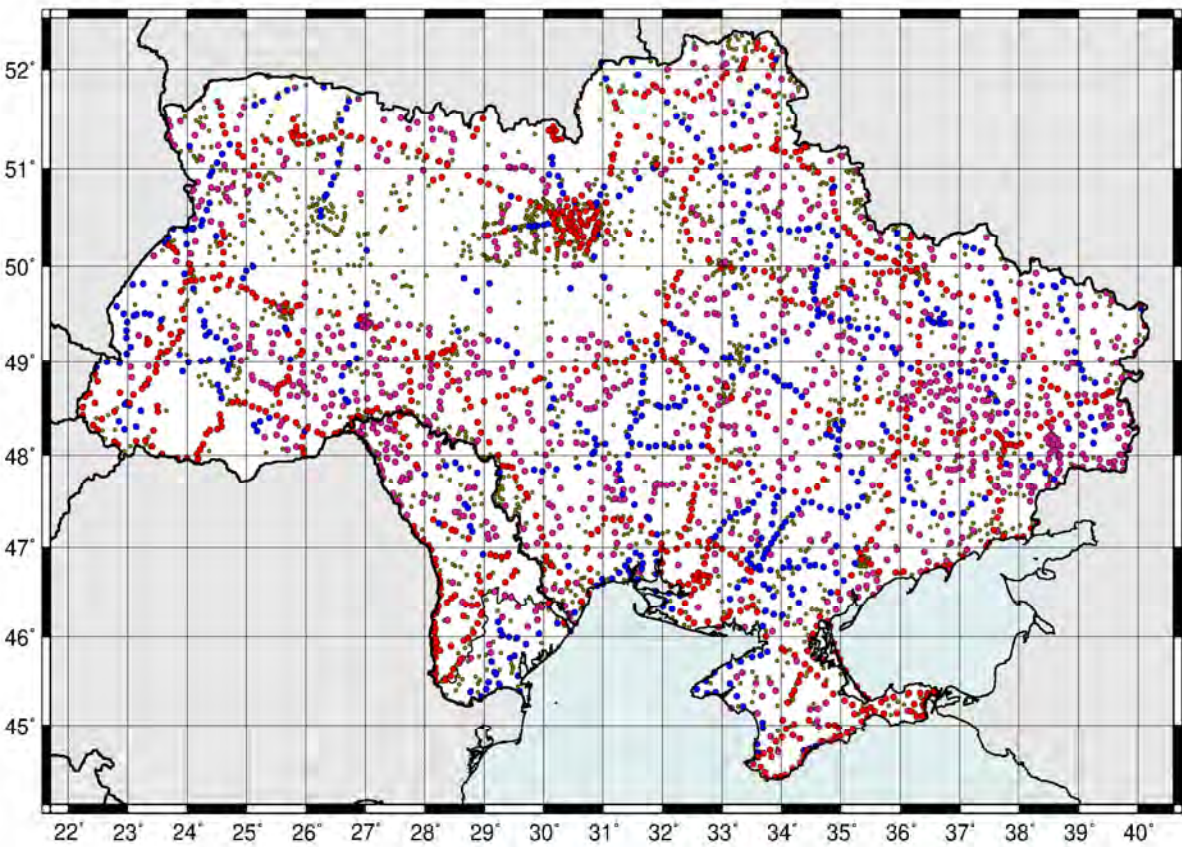


Fig. 3. Distribution of GPS/leveling data of the I (●), II (●), III (●), and IV (●) orders

The second set (Set 2) of free air gravity anomalies at the points of the 2'×3' grid was derived from the digitized Bouguer anomalies covering the Ukraine and Moldova area (accuracy ~ 1–2 mGal) with added gravimetric networks 1st, 2nd, and 3rd orders having the accuracy about 0.005, 0.01, and 0.05 mGal correspondingly. Then classical terrain reduction was applied for the conversion of the free air anomalies Δg to the Faye anomalies Δg_F . This reduction was based on the DTM SRTM3 given for a 3"×3" grid [Jarvis et al., 2006]. Subsequently, the Faye anomalies Δg_F were adopted as a basic initial information for quasigeoid computations. Note that the classical terrain reduction was derived by using the numerical integration up to a radius of 167 km and has maximal values of ca. 50 mGal in the Ukraine area considered.

Finally, Fig. 3 illustrates the Set 3 containing 4070 points of GPS/leveling data of different orders covering the Ukraine and Moldova area. Set 3 was formed after the detection of gross errors in GPS/leveling data given in the Baltic 1977 height system.

3. Gravimetric and combined quasigeoid solutions

The traditional remove-restore technique was applied to eliminate the long wavelength constituent of the gravity field. It was represented by the EGM2008 gravity field model [Pavlis et al., 2008] used up to degree and order 720. In the case of gravimetry-only data the residuals $\delta\Delta g$ were initially computed as

$$\delta\Delta g = \Delta g_F - \Delta g_{EGM2008}, \quad (1)$$

where Δg_F is the Faye anomaly; $\Delta g_{EGM2008}$ is the EGM2008 gravity anomaly up to degree and order 720. Then the prediction of the residual anomaly height $\delta\zeta_P$ at a point P inside the studying area was made by applying the collocation with regularization

$$\delta\zeta_P = \mathbf{C}_{\delta\zeta, \delta\Delta g} (\mathbf{C} + \alpha \mathbf{C}_{nn})^{-1} \mathbf{1}, \quad (2)$$

where in case of gravimetry data, $\mathbf{1}$ is the q -vector consisting of the components $\delta\Delta g_i$ ($i=1, 2, \dots, q$); q is a number of observations; \mathbf{C} is the $(q \times q)$ – covariance matrix of the residual gravity anomalies $\delta\Delta g$; $\mathbf{C}_{\Delta g, \delta\zeta}$ is the $(1 \times q)$ – cross-covariance matrix between $\delta\Delta g$ and $\delta\zeta$; \mathbf{C}_{nn} is the $(q \times q)$ – covariance matrix of the measurement noise \mathbf{n} and α is the regularization parameter or weight factor constraining the variability of the solution [Neyman, 1979; Moritz, 1980]. Obviously, the collocation method corresponds to $\alpha=1$ in Eq. (2).

After applying the regularization/collocation via Eq. (2), the anomaly heights ζ at the chosen grid are restored in the following way

$$\zeta = \zeta_{EGM2008} + \delta\zeta, \quad (3)$$

where $\zeta_{EGM2008}$ is the contribution of the EGM2008 gravity field model to quasigeoid heights to the same order. For further use of Eq. (2), the following two problems have to be solved: (a) building the ACF (analytical covariance function) $K(P, Q)$ of the anomalous potential T ; (b) the choice of an appropriate method for the computation of the regularization parameter α .

In this study we apply reproducing kernels $K(P, Q)$, which are represented by singular point harmonic functions only [Marchenko and Lelgemann, 1998]:

$$K_n(P, Q) = \left[\frac{GM}{R} \right]^2 \beta_n \sigma^{n+1} v_n, \quad \sigma = \frac{R_B^2}{r_P r_Q}, \quad (4)$$

where R is the Earth's mean radius; R_B is the Bjerhammar's sphere radius; r_P and r_Q are the geocentric distances to the external points P and Q ; GM is the product of the gravitational constant G and the mass M of the Earth; v_n is the dimensionless potential of a radial multipole of degree n and β_n represents a dimensionless coefficient [Marchenko, 1998; Marchenko et al., 2001]. The traditional determination of the parameter α in Eq. (2) requires a specific iterative process and the inversion of a matrix with a dimension equal to the number q of observations [Neyman, 1979]. So, when a number of observations is large we come to a time consuming procedure. To avoid this difficulty we may use an estimate

$$\alpha = 1 + \sqrt{1 + \text{Trace}(\mathbf{C}\mathbf{C}_{nn}) / \text{Trace}(\mathbf{C}_{nn}\mathbf{C}_{nn})}. \quad (5)$$

verified in [Marchenko et al., 2001; Marchenko, 2003] for the inversion of SSH altimetry data into Δg .

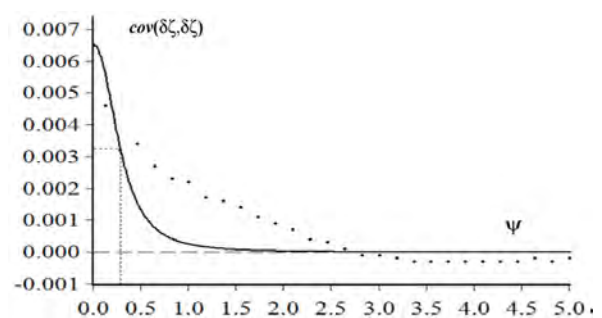


Fig. 4. Empirical covariance function (dotted curve) and analytical covariance function (solid curve) of the residuals $\delta\zeta$ [m²], ψ is the spherical distance [in degree]

Recalling now the processing of BGI gravimetry and satellite altimetry data, in the following we used the Set 1 (i.e. the 2'×2' grid of gravity anomalies) together with the Set 2 and the Set 3 of GPS/leveling data for constructing the gravimetric quasigeoid UQG2012 by means of the regularization/collocation

method. The empirical covariance function (ECF) was computed on the basis of the residual gravity anomalies $\delta\Delta g$. It was then approximated by some specific reproducing kernels or analytical covariance functions ACF, derived from radial multipole potentials by the Kelvin transformation [Marchenko, 1998], that provide the covariance propagation in \mathbf{R}^3 to geoid heights and other functionals of the anomalous potential. The modified Poisson kernel without zero degree harmonics showed the best fit to ECF [Marchenko and Lelgemann, 1998]. Therefore, it was selected (Fig. 4). It should be mentioned that in contrast to the covariance function of

residual gravity anomalies the empirical covariance function of residual geoid heights represents the usual type of such empirical functions with probable remain systematic parts in these data.

The comparison of the UQG2012 solution with independent GPS/leveling control points demonstrates a good agreement with St. Dev. < 4 cm. It is worth mentioning, that only differences between GPS/leveling data of 1st, 2nd, and 3rd orders and UQG2012 correspond to the same level of agreement (5 cm in terms of r.m.s deviation) as in the case of independent GPS/leveling control points (Table 1).

Table 1

Differences between GPS/leveling points and UQG2012

Statistics (cm)	GPS/leveling points			
	I order	II order	III order	All points
Min	-8,1	-6,7	-17,9	-22,9
Max	6,2	6,8	17,6	23,6
Mean	-0,3	-0,1	0,9	0,1
Std. dev.	1,4	1,6	5,5	5,4

Table 2

Differences between GPS/leveling points and EGM2008

Statistics (cm)	GPS/leveling points			
	I order	II order	III order	All points
Min	-38,0	-30,3	-28,0	-38,0
Max	33,4	25,7	25,6	33,4
Mean	1,7	1,6	2,1	1,1
Std. dev.	8,6	7,5	8,5	8,7

An additional evaluation of the UQG2012 solution was deduced from its comparison with the recent high-resolution gravity field model EGM2008 up to degree 2190. For proper comparison a simplest datum shift transformation was applied, statistics are given in Table 2. Major differences are observed in

the Black Sea basin, Carpathians and Crimea area with values greater than 50 cm, together with smaller differences in a few scattered zones. Nevertheless, the deviation between GPS/leveling points of 1st and 2nd orders and the UQG2012 quasigeoid in the Carpathian and Crimean regions are smaller than ± 10 cm.

Table 3

Differences between GPS/leveling points and EGG08

Statistics (cm)	GPS/leveling points			
	I order	II order	III order	All points
Min	-4,7	-0,6	-3,7	-7,9
Max	54,3	51,8	55,7	55,7
Mean	25,6	26,0	26,8	25,5
Std. dev.	7,9	7,6	9,3	8,7

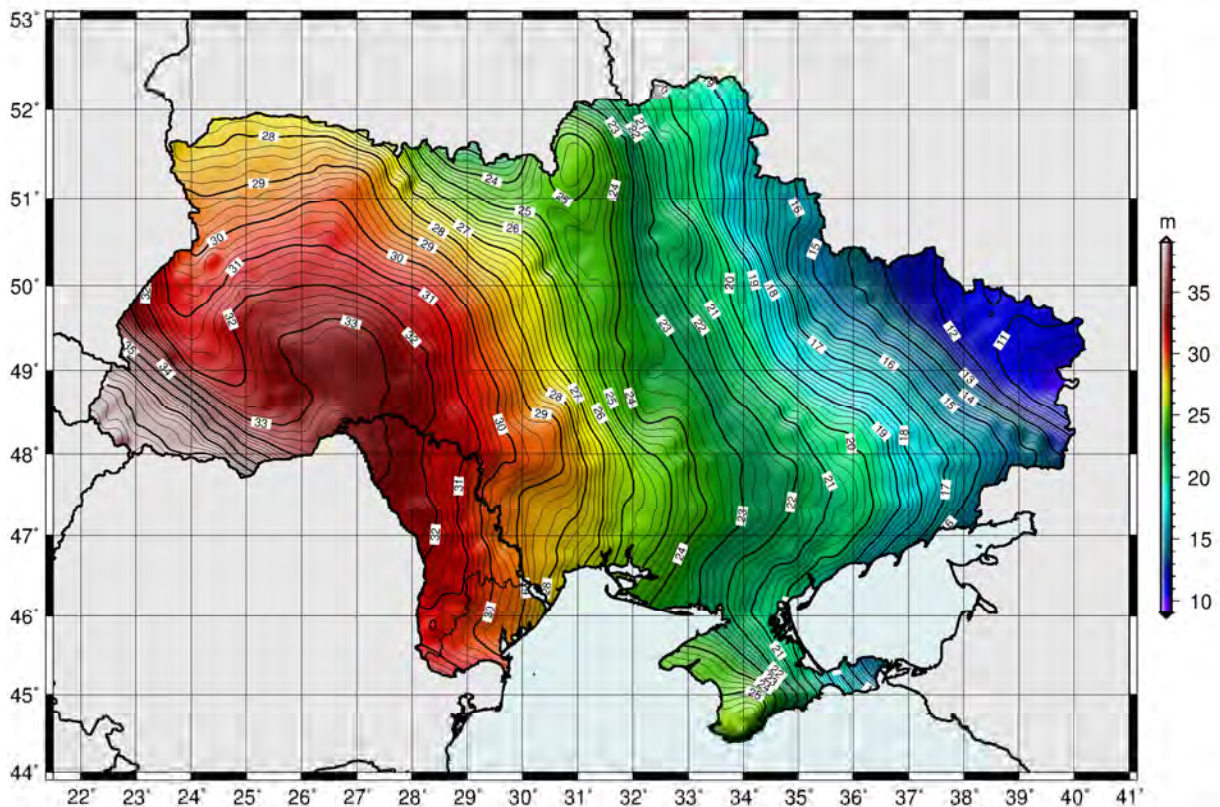


Fig. 5. Heights [m] of the UQG2012 quasigeoid solution

Another evaluation rests on the comparison with the EGG08 geoid model [Denker et al., 2007]. In this connection we have to note that the EGG08 quasigeoid is referred to the Amsterdam tide gauge. Therefore, one can observe a shift of 25 cm concerning GPS/leveling control points which are referred to the Baltic 1977 system (Table 3). The resulting quasigeoid solution UQG2012 for the Ukraine and Moldova area is shown in the Fig.5.

Conclusions

Considering the results discussed above we can formulate the following conclusions.

- The application of the regularization method to updated satellite altimetry, gravimetry, terrain and GPS/leveling data leads to a significantly improved quasigeoid solution UQG2012 for the Ukraine and Moldova area.
- A comparison of the UQG2012 model with independent GPS/leveling data given in the Baltic height system shows a good agreement with standard deviation < 4 cm. This noise level corresponds to an expected quasigeoid accuracy of about 5 cm for the Ukraine and Moldova area.
- In contrast to previous geoid models the combined UQG2012 solution provides considerably better agreement with the independent GPS/leveling control stations since Std. Dev. < 4 cm represents an improvement by 50%. This progress can be partly explained by the application of the 3"×3" DTM

SRTM3 (instead of the 1'×1' DTM GEBCO) for terrain reductions as well a renewed GPS-Leveling and gravity data.

- A comparison of the UQG2012 quasigeoid with the global gravity field model EGM2008 up to degree 2190 leads to an agreement within ± 10 cm (in terms of standard deviation) and differences having values greater than 50 cm in the Black Sea area, and Carpathian and Crimean Mountains.

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РОЗВ'ЯЗКИ КВАЗІГЕОІДА ДЛЯ ТЕРИТОРІЇ УКРАЇНИ

Мета. Розв'язок UQG2012 регіонального квазігеоїда з точністю більше ніж 2 см щодо даних GPS-нівелювання 1-го і 2-го порядку побудовано за допомогою методу середньої квадратичної колокації. У першій ітерації гравіметричний квазігеоїд UQG2011 обчислено за даними аномалій Фая для подальшого виявлення грубих помилок у даних GPS-нівелювання. Редукція за рельєф обчислена на основі 3"×3" цифрової моделі місцевості SRTM3. **Наукова новизна та практична цінність.** Так, остаточне рішення складається з UQG2012 аномалій сили тяжіння і квазігеоїда висот у вузлах 2'×3' рівномірної сітки, оцінених за допомогою методу колокації, застосованого до набору 4070 GPS-визначених висот квазігеоїда, плюс зазначених вище гравіметричних даних. Порівняння квазігеоїда UQG2012 з усіма незалежними даними GPS-нівелювання (заданих у системі Балтійської 1977 висот) показує відповідність $RMS < 4$ см. Цей рівень шуму відповідає розрахунковій точності квазігеоїда UQG2012 для України та Молдови, ніж 4–5 см щодо GPS-нівелювання точок різних класів. Оцінка рішення UQG2012 з незалежними даними GPS-нівелювання 1 і 2 класів дає значно кращу згоду з середньоквадратичним відхиленням близько 1,5 см. Нарешті, порівняння з Європейським квазігеоїдом EGG08 призводить до відмінностей близько 20–50 см у деяких районах і загалом до середнього зсуву 25 см, викликаних різними системами висот, що використовуються.

Ключові слова: регіональний квазігеоїд; гравіметрія; альтиметрія; ГНСС/нівелювання.

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РЕШЕНИЯ КВАЗИГЕОИДА ДЛЯ ТЕРИТОРИИ УКРАИНЫ

Цель. Решение UQG2012 регионального квазигеоида с точностью лучше, чем 2 см по отношению к данным GPS-нивелирования 1-го и 2-го порядка было построено с помощью метода средней квадратичной коллокации. В первой итерации гравиметрический квазигеоид UQG2011 был вычислен по данным аномалий Фая для дальнейшего выявления грубых ошибок в данных GPS-нивелирования. Редукция за рельеф основана на 3"×3"цифровой модели местности SRTM3. **Научная новизна и практическая ценность.** Таким образом, окончательное решение состоит из UQG2012 аномалий силы тяжести и квазигеоида высот в узлах 2'×3' равномерной сетки, оцененных с помощью метода коллокации, примененного к набору 4070 пунктов GPS-нивелирования высоты квазигеоида, плюс указанных выше гравиметрических данных. Сравнение квазигеоида UQG2012 со всеми независимыми данными GPS-нивелирования (заданных в системе Балтийской +1977 высот) показывает хорошее согласие с $RMS < 4$ см. Этот уровень шума соответствует расчетной точности квазигеоида UQG2012 для Украины и Молдовы, чем 4–5 см по отношению к GPS-нивелированию точек разных классов. Оценка решения UQG2012 с независимыми данными GPS-нивелирования 1 и 2 классов дает значительно лучшее согласие с среднеквадратичным отклонением около 1,5 см. Наконец, сравнение с Европейским квазигеоидом EGG08 приводит к различиям около 20–50 см в некоторых районах и в общем к среднему сдвигу 25 см, вызванных различными системами используемых высот.

Ключевые слова: Региональный квазигеоид; гравиметрия; альтиметрия; ГНСС / нивелирование.

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