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DETERMINATION OF THE HORIZONTAL STRAIN RATES TENSOR IN WESTERN UKRAINE

Doppler Orbitography and Radio-positioning Integrated by Satellite (CORS) observations from 37 Global Navigation Satellite System (GNSS) stations located in the Western Ukraine area were processed using Bernese Processing Engine module (BPE) of Bernese GNSS Software version 5.2 for a time span of about 2.5 years. To get a better agreement for constrains, the IGS stations closest to the surrounding area of study were chosen with fixed coordinates of ITRF2008 at epoch 2005.0. Eastern and Northern components of velocities of GNSS observations from these 37 permanent stations, calculated from GNSS measurements, were used to construct a 2D model of horizontal strain rates field for the area. This study is presented in three parts. Firstly, two exact solutions for the components of the 2D strain rate tensor derived on the geosphere based on solving the eigenvalues – eigenvectors problem were analyzed, including skew symmetric rotational rate tensor. Secondly, based on the most simple and useful formulas from the first stage, a rigorous estimation of the accuracy of components of the 2D strain rate tensor were obtained based on the covariance propagation rule. Finally, the components of the 2D strain rate tensor, dilatation rate and components of the sheer rate tensor in the region were computed. A model of the rotation rate tensor was constructed for the described area, which led to the conclusion that the region of study should be interpreted as a deformed territory. Based on the computations from the GNSS-data model of components of horizontal deformations, the rates of principal values and rates of principal axes of the Earth's crust deformation were found. To be consistent, the main tectonic formations are shown as the background intensity of different components of velocities, the rotation rate and strain rate tensors. Topographic features of the region were based on the SRTM-3 model (Shuttle Radar Topography Mission) with resolution 3″×3″. At the first sight, the maximum sheer rates have greatest values in the areas located around the Ukrainian Carpathians. The dilatation rate has also a similar distribution. Nevertheless, because in the paper only eigenvalue – eigenvector problem without accuracy estimation has been considered, which possibly leads to doubtful conclusions regarding interpretation and requires an additional solution of a purely mathematical problem. The full covariance matrix of the strain rate tensor should be found based on given full covariance matrix of the velocity components obtained by Bernese software. As a matter of fact, the study region is very complex in terms of crustal movements, which, according to the results obtained, require further densification of permanent GNSS stations.

Key words: Horizontal velocity; strain rate tensor; dilatation rate; maximum sheer rate tensor; accuracy estimation; skew symmetric rotational rate tensor.

Introduction

The deformations of the Earth's crust caused by the processes of the deep earth dynamics arose because of the translational-rotating motion of the planet in space. Such deformations are classified both in terms of their changes in time, and in the distribution of various spatial displacements. In particular, they can be age-related, periodic and occasional, and in addition, they can be divided into global, regional, and local deformations. Our knowledge of the Earth's crustal movements is strongly dependent on their nature and the period of deformation determinations obtained from various measurements [Minster & Jordan, 1978; DeMets, et al. 1990; DeMets, et al., 1994; England, Molnar, 1997; Kreemer, et al., 2000, Crespi, et al., 2000; Bird, 2003; et al.]. Traditionally, studying the deformations of the Earth's crust involved investigating the horizontal and vertical components of the deformation field. In principle, the deformation analysis became a mostly geodetic task using satellite geodesy. These allowed the monitoring and determining with high accuracy the three-dimensional deformation field by means of VLBI (Very Long Baseline Interfero-

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meters), SLR (Satellite Laser Ranging), DORIS (Doppler Orbitography and Radio-positioning Integrated by Satellite), GNSS (Global Navigation Satellite System), and InSAR (Interferometric Synthetic Aperture Radar). Development of these technologies cannot occur without the precise definition and implementation of the Earth's coordinate system to study the deformations of the Earth's crust, as reported in the IERS Conventions 2010 by [Petit, Luzum]. Measurements from the moderately dense network of GNSS stations were used for this study in the Western Ukraine region.

Deformation analysis represents a fundamental tool for solutions of the problems of modern geodynamics for the study of spatial and temporal changes of deformation fields and modern movements of the Earth's crust. Due to tectonic processes, their peculiarities can be explained by analyzing long-term GNSS observations in different regions of the world. Therefore, today such investigations contain a common application of the experimental study of deformations using the latest GNSS technologies, in particular, a traditional approach in geophysics. The determination of deformations of the Earth's crust, which is devoted to a very large number of scientific works every year, is usually based on the mathematical approach having a tensor nature. From the outcome of such investigations [see, for example in separate papers of, Crespi et al., 2000; Kreemer et al., 2000; Marchenko, 2003; Vanichek, et al., 2008; Marchenko et al., 2010] now already stated that it is possible to calculate the 2D and 3D strain rate tensor with 2D and 3D rigorous accuracy estimation [Marchenko, 2003; Marchenko et al., 2010], analysis of the deformation field components, and the construction of mathematical models of active fault zones. Such a study of deformation processes using GNSS observations leads to the refinement of known tectonic plates.

As will be shown below in later sections, for the deformation analysis of the Earth's crust, the additional requirement requires the determination of partial derivatives of the vector functions of the strain rates. In the ideal case, these functions should be given continuously in the space-time domain, which, however, is not achieved by geodetic measurements that have discrete nature in space and time. Since modern tectonics are generally determined from geophysical and geodetic measurements, they also have a discrete nature. For this reason, the initial data also require continuous nature in space and time and should be evaluated by means of approximation by unknown functions based on a known discrete distribution, which represents a problem having a unique solution. As was noted by Juliette et al., 2006, this problem is nothing else but preprocessing part of the deformation analysis and can be solved by either a finite element method or such worldwide approach as the least square collocation.

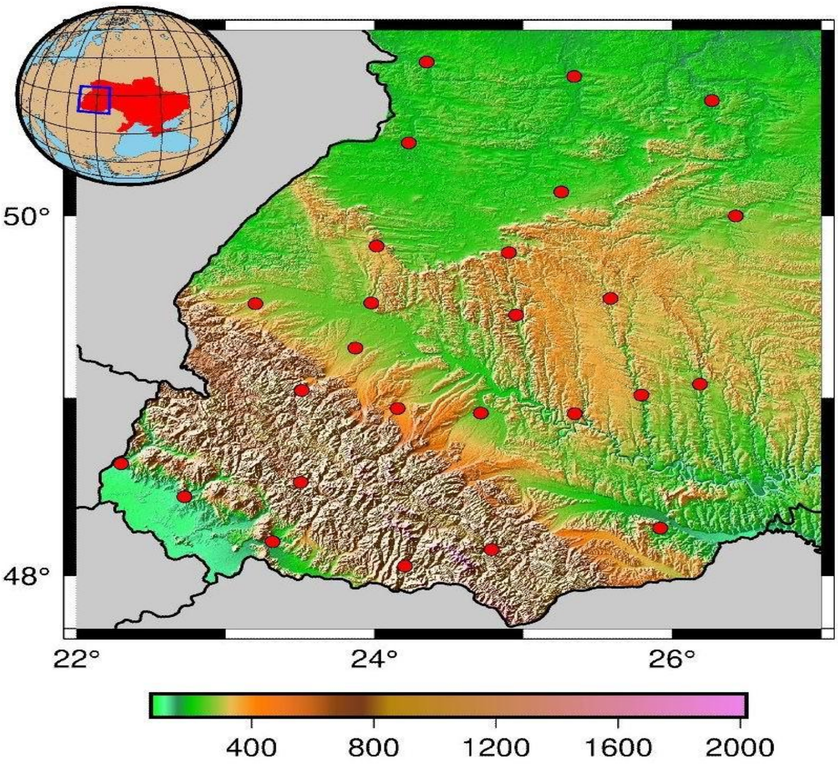

Fig. 1. Distribution of 26 GNSS station and topographical heights [m] according to STRM-3 model

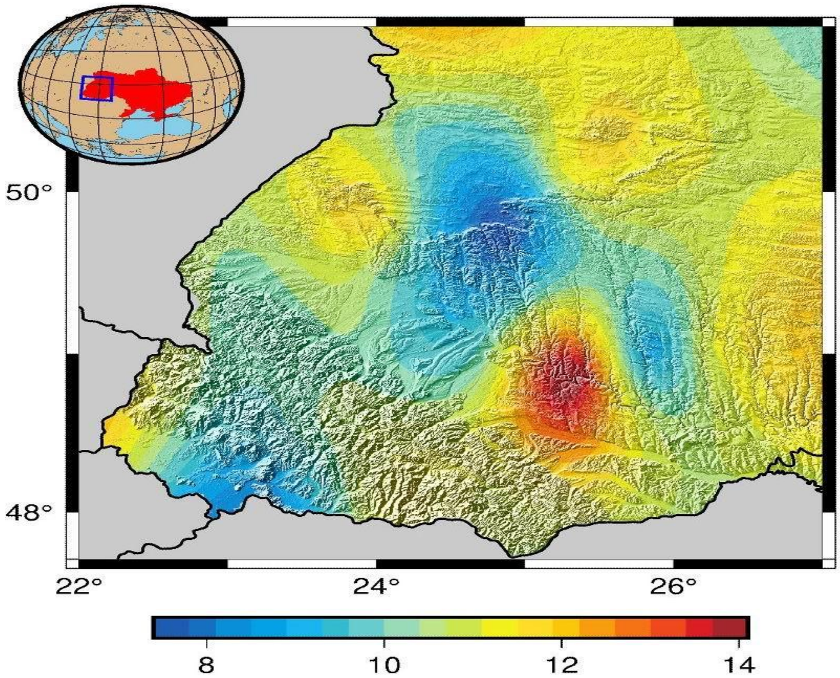

Fig. 2. Eastern V_E component [mm/yr] of velocity vectors in the ITRF2008 system at epoch 2005.0

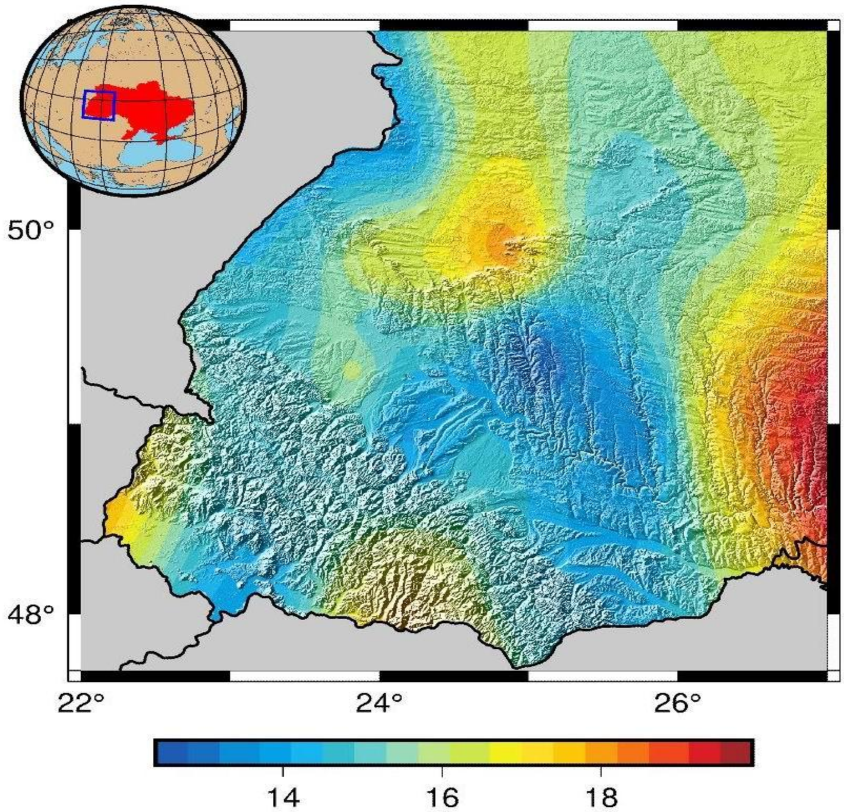

Fig. 3. Northern V_N component [mm/yr] of velocity vectors in the ITRF2008 system at epoch 2005.0

Data

Continuous observations CORS from 37 GNSS stations were processed using Bernese Processing Engine module (BPE) of Bernese GNSS Software version 5.2 for the time span of about 2.5 years. To get a better agreement for constrains the IGS stations closest to the surrounding area of study were chosen with fixed coordinates of ITRF2008 at epoch 2005.0. Fig. 1 demonstrate 26 permanent stations in the West Ukraine area called here as Set 1. Set 2 represents 11 stations, which are surrounding the study region. As well-known, the studied area is characterized by complexity of tectonic and geological structures. For better understanding, on the Fig. 1 we give additionally the topographic features of the region based on the SRTM-3 model (Shuttle Radar Topography Mission) with high resolution 3″×3″. Obtained coordinates of 37 stations and 2D velocities were applied in the following as input data to calculate the strain rate tensor and rotational rate tensor. Figs. 2 and 3 illustrate the eastern V_E and northern V_N components derived by BPE velocity vectors in the time span about 2.5 years calculated with respect to the ITRF2008 system at given epoch 2005.0 yr. The main tectonic formations are shown as the background intensity of different components of eastward V_E and northward V_N velocities based on the SRTM-3 model.

Strain rate and rotations rate tensors

Based on the general equations for the determination of rotation tensor on the spherical Earth, let's assume that for each station vectors of the rectangular coordinates (x_j, y_j, z_j) and corresponding velocity (V_x^j, V_y^j, V_z^j) are known in ITRF system. Transformation to the local NEU geocentric coordinates $(\varphi_j, \lambda_j,\ \mathrm{R})$, having the positive directions to North, East and Up, is well known and can be described for the velocities (V_N^j, V_E^j, V_r^j) by the following rule:

$$\mathbf{V}_{\mathrm{NEU}} = \mathbf{R}_{\varphi\,\lambda} \cdot \mathbf{V}_{xyz}\,, \tag{1}$$

where the rotation matrix $\mathbf{R}_{\varphi\,\lambda}$ is applied for the transformation from the global right to the local left coordinate system NEU with the axis direction North–East–Up. Obviously that the following formulas for (2) and (3) are hold:

$$\mathbf{R}_{\varphi\,\lambda} = \begin{pmatrix} -\sin\varphi\cos\lambda & -\sin\varphi\sin\lambda & \cos\varphi \\ -\sin\lambda & \cos\lambda & 0 \\ \cos\varphi\cos\lambda & \cos\varphi\sin\lambda & \sin\varphi \end{pmatrix}, \tag{2}$$

$$\mathbf{V}_{\mathrm{NEU}} = \left[V_N^j,\ V_E^j,\ V_r^j\right]^{\mathrm{T}},$$

$$\mathbf{V}_{xyz} = \left[V_x^j,\ V_y^j,\ V_z^j\right]^{\mathrm{T}}. \tag{3}$$

Now if we assume that studying strains are infinitesimal, the corresponding tensor of second degree can be additively decomposed into ε_{ij} infinitesimal strain rate tensor and ω_{ij} as the rotation rate (vorticity) tensor.

According to Hains, Holt, (1993) the horizontal strain rate field may be inverted if the rotation vector function $\mathbf{\Omega}(\mathbf{r})$ is known that expresses continuous horizontal velocity field on a sphere:

$$\mathbf{v} = R\left[\mathbf{\Omega}(\tilde{\mathbf{r}}) \times \tilde{\mathbf{r}}\right], \tag{4}$$

where R is the Earth's radius; $\mathbf{r}$ is the unite radial vector. The equation (4) is crucial in Hains, Holt (1993) theory and allows straightforward determination of the horizontal velocity field on a sphere. Thus the components of the strain rate tensor $\mathbf{S}_V$ given by (Haines, Holt, 1993; Kreemer, 2000) for the 2D space read:

$$\left.\begin{aligned} \dot{\varepsilon}_{\lambda\,\lambda} &= \frac{\tilde{\mathbf{n}}}{\cos\varphi}\frac{\partial\mathbf{\Omega}(\tilde{\mathbf{r}})}{\partial\lambda} + \frac{V_r}{R} = \frac{\tilde{\mathbf{n}}}{\cos\varphi}\frac{\partial\mathbf{\Omega}(\tilde{\mathbf{r}})}{\partial\lambda}, \\ \dot{\varepsilon}_{\varphi\,\varphi} &= -\tilde{\mathbf{e}}\frac{\partial\mathbf{\Omega}(\tilde{\mathbf{r}})}{\partial\varphi} + \frac{V_r}{R} = -\tilde{\mathbf{e}}\frac{\partial\mathbf{\Omega}(\tilde{\mathbf{r}})}{\partial\varphi}, \\ \dot{\varepsilon}_{\varphi\,\lambda} &= \frac{1}{2}\left(\tilde{\mathbf{n}}\frac{\partial\mathbf{\Omega}(\tilde{\mathbf{r}})}{\partial\varphi} - \frac{\tilde{\mathbf{e}}}{\cos\varphi}\frac{\partial\mathbf{\Omega}(\tilde{\mathbf{r}})}{\partial\lambda}\right). \end{aligned}\right\} \tag{5}$$

where V_r is the velocity in radial direction; $\mathbf{\Omega}(\tilde{\mathbf{r}})$ is the for the chosen patch or selected plate considered as a rigid body.

Two vectors $\mathbf{r} = (R, \varphi, \lambda)$ and $\mathbf{r}_0 = (R, \varphi_0, \lambda_0)$ given on the spherical Earth with the radius R and the local directions $(\tilde{\mathbf{n}}, \tilde{\mathbf{e}}, \tilde{\mathbf{r}})$ and $(\tilde{\mathbf{n}}_0, \tilde{\mathbf{e}}_0, \tilde{\mathbf{r}}_0)$ to the north, east and vertical Up, can be expressed in the following form:

$$\tilde{\mathbf{n}} = \left[-\sin\varphi\cos\lambda,\ -\sin\varphi\sin\lambda,\ \cos\varphi\right], \tag{6}$$

$$\tilde{\mathbf{e}} = \left[-\sin\lambda,\ \cos\lambda,\ 0\right], \tag{7}$$

$$\tilde{\mathbf{r}} = \left[\cos\varphi\cos\lambda,\ \cos\varphi\sin\lambda,\ \sin\varphi\right]. \tag{8}$$

In the first approximation the linear velocity $\mathbf{v}$ at the point $\mathbf{r}$ can be stated via the strain rate tensor $\nabla\mathbf{v}(\mathbf{r})$, determined through Hamilton operator ∇ [Ward, 1998]:

$$\mathbf{v}(\mathbf{r}) = \mathbf{v}(\mathbf{r}_0) + (\mathbf{r} - \mathbf{r}_0)\nabla\mathbf{v}(\mathbf{r}_0)\,, \tag{9}$$

Considering hypothetically that movements take place in the tangent plane to the Earth's sphere is true, then from the known relationship between the linear and angular velocity one gets

$$\mathbf{v} = \mathbf{\Omega}(\mathbf{r}) \times \mathbf{r}\,. \tag{10}$$

Moreover, taking into account (10), equation (9) can be transformed as follows:

$$\mathbf{v} = \mathbf{\Omega}(\mathbf{r}_0) \times \mathbf{r}_0 + (\mathbf{r} - \mathbf{r}_0)\cdot\left[\nabla\mathbf{\Omega}(\mathbf{r}_0) \times \mathbf{r}_0\right]. \tag{11}$$

The first term of the equation (11) represents rotation around the pole $\mathbf{\Omega}(\mathbf{r}_0)$, and the tensor $\left[\nabla\mathbf{\Omega}(\mathbf{r}_0) \times \mathbf{r}_0\right]$ can be additively decomposed into the strain rate tensor $\mathbf{S}_V(\mathbf{r}_0)$ and rotation rate tensor

$\mathbf{R}_V(\mathbf{r}_0)$. If $\mathbf{\Omega}(\mathbf{r}) = \text{const}$, then $\nabla\mathbf{\Omega}(\mathbf{r}_0) = 0$ and tensors $\mathbf{S}_V(\mathbf{r}_0)$ and $\mathbf{R}_V(\mathbf{r}_0)$ cannot be determined. For the transformation from the 3D space to the 2D space as surface of the sphere, it is sufficient for $\mathbf{\Omega}(\mathbf{r})$ to consider dependence from two polar coordinates only

$$\mathbf{\Omega}(\mathbf{r}) = \mathbf{\Omega}(\varphi, \lambda), \tag{12}$$

Then the horizontal strain rate tensor and accuracy of it constituents can be estimated by neglecting the velocity V_r in radial direction in (Eq. 5) and using the known eastern V_E and northern V_N velocities derived from GPS observations. This assumes the vector $\left[V_N^j,\ V_E^j\right]^{\mathrm{T}}$ of a horizontal velocity or residual velocity at the *j* number of geodetic point in the north V_N^j and the east V_E^j directions in the NEU local coordinate system (as example, after removing of the NUVEL-1A model). In such a case unknown parameters could be considered as infinitesimal values and should be determined by elements of the symmetric strain rate tensor $\mathbf{S}_V$:

$$\mathbf{S}_V = \begin{bmatrix} \dfrac{\partial V_N}{\partial \varphi} & \dfrac{1}{2}\left(\dfrac{\partial V_N}{\partial \lambda} + \dfrac{\partial V_E}{\partial \varphi}\right) \\ \dfrac{1}{2}\left(\dfrac{\partial V_N}{\partial \lambda} + \dfrac{\partial V_E}{\partial \varphi}\right) & \dfrac{\partial V_E}{\partial \lambda} \end{bmatrix} = \begin{bmatrix} \dot{\varepsilon}_{\varphi\varphi} & \dot{\varepsilon}_{\varphi\lambda} \\ \dot{\varepsilon}_{\varphi\lambda} & \dot{\varepsilon}_{\lambda\lambda} \end{bmatrix}, \tag{13}$$

and skew symmetric tensor $\mathbf{R}_V$ or rotation rate (vorticity) tensor:

$$\mathbf{R}_V = \begin{bmatrix} 0 & \dfrac{1}{2}\left(\dfrac{\partial V_N}{\partial \lambda} - \dfrac{\partial V_E}{\partial \varphi}\right) \\ \dfrac{1}{2}\left(\dfrac{\partial V_N}{\partial \lambda} - \dfrac{\partial V_E}{\partial \varphi}\right) & 0 \end{bmatrix} = \dot{\omega}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}. \tag{14}$$

where $\dot{\omega}$ is the rotation rate of the region, assumed as a rigid body. Obviously the rotation rate is adopted here as a function of spherical coordinates. The strain rate tensor $\mathbf{S}_V$ and the rotation rate tensor $\mathbf{R}_V$ are then given by the formulas (13) and (14), if the velocity vector $\mathbf{v} = [V_N, V_E]^{\mathrm{T}}$ consists of the northern V_N and the eastern V_E components.

Thus, it becomes necessary to determine the fundamental function $\mathbf{\Omega}(\tilde{\mathbf{r}})$ in the considered approach. Generally all geodetic and geological data require some preliminary analysis and prediction to the nodes of selected usually uniform grid to determine the derivatives presented in equation (5)–(14). In addition, the elastic properties of a tectonic plate can be modeled through spatial derivatives of a function $\mathbf{\Omega}(\tilde{\mathbf{r}})$, which are equal to zero in (5) for such areas, which are located on the same plate and with the same function $\mathbf{\Omega}(\tilde{\mathbf{r}})$ dependent from the Euler pole. Generally speaking, in the frame of considered theory, any area for which, $\mathbf{\Omega}(\tilde{\mathbf{r}}) = \text{const}$ can be interpreted as a rigid plate or region.

Solution of eigenvalues – eigenvectors problem

To analyze the solution for the eigenvalues and eigenvectors of the symmetric tensor $\mathbf{S}_V$, given by the expression (13), we recall that there are two different approaches. In order to select an optimal version for formulas and further accuracy estimation we will consider these two different solutions. According to the well known first solution given for strain tensor (see, for example, Vaníček, et al., 2008) the invariants of the matrix (13) can be calculated as follows:

$$I_1 = \text{Trace}(\mathbf{S}_V) = \dot{\varepsilon}_{\varphi\varphi} + \dot{\varepsilon}_{\lambda\lambda} = 2\dot{\chi}, \tag{15}$$

$$I_2 = \text{Det}(\mathbf{S}_V) = \dot{\varepsilon}_{\varphi\varphi}\dot{\varepsilon}_{\lambda\lambda} - \dot{\varepsilon}_{\varphi\lambda}^2, \tag{16}$$

and used to solve the characteristic equation

$$\Lambda^2 - I_1\Lambda + I_2 = 0. \tag{17}$$

The solution of the equation (17) leads to the invariants I_1 and I_2 of the matrix (13). Two eigenvalues Λ_1 and Λ_2 are obtained as a solution of this quadratic equation:

$$\Lambda_1 = \frac{I_1 + \upsilon}{2} = \dot{\chi} + \upsilon/2,\ \Lambda_2 = \frac{I_1 - \upsilon}{2} = \dot{\chi} - \upsilon/2, \tag{18}$$

where we suppose that $\Lambda_1 > \Lambda_2$ and $\upsilon = \Lambda_1 - \Lambda_2$ is the roots difference of the equation (17) or the so-called rate of maximum shear, which is determined based on invariants (15) and (16) and the corresponding elements of the strain rate tensor (by substituting for Λ_1 and Λ_2 in equation for $\upsilon = \Lambda_1 - \Lambda_2$) we get:

$$\upsilon = \sqrt{I_1^2 - 4I_2} = \sqrt{(\dot{\varepsilon}_{\varphi\varphi} - \dot{\varepsilon}_{\lambda\lambda})^2 + 4\dot{\varepsilon}_{\varphi\lambda}^2}. \tag{19}$$

It is evident that two principal axes represent such directions of strain rate tensor that characterize the maximum and minimum axes corresponding to the expansion Λ_1 and compression Λ_2 of some chosen certain area of study and can be found based on known values Λ_1 and Λ_2. Usually an eigenvector problem is solved in digital form.

Nevertheless we shall discuss another approach. Henceforth the eigenvalues (18), which correspond to certain vectors, can be obtained together with directions of the eigenvectors in a closed form that is necessary for further accuracy estimation of all components of the eigenvalue-eigenvector problem. For this, we first recall that the principal axes of our tensor (13) coincide with the principal directions of the so-called tensor-deviator, which is defined not only by symmetric properties of the strain rate tensor (13), but also by a zero trace $\text{Trace}(\mathbf{S}_V)$. For instance, among other types of tensors-deviators that have already been studied in terms of the derivation of analytic solutions of the eigenvalues – eigenvectors problem are the Earth's inertia tensor and well-known from GOCE satellite mission gravitational gradient tensor [Moritz, & Muller, 1987; Marchenko, & Schwintzer, 2003; Marchenko, 2003; Marchenko, et al., 2016]. Coming to the corresponding transformation through $\mathrm{Trace}(\mathbf{S}_V)$ of the tensor (13) to the deviator $\mathbf{D}_V$ it is easily seen:

$$\mathbf{S}_V = \mathbf{D}_V + \frac{\mathrm{Trace}(\mathbf{S}_V)}{2},$$

$$\mathbf{D}_V = \mathbf{S}_V - \frac{\mathrm{Trace}(\mathbf{S}_V)}{2} = \mathbf{S}_V - \dot{\chi}. \quad (20)$$

Equations (20) provide the desired matrix – deviator $\mathbf{D}_V$:

$$\mathbf{D}_V = \frac{1}{2}\begin{bmatrix} \dot{\varepsilon}_{\varphi\varphi} - \dot{\varepsilon}_{\lambda\lambda} & 2\dot{\varepsilon}_{\varphi\lambda} \\ 2\dot{\varepsilon}_{\varphi\lambda} & \dot{\varepsilon}_{\lambda\lambda} - \dot{\varepsilon}_{\varphi\varphi} \end{bmatrix}. \quad (21)$$

The solution of eigenvalues – eigenvectors problem for the deviator (21) is straightforward, since the invariants have the simplest form

$$i_1 = \mathrm{Trace}(\mathbf{D}_V) = 0,$$

$$i_2 = \mathrm{Det}(\mathbf{D}_V) = -\left[(\dot{\varepsilon}_{\varphi\varphi} - \dot{\varepsilon}_{\lambda\lambda})^2/4 + \dot{\varepsilon}_{\varphi\lambda}^2\right], \quad (22)$$

that allows solving the corresponding quadratic equation and finding both roots and its restoring to the original equation (17):

$$\lambda^2 + i_2 = 0,$$

$$\left.\begin{matrix}\lambda_1\\ \lambda_2\end{matrix}\right\} = \pm\sqrt{(\dot{\varepsilon}_{\varphi\varphi} - \dot{\varepsilon}_{\lambda\lambda})^2/4 + \dot{\varepsilon}_{\varphi\lambda}^2} = \pm\upsilon/2, \quad (23)$$

$$\left.\begin{matrix}\Lambda_1\\ \Lambda_2\end{matrix}\right\} = \dot{\chi} + \left\{\begin{matrix}\lambda_1\\ \lambda_2\end{matrix}\right\} = \dot{\chi} + \left\{\begin{matrix}\upsilon/2\\ -\upsilon/2\end{matrix}\right. . \quad (24)$$

After some algebraic manipulations formulas (20)–(24) provide the important practical aspect. The tensor $\mathbf{S}_V$ can be written now in the following way

$$\mathbf{S}_V = \frac{1}{2}\begin{bmatrix} \dot{\chi} - \dot{\gamma}_1 & \dot{\gamma}_2 \\ \dot{\gamma}_2 & \dot{\chi} + \dot{\gamma}_1 \end{bmatrix}, \quad (25)$$

where

$$\dot{\chi} = (\dot{\varepsilon}_{\varphi\varphi} + \dot{\varepsilon}_{\lambda\lambda})/2,$$

$$\dot{\gamma}_1 = \dot{\varepsilon}_{\lambda\lambda} - \dot{\varepsilon}_{\varphi\varphi}, \qquad \dot{\gamma}_2 = 2\dot{\varepsilon}_{\varphi\lambda}, \quad (26)$$

In the equation (26) $\dot{\chi}$ is the dilation rate or the rate of average expansion (compression) of the region surface; $\dot{\gamma}_1$ and $\dot{\gamma}_2$ represent the rate of components of the total rate $\dot{\gamma}$ in a studying area. It is obvious that the rate that this rate $\dot{\gamma}$ can be derived from $\dot{\gamma}_1$ and $\dot{\gamma}_2$:

$$\dot{\gamma} = \sqrt{\dot{\gamma}_1^2 + \dot{\gamma}_2^2}. \quad (27)$$

Thus, in the relationships (5), (13) and (25), the last representation of the tensor $\mathbf{S}_V$ becomes especially important, since it enables one to obtain a solution of the characteristic equation (17) for the tensor (25) in the most appropriate form

$$\left.\begin{matrix}\Lambda_1 = (\dot{\chi} + \dot{\gamma})/2\\ \Lambda_2 = (\dot{\chi} - \dot{\gamma})/2\end{matrix}\right\}. \quad (28)$$

According to Vaníček et al., (2008), for the study of deformation field, the so-called maximum displacement $\upsilon = \Lambda_1 - \Lambda_2$ is used as an invariant characteristic in the form (19). It is easily seen from (28) and $\upsilon = \dot{\gamma}$, these concepts are algebraically identical in the case of strain tensor and strain rate tensor considered here in the 2D space. In our case the parameter $\upsilon = \dot{\gamma}$ is nothing else but the rate of maximum shear.

Now comes important step for the determination of the eigenvectors (also called by principal vectors) in view of the fact that the eigenvalues Λ_1 and Λ_2 of the tensors (13) or (25) correspond to their principal vectors $\mathbf{\Lambda}_1$ and $\mathbf{\Lambda}_2$ respectively. Remembering the definition of Λ_1 and Λ_2 these vectors can be found as a nontrivial solution of the homogeneous (singular) system of algebraic equations

$$(\mathbf{S}_V - \Lambda_j\mathbf{I})\cdot\mathbf{\Lambda}_j = 0, \quad (29)$$

where $\mathbf{I}$ is the (2×2) unit matrix. Consider the matrix of the system (29) in the vector form

$$\mathbf{S}_V - \Lambda_j\mathbf{I} = \left[\mathbf{s}_1 - \Lambda_j\mathbf{e}_1,\ \mathbf{s}_2 - \Lambda_j\mathbf{e}_2\right], \quad (30)$$

where each auxiliary vector $\mathbf{s}_i$ represents the i-th column of the matrix $\mathbf{S}_V$:

$$\mathbf{s}_1 = \frac{1}{2}\begin{bmatrix} \dot{\chi} - \dot{\gamma}_1 \\ \dot{\gamma}_2 \end{bmatrix}, \quad \mathbf{s}_2 = \frac{1}{2}\begin{bmatrix} \dot{\gamma}_2 \\ \dot{\chi} + \dot{\gamma}_1 \end{bmatrix}. \quad (31)$$

Here $\mathbf{e}_1$ and $\mathbf{e}_2$ are the unit vectors in the adopted horizontal local coordinate system.

Thus, the system of linear equations (29) provides the following two conditions of orthogonality

$$(\mathbf{s}_i - \Lambda_j\mathbf{e}_i,\ \mathbf{\Lambda}_j) = 0, \quad (i = 1,2),$$

$$(j = \mathrm{const}). \quad (32)$$

Therefore, each eigenvector $\mathbf{\Lambda}_j$ will be normal to a plane, in which all-auxiliary vectors $\mathbf{s}_i - \Lambda_j\mathbf{e}_i$ belongs to each fixed $j = \mathrm{const}$. The transformation of the matrix $\mathbf{S}_V - \Lambda_j\mathbf{I}$ into the system of principal axes ($\mathbf{\Lambda}_1$, $\mathbf{\Lambda}_2$) leads to the relationship

$$\mathbf{S}_V - \Lambda_j\mathbf{I} \Rightarrow \begin{pmatrix} \Lambda_1 - \Lambda_j & 0 \\ 0 & \Lambda_2 - \Lambda_j \end{pmatrix},$$

$$\mathrm{rank}(\mathbf{S}_V - \Lambda_j\mathbf{I}) = 1 \text{ where } \Lambda_1 > \Lambda_2, \quad (33)$$

for each (j = 1, 2). The result (33) reflects the following fact: there is only one linearly independent vector in the set $\mathbf{s}_i - \Lambda_j\mathbf{e}_i$ for each fixed (j = 1, 2). Thus, we can obtain an eigenvector $\mathbf{\Lambda}_j$ as a vector product of the corresponding linearly independent vectors $\mathbf{s}_i - \Lambda_j\mathbf{e}_i$. Simplest general solution can be formed by analogy with a three-dimensional case [Marchenko, 2003] by calculating the following vector product, which coincide with the eigenvectors but are unnormalized,

$$\mathbf{Z}_j = (\mathbf{s}_1 - \Lambda_j \mathbf{e}_1) \times (\mathbf{s}_2 - \Lambda_j \mathbf{e}_2) \ . \qquad (34)$$

After standard normalization of each vector $\mathbf{Z}_j$ one gets

$$\mathbf{\Lambda}_j = \mathbf{Z}_j / \sqrt{(\mathbf{Z}_j, \mathbf{Z}_j)} \ . \qquad (35)$$

As a result, transformation (34) allows us to represent every non-unique eigenvector in the simplest form

$$\mathbf{Z}_j = \mathbf{P} + \Lambda_j \mathbf{s} + \Lambda_j^2 \mathbf{e} \ , \qquad (36)$$

where

$$\mathbf{P} = \mathbf{s}_1 \times \mathbf{s}_2 \, , \ \mathbf{s} = \mathbf{s}_1 + \mathbf{s}_2 \, , \ \mathbf{e} = \mathbf{e}_1 + \mathbf{e}_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}^{\mathrm{T}} . \quad (37)$$

Thus, the resulting equations (35)–(37) give a rigorous solution of the problem. However, for a strain rate tensor, which is studied in the 2D case, there are significantly simpler dependencies that allow further accuracy estimation of eigenvalues and principal axes using known velocities field and its covariance matrix from GPS-data processing. Note that the eigenvalues Λ_1 and Λ_2 of the tensors (13) or (25) assumed as principal values of strain rate tensor that correspond to the principal vectors of the maximum $\mathbf{\Lambda}_1$ (minimum $\mathbf{\Lambda}_2$) extension (compression) in these principal directions.

In this case, these vectors, considered as axial, correspond to principal directions. Simplest formulas represents the azimuth α_1 of the first principal axis $\mathbf{\Lambda}_1$, calculated by the formula (38), and the azimuth α_2 of the second principal axis $\mathbf{\Lambda}_2$, which is determined from the condition that the principal axes are perpendicular to each other:

$$\alpha_1 = \frac{1}{2} \arctan\left(\frac{\dot{\gamma}_2}{\dot{\gamma}_1} \right) , \ \alpha_2 = \alpha_1 + \pi/2 \, . \qquad (38)$$

The azimuth $\beta = \alpha_1 + \pi/4$ corresponds also to axial vector in the direction of the maximum shear and equivalent to the bisector of the angle between the principal axes $\mathbf{\Lambda}_1$ and minimal $\mathbf{\Lambda}_2$. Equation (38) together with the solution (25)–(28) of the eigenvalues problem allows us to proceed to a rigorous estimation of the accuracy of the parameters from given GPS observation.

Error propagation for the eigenvalues – eigenvectors problem

According to [Marchenko, 2003; Marchenko, et al., 2010], the formulas for the accuracy estimation of the eigenvalues and eigenvectors can be obtained via error propagation if the input data represented in following form

$$\mathbf{T} = \left[\dot{e}_{\varphi\varphi}, \, \dot{e}_{\lambda\lambda}, \, \dot{e}_{\varphi\lambda} \right]^{\mathrm{T}} , \qquad (39)$$

together with the known covariance matrix $\mathbf{C}_{\mathrm{TT}}$ of the components (39) of the strain rate tensor. Taking into account that adopted functional dependence for the calculation of eigenvalues is the relationships (28), which are based on the vector

$$\mathbf{t} = \left[\dot{\chi}, \, \dot{\gamma}_1, \, \dot{\gamma}_2 \right]^{\mathrm{T}} = \left[(\dot{e}_{\varphi\varphi} + \dot{e}_{\lambda\lambda})/2, \ \dot{e}_{\lambda\lambda} - \dot{e}_{\varphi\varphi}, \ 2\dot{e}_{\varphi\lambda} \right]^{\mathrm{T}} , \qquad (40)$$

we come to the additional task of preliminary accuracy estimation of the components of the vector $\mathbf{t}$. Therefore, using the covariance propagation rule the matrix (41) of the following partial derivatives is necessary

$$\frac{\partial \mathbf{t}}{\partial \mathbf{T}} = \begin{pmatrix} 1/2 & 1/2 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} , \qquad (41)$$

When (41) is given, the full covariance matrix $\mathbf{C}_{\mathrm{tt}}$ of the vector $\mathbf{t}$ can be found by means of error propagation rule

$$\mathbf{C}_{\mathrm{tt}} = \frac{\partial \mathbf{t}}{\partial \mathbf{T}} \mathbf{C}_{\mathrm{TT}} \left(\frac{\partial \mathbf{t}}{\partial \mathbf{T}} \right)^{\mathrm{T}} . \qquad (42)$$

For further accuracy estimation of the eigenvalues

$$\boldsymbol{\lambda} = \left[\Lambda_1, \ \Lambda_2 \right]^{\mathrm{T}} , \qquad (43)$$

we recall that each eigenvalue can be represented as a dependence on two parameters only: the dilation rate $\dot{\chi}$ and two components $\dot{\gamma}_1$ and $\dot{\gamma}_2$ allowing to obtain the rate of the maximum shear $\dot{\gamma} = (\dot{\gamma}_1^2 + \dot{\gamma}_2^2)^{1/2}$ (27). It should be noted that accuracy of the dilatation rate $\dot{\chi}$ and accuracy of the components $\dot{\gamma}_1$ and $\dot{\gamma}_2$ (21) are derived from (42) and the variance $\mathrm{var}(\dot{\gamma})$ of the parameter $\dot{\gamma}$ can be found in the following way

$$\mathrm{var}(\dot{\gamma}) = \frac{\partial \dot{\gamma}}{\partial \mathbf{t}} \mathbf{C}_{\mathrm{tt}} \left(\frac{\partial \dot{\gamma}}{\partial \mathbf{t}} \right)^{\mathrm{T}} = \frac{\partial \dot{\gamma}}{\partial \mathbf{t}} \frac{\partial \mathbf{t}}{\partial \mathbf{T}} \mathbf{C}_{\mathrm{TT}} \left(\frac{\partial \dot{\gamma}}{\partial \mathbf{t}} \frac{\partial \mathbf{t}}{\partial \mathbf{T}} \right)^{\mathrm{T}} ,$$

$$\frac{\partial \dot{\gamma}}{\partial \mathbf{t}} = \left[0, \ \frac{\dot{\gamma}_1}{\dot{\gamma}}, \ \frac{\dot{\gamma}_2}{\dot{\gamma}} \right] . \qquad (44)$$

Differentiating (43), according to usual rules, we get the matrix of partial derivatives from the eigenvalues vector $\boldsymbol{\lambda}$ with respect to the components of the vector $\mathbf{t}$ (42) and the complete covariance matrix $\mathbf{C}_{\lambda\lambda}$:

$$\frac{\partial \boldsymbol{\lambda}}{\partial \mathbf{t}} = \begin{bmatrix} \frac{1}{2} & \frac{\dot{\gamma}_1}{2\dot{\gamma}} & \frac{\dot{\gamma}_2}{2\dot{\gamma}} \\ \frac{1}{2} & -\frac{\dot{\gamma}_1}{2\dot{\gamma}} & -\frac{\dot{\gamma}_2}{2\dot{\gamma}} \end{bmatrix} ,$$

$$\mathbf{C}_{\lambda\lambda} = \frac{\partial \boldsymbol{\lambda}}{\partial \mathbf{t}} \mathbf{C}_{\mathrm{tt}} \left(\frac{\partial \boldsymbol{\lambda}}{\partial \mathbf{t}} \right)^{\mathrm{T}} = \frac{\partial \boldsymbol{\lambda}}{\partial \mathbf{t}} \frac{\partial \mathbf{t}}{\partial \mathbf{T}} \mathbf{C}_{\mathrm{TT}} \left(\frac{\partial \boldsymbol{\lambda}}{\partial \mathbf{t}} \frac{\partial \mathbf{t}}{\partial \mathbf{T}} \right)^{\mathrm{T}} . \qquad (45)$$

Accuracy estimation of the directions of the principal axes reduces in the evaluation of the accuracy of azimuths (38), which correspond to the maximum $\mathbf{\Lambda}_1$ and minimum $\mathbf{\Lambda}_2$ directions. Applying the covariance propagation rule to the first of the relations (38), variance of azimuth of the first principal direction can be obtained

$$\frac{\partial \alpha_1}{\partial \mathbf{t}} = \left[0 \quad -\frac{\dot{\gamma}_1}{2\dot{\gamma}} \quad \frac{\dot{\gamma}_2}{2\dot{\gamma}} \right],$$

$$\mathrm{var}(\alpha_1) = \frac{\partial \alpha_1}{\partial \mathbf{t}} \mathbf{C}_{\mathrm{tt}} \left(\frac{\partial \alpha_1}{\partial \mathbf{t}} \right)^{\mathrm{T}} =$$

$$= \frac{\partial \alpha_1}{\partial \mathbf{t}} \frac{\partial \mathbf{t}}{\partial \mathbf{T}} \mathbf{C}_{\mathrm{TT}} \left(\frac{\partial \alpha_1}{\partial \mathbf{t}} \frac{\partial \mathbf{t}}{\partial \mathbf{T}} \right)^{\mathrm{T}}. \qquad (46)$$

Variance of the azimuth α_2 of the second principal axis and the azimuth β of the direction of maximum shear will be equivalent, since partial derivatives of these three parameters coincide.

Estimation of the strain rate tensor in the West Ukraine area

The initial eastern V_E (Fig. 2) and northern V_N (Fig. 3) linear velocity were found from the 37 GNSS stations located in the study area using Bernese Processing Engine module (BPE) of Bernese GNSS Software version 5.2 for the time span about 2.5 years. To get a better accordance for constrains the IGS stations closest to a vicinity of study location was included with additional fixed coordinates of ITRF2008 at epoch 2005.0. Therefore, these velocities V_E and V_N also related to the ITRF2008 system (epoch 2005.0) as source information. See Figs. 2 and 3.

The components $\dot{\varepsilon}_{\lambda\lambda}$, $\dot{\varepsilon}_{\varphi\varphi}$, $\dot{\varepsilon}_{\varphi\lambda}$ of the strain rate tensor were computed straightforward through numerical differentiation using unites [µstrain/yr= = 10^{-6}/year]. Then formulas (24)–(27) allow to calculate the dilation rate $\dot{\chi}$ and the rate components $\dot{\gamma}_1$, $\dot{\gamma}_2$ of the tensor – deviator. Obviously the eigenvalues and eigenvectors can be derived from formulas (28) and (38) respectively. Figure 6 illustrates the eigenvectors obtained from (38) in the points of GNSS stations location. Then the components $\dot{\varepsilon}_{\lambda\lambda}$, $\dot{\varepsilon}_{\varphi\varphi}$, $\dot{\varepsilon}_{\varphi\lambda}$ of strain rate tensor and the component $\dot{\omega}$ of rotational rate tensor have been calculated based on formulas (13) and (14) respectively. We omit here the parameters $\dot{\varepsilon}_{\lambda\lambda}$, $\dot{\varepsilon}_{\varphi\varphi}$, $\dot{\varepsilon}_{\varphi\lambda}$ and give the value $\dot{\omega}$ illustrated by Fig. 4. After determining the components $\dot{\varepsilon}_{\lambda\lambda}$, $\dot{\varepsilon}_{\varphi\varphi}$, $\dot{\varepsilon}_{\varphi\lambda}$ of the strain rate tensor, formulas (24)–(27) allow easy calculation of the maximum Λ_1 and minimum Λ_2 eigenvalues given in the local NEU system (ITRF2008 frame), the dilation rate $\dot{\chi}$, the rate components $\dot{\gamma}_1$, $\dot{\gamma}_2$ of the tensor – deviator and directions of the rate of the principal deformations $\mathbf{\Lambda}_1$ and $\mathbf{\Lambda}_2$ as eigenvectors. It is evident that the maximum and minimum eigenvalues should be derived from formula (28).

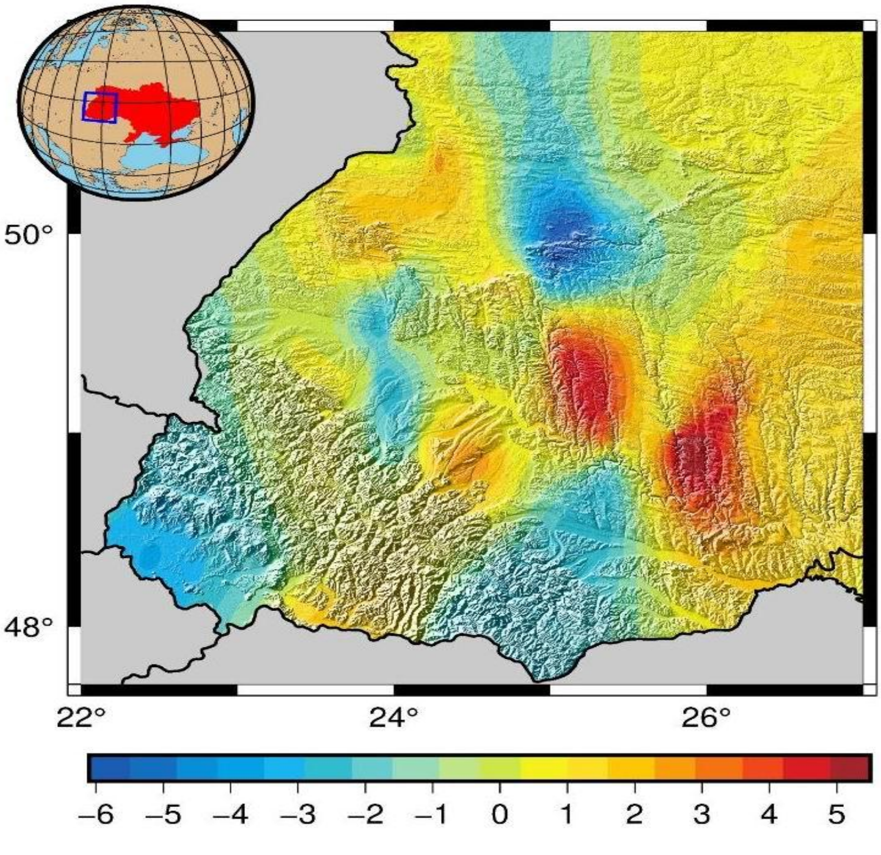

Fig. 4. Basic component of rotation rate tensor $\dot{\omega}$ [10^{-6}/year] (rotation rates of the region)

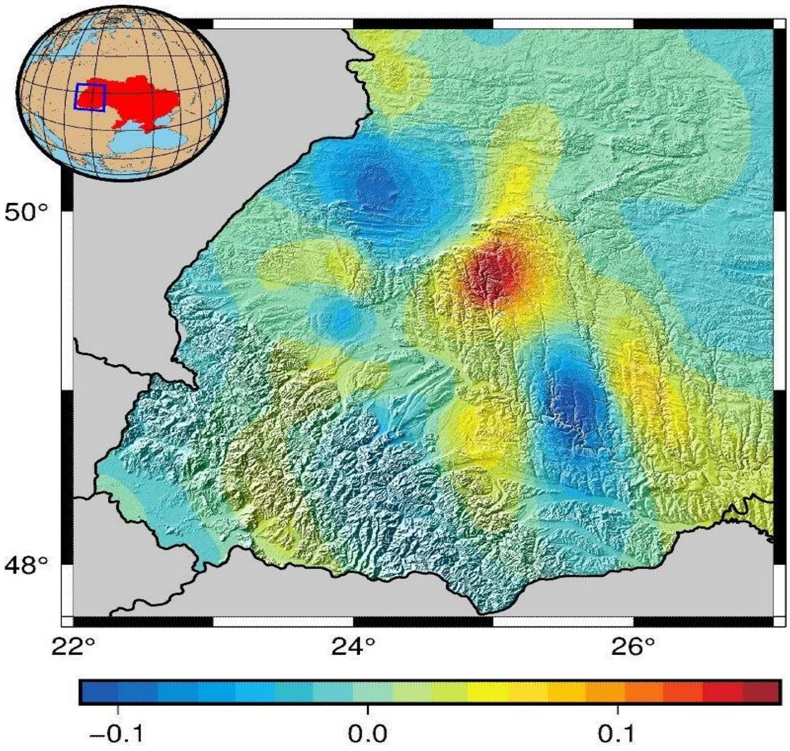

Fig. 5. Dilatation rate [μstrain/yr] based on principal deformations corresponding the expansion $\mathbf{\Lambda}_1$ and compression $\mathbf{\Lambda}_2$

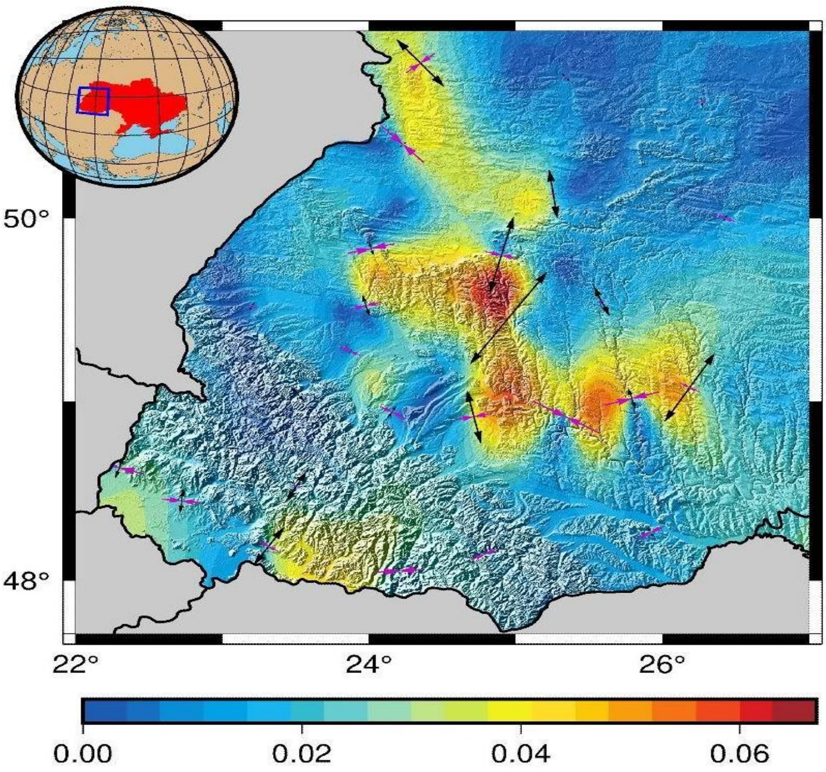

Fig. 6. Maximal shear rate [μstrain/year]; directions of the principal deformations (↔) $\mathbf{\Lambda}_1$ (expansion) and (→←) $\mathbf{\Lambda}_2$ (compression)

Fig. 5 demonstrates the dilation rate $\dot{\chi}$. Maximum sheer rate vector $\dot{\gamma}$ with directions of the same principal axes is presented in fig. 6, which correspond to the maximum $\mathbf{\Lambda}_1$ and minimum $\mathbf{\Lambda}_2$ eigenvectors or principal deformation rate in the region. To be consistent, the main tectonic formations are shown on all Fig. 2–6 as the background intensity of different components of the velocity components, rotation rate and strain rate tensors. Topographic features of the region were based on the SRTM-3 model (Shuttle Radar Topography Mission) with resolution 3″×3″.

Summary and Conclusion

In summary we can conclude.

1. GNSS observations from the 37 stations located in the Western Ukraine area were processed using Bernese GNSS Software version 5.2 for the time span about 2.5 years. Therefore, coordinates and velocities of 37 GNSS stations have been calculated. Those results were used to construct the 2D model of horizontal strain rates in the region of Western Ukraine, including a part of the Carpathian Mountains.
2. Then after densification a digital model of linear horizontal velocities of the Earth's crust movements for the Western Ukraine area was calculated. Two well-known methods for analytical solution of the eigenvalues – eigenvector problem for the 2D strain rate tensor are analyzed, and their identity is shown. Simplest formulas were chosen for further use in calculations and rigorous accuracy estimation.
3. For better understanding, the basic tectonic formations are shown as the background intensity of different components of velocity, the rotation rate, and strain rate tensors. Topographic features of the region were based on the SRTM-3 model (Shuttle Radar Topography Mission) with high resolution 3″×3″ (Fig. 1). A model of the rotation rate tensor was constructed for the region of Western Ukraine, which leads to the following conclusions. According to the classical approach, it is assumed that each tectonic plate should be rigid (having the same linear velocities for each sub-region lying on the same plate), then any area for which the condition $\dot{\omega} = 0$ is fulfilled (linear velocity = const), considered as a non-deformed region. This condition $\dot{\omega} \neq 0$ is not fulfilled for the Western Ukraine area.
4. Based on the computations from GPS-data model of components of horizontal deformations, the rates of principal values and rates of prin
5. cipal axes of the Earth's crust deformation were found. At the first sight, it should be pointed out that the maximum sheer rates have greatest values in the areas located around the Ukrainian Carpathians. The dilatation rate has also a similar distribution.
6. However, this paper deals only with the problem of eigenvalues – eigenvectors without estimation of accuracy, which may lead to doubtful conclusions about interpretation and require additional solution of a purely mathematical problem. The complete covariance matrix $\mathbf{C}_{\mathrm{TT}}$ of the strain rate tensor must be found from the given covariance matrix of the velocity components obtained by Berne software. This problem was omitted in the article due to possible further development.

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ОЦІНЮВАННЯ ТЕНЗОРА ШВИДКОСТЕЙ ГОРИЗОНТАЛЬНИХ ДЕФОРМАЦІЙ У РЕГІОНІ ЗАХІДНОЇ УКРАЇНИ

Дані GNSS спостережень (CORS) з 37 станцій, розташованих у районі Західної України, обробленО за допомогою модуля Bernese Processing Engine (BPE) Бернського програмного забезпечення GNSS версії 5.2 протягом періоду часу близько 2,5 року. Щоб досягти кращої згоди, вибрано станції IGS, найближчі до навколишнього району дослідження, з фіксованими координатами ITRF2008 в епоху 2005.0. Східна та північна складові швидкості спостережень GNSS з цих 37 постійних станцій, обчислені за результатами вимірювань GNSS, використані для побудови двовимірної моделі поля горизонтальних деформацій цієї місцевості. Це дослідження складається з трьох частин. По-перше, проаналізовано два точні рішення для компонентів 2D тензора швидкостей деформацій, отримані на геосфері на основі розв'язання власних величин – задачі власних векторів,ураховуючи симетричний тензор швидкості обертання. По-друге, на основі найпростіших і найкорисніших формул з першого етапу виконано строге оцінювання точності компонентів 2D тензора швидкостей деформацій на основі правила поширення коваріацій. Нарешті, обчислено компоненти 2D тензора швидкості деформації, швидкості дилатації та компоненти тензора рівних швидкостей в області. Для описаної області побудовано модель тензора швидкості обертання. Це привело до висновку, що область дослідження слід інтерпретувати як деформовану територію. На основі обчислень з GNSS-моделі цих компонентів горизонтальних деформацій встановлено норми основних значень та швидкості основних осей деформації земної кори. Основні тектонічні утворення показано як фонову інтенсивність різних компонентів швидкостей, швидкість обертання та тензори швидкості деформації. Топографічні особливості регіону ґрунтувались на моделі SRTM-3 (місія з топографії Shuttle) з роздільною здатністю 3″×3″. На перший погляд, найбільші значення отримано в районах, розташованих навколо Українських Карпат. Швидкість дилатації також має подібний розподіл. Тим не менше, оскільки в роботі обчислено лише власні числа та власні вектори без оцінки точності, це може призвести до сумнівних висновків щодо інтерпретації результатів і вимагає додаткового розв'язання суто математичної задачі. Потрібно знайти коваріаційну матрицю тензора деформації на основі заданої коваріаційної матриці компонентів швидкості, одержаних програмним забезпеченням Bernese. Оскільки досліджуваний регіон є дуже складним, то за отриманими результатами необхідне подальше ущільнення перманентних станцій GNSS.

Ключові слова: тензор швидкостей горизонтальних деформацій; швидкість дилатації; тензор швидкостей максимального зсуву; оцінювання точності.

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