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Liubov ZHURAVCHAK[1], Nataliya ZABRODSKA[2]

[1] Lviv Polytechnic National University, 12, S. Bandery Str., Lviv, 79013, Ukraine, tel. +38(032)2582578, e-mail: liubov.m.zhuravchak@lpnu.ua, https://orcid.org/0000-0002-1444-5882
[2] Carpathian branch of S. Subbotin Institute of Geophysics of National Academy of Sciences of Ukraine, 3-b, Naukova Str., Lviv, 79060, Ukraine, tel. +38(032)2648563, e-mail: nataliya.zabrodska@gmail.com, https://orcid.org/0009-0001-8850-6721

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ALGORITHM FOR DETERMINING INCLUSION PARAMETERS IN SOLVING INVERSE PROBLEMS OF GEOELECTRICAL EXPLORATION USING THE PROFILING METHOD

The paper aims to develop an algorithm for recognizing the physical and geometric parameters of inclusion, using indirect methods of boundary, near-boundary, and partially-boundary elements based on the data of the potential field. Methodology. The direct and inverse two-dimensional problems of the potential theory concerning geophysics were solved when modeling the earth's crust with a piecewise-homogeneous half-plane composed of a containing medium and inclusion that are an ideal contact. To construct the integral representation of the solution of the direct problem, a special fundamental solution for the half-plane (Green's function) of Laplace's equation, which automatically satisfies the zero-boundary condition of the second kind on the day surface, and a fundamental solution for inclusion were used. To find the intensities of unknown sources introduced in boundary, near-boundary, or partially-boundary elements, the collocation technique was used, i.e. the conditions of ideal contact are satisfied in the middle of each boundary element. After solving the obtained SLAE, the unknown potential in the medium and inclusion and the flow through their boundaries are found, considering that the medium and inclusion are considered as completely independent domains. Results. The computational experiment for the task of electric prospecting with a constant artificial field using the resistance method, in particular, electrical profiling, was carried out, while focusing on the physical and geometric interpretation of the data. Initial approximations for the electrical conductivity of the inclusion, its center of mass, orientation and dimensions are determined by the nature of the change in apparent resistivity. To solve the inverse problem two cascades of iterations are organized: the first one is to specify the location of the local heterogeneity and its approximate dimensions, the second one is to specify its shape and orientation in space. At the same time, the minimization of the functional considered on the section of the boundary, where an excess of boundary conditions is set, is carried out. Originality. The method of boundary integral equations is shown to be effective for constructing numerical solutions of direct and inverse problems of potential theory in a piecewise homogeneous half-plane, using indirect methods of boundary, near-boundary, and partial-boundary elements as variants. Practical significance. The proposed approach for solving the inverse problem of electrical exploration with direct current is effective because it allows fora step-by-step, “cascade” recognition of the shape, size, orientation, and electrical conductivity of the inclusion. We follow the principle of not using all the details of the model and not attempting to recognize parameters with little effect on the result, especially with imprecise initial approximations.

Key words: mathematical modeling, potential theory, direct problem, inverse problem, indirect near-boundary element method, partially-boundary elements, piecewise homogeneous medium, electrical profiling.

Introduction

The study of the earth's interior by geological and geophysical methods provides a basis for elucidating fundamental questions of geodynamic processes, which primarily relate to understanding how deep high-temperature fluids are formed and penetrate the earth's crust. They are an important source for all subsequent processes of formation of carbon, sulfide, and iron-containing metasomatites, as well as the formation of ore and oil and gas deposits. As is known, fluids penetrate through the lithosphere by draining deep zones of high permeability, which often correspond to deep faults. Studies show that areas of articulation of various types of tectonic plates (oceanic, continental or intracontinental) are characterized by electrically conductive structures. The nature of such deep regional anomalies is not explained necessarily by partial melting, they can be the result of transportation of fluids and, accordingly, ore components from the crust and mantle during tectonomagmatic activation. The hydrogen and carbon present in the earth's crust and upper mantle can shift within the contact zones of geological formations of various ages. This movement leads to an increase in electrical conductivity, allowing us to identify areas that are likely to contain valuable mineral deposits. Articulation areas of tectonic plates of different ages are studied experimentally within the area magnetotelluric and magnetovariation methods [Nikolaev et al., 2019].

When studying geodynamic processes in the earth's crust, generated by natural or artificial force fields of various physical nature, numerous physical and mechanical effects that appear on its surface as the result of changes in its structure are analyzed. The analysis includes the search for non-homogeneous objects such as falls, cavities, various caves, landslides, hydrocarbon or ore deposits. It also involves localizating and determining the physical properties of these objects, as well as monitoring the territories where they are located. This is one of the tasks of geophysical research methods, particularly, electrical prospecting.

The advantages of methods that utilize natural and artificial potential fields (gravitational, magnetic, electric, thermal, filtering) to detect object heterogeneity include the ease of implementation in field or experimental conditions and their economic feasibility. These methods do not require special expensive equipment. At the same time, the mathematical models of steady-state processes used in these methods consist of Laplace or Poisson equations supplemented with boundary conditions of the first, second, or third kind and mixed, and are well studied [Lv, et al., 2023; Milson, Eriksen, 2011; Pierre van Baal, 2014; Qu, et al., 2015; Zhdanov, 2009].

Isolating and detecting physical anomalies is a complex mathematical and technical problem, since they are present against a backdrop of irregular and often turbulent natural and man-made disturbances, such as variations in the upper layers of the earth, uneven terrain, space, atmospheric, climatic, and industrial factors. At the same time, interference of fields of various nature, which is both a simple superposition of field parameters and their complex nonlinear interactions, is always observed. Anomalies manifested as changes in the physical characteristics of the object. For example, the gravitational field depends on the change in the density of rocks, the magnetic field – on the magnetic susceptibility and residual magnetization of its components, the electrical field depends on specific electrical resistance, and temperature depends onthermal properties, particularly thermal conductivity.

Analytical or numerical solutions of direct problems involve determining the parameters of the physical field based on known physical characteristics, size, and shape of the components of the object. These solutions, can be found unambiguously, although sometimes they may require using complex algorithms [Brebbia, et al., 2012; Foks, et al., 2014; Zhang, et al., 2013]. At the same time, the same distribution of physical field parameters can correspond to different ratios of physical characteristics and sizes of the object's components. In other words, finding a solution to the inverse problem of mathematical physics (determining the dimensions of the components of the object and their physical characteristics according to the observed field) is much more difficult due to its ambiguity [Mikheeva, et al., 2023; Mukanova & Modin, 2018].

The interpretation of gravity, magnetic, and electrical anomalies has many common features. This is explained by the similarity of the basic laws of interaction of gravitational, magnetic, and electric masses (Newton's, Coulomb's, and Ohm's laws), which led to the establishment of mathematical relationships between gravitational, magnetic, and electric potentials. However, despite similarities, there are also differences in the nature and morphology of gravitational, magnetic, and electrical anomalies [Zhou, et al., 2023; Li, et al., 2022]. Anomalous objects in gravity prospecting are unipolar, that is, they form either positive or negative anomalies. Anomaly objects in magnetic exploration are bipolar, since each magnetized domain can form both positive and negative anomalies. Therefore, the structure of the anomalous magnetic field is more complex than that of the gravitational one. It is further complicated by the different length of the domains in the direction of magnetization, its different angle, the presence of induction and residual rock magnetization. The form and intensity of anomalies, and therefore the effectiveness of electrical profiling (EP) as a method of electrical prospecting with direct current, depend on the various natural and technical factors. These include the appropriate method selection, prospecting depth, the observation system, the intensity of the primary (feed) field and its polarization. This involves the direction of the electric field vector relative to the extension of objects. For example, when this vector direction coincides with the extension of objects, maximum secondary magnetic fields are induced in conductive domains. And when it is perpendicular to the extension, maximum conductive anomalies of secondary electric fields are observed. The methodology, or the theory of rational interpretation, is the same for all electrical prospecting methods. However, the geological-geophysical interpretation, as well as the field of applications, are different. The physical-mathematical quantitative interpretation of these methods, which boils down to solving the inverse problem, is well developed only for one-dimensional (horizontally layered) models of environments. Interpretation of electric fields with the help of modern computers is carried out with greater accuracy, objectivity, and speed. Among the many algorithms for solving the inverse problem of electrical prospecting, algorithms of various selection options have become the most popular.

Quantitative interpretation of EP data is a complex and imprecise process. Therefore, it makes sense to talk only about semi-quantitative interpretation, whose main task is to determine the epicenter of the reconnaissance object, that is, the area under which it is located, as well as to assess the shape and depth of its location, sometimes dimensions, physical and geological nature of anomalies. It begins with the selection of physical and geological models that can be used to approximate exploration objects. They include media contacts, thick and thin layers, isometric (spherical), elongated (lens-like, cylinder-like) objects, etc. Solving direct and especially inverse problems by mathematical and physical modeling methods for the listed models is more difficult than for vertical electrical soundings. The effectiveness of EP is determined not only by the presence of favorable geoelectrical conditions and a successful choice of method but also by a sufficient amount of additional geological and geophysical information. In particular, depending on the physical properties of the rocks, it is advisable to carry out EP together with magnetic exploration, thermal exploration, or radiometry. To interpret the results of the EP, a priori data, geological sections, and maps are needed, which, in turn, are refined after the EP is carried out.

Purpose

The paper aims to develop an algorithm for recognizing the physical and geometric parameters of inclusion, using indirect methods of boundary, near-boundary, and partially-boundary elements based on the data of the potential field.

Methodology

Problem formulation

Let it be necessary to determine the geometric parameters of inclusion according to the nature of the flow of the potential field on the electrically insulated boundary $\partial\Omega = \{(x_1, x_2): -\infty < x_1 < \infty, x_2 = 0\}$ of the half-plane, which occupies the domain $\Omega = \mathbf{R}^{2-} = \{(x_1, x_2): -\infty < x_1 < \infty, -\infty < x_2 < \infty)$ in the Cartesian coordinate system (x_1, x_2). We assume that the flow of the potential field is equal to zero everywhere on $\partial\Omega$, except points A $(x_{1A}, 0)$ and B $(x_{1B}, 0)$, where current sources are located, feeding electrodes with known constant intensities g_A and g_B, respectively. In addition, there is a section $\partial\Omega_b \subset \partial\Omega$ on which we additionally know the value of the potential *u*.

We assume that the potential $u_0(x)$ of the stationary electric field inside the half-plane satisfies the equation

$$P(u_0(x)) = \Delta u_0(x) = \sigma_0 \left(\frac{\partial^2 u_0(x)}{\partial x_1^2} + \frac{\partial^2 u_0(x)}{\partial x_2^2} \right) = -g_0(x)\chi_g(x), \ x \in \Omega_0, \qquad (1)$$

everywhere except an inclusion Ω_1 $(\Omega_1 \subset \Omega)$. In the domain Ω_1 the environment is homogeneous, but different from that in which the operator $P(u_0(x))$ operates, therefore, the process in it is described by the equation

$$P(u_1(x)) = \Delta u_1(x) = \sigma_1 \left(\frac{\partial^2 u_1(x)}{\partial x_1^2} + \frac{\partial^2 u_1(x)}{\partial x_2^2} \right) = 0, \ x \in \Omega_1. \qquad (2)$$

Here σ_s $(s = 0,1)$ is a constant physical characteristic (conductivity coefficient), $\chi_g(x)$ is a characteristic function of the domain $\Omega_g \subset \Omega_0$, $x = (x_1, x_2)$.

For the mathematical formulation of the excess of boundary conditions, we consider that a boundary condition of the second kind is set on the boundary $\partial\Omega$, and a boundary condition of the first kind is also set on the section $\partial\Omega_b$:

$$q_0(x) = -\sigma_0 \frac{\partial u_0(x)}{\partial \mathbf{n}(x)} = 0, \ x \in \partial\Omega = \partial\Omega^{(2)} \cup \partial\Omega_b, \qquad (3)$$

$$u_0(x) = u_b(x), \ x \in \partial\Omega_b, \qquad (4)$$

where $\mathbf{n}(x) = (n_1(x), n_2(x))$ is a uniquely defined external unit normal to the boundary $\partial\Omega_0 = \partial\Omega \cup \partial\Omega_1$.

The choice of sources on the boundary of the half-plane in the form $g_0(x) = g_0(\xi_A) + g_0(\xi_B)$, where $g_0(\xi_A) = 2g_A / \sigma_0, g_0(\xi_B) = 2g_B / \sigma_0$, ensures the fulfillment of condition (3).

Note that there must be an empty set for the correct statement of direct problems of mathematical physics $\partial\Omega_b$. When solving inverse problems, the presence of $\partial\Omega_b$ is mandatory, and the quality and reliability of the result is higher when the area $\partial\Omega^{(2)}$ is smaller, that is, it is the best one when the condition $\partial\Omega^{(2)} = \varnothing$ is fulfilled.

The ideal contact conditions are set at the media interface $\partial\Omega_1$:

$$u_0(x) = u_1(x),$$

$$-\sigma_0 \frac{\partial u_0(x)}{\partial \mathbf{n}(x)} = -\sigma_1 \frac{\partial u_1(x)}{\partial \mathbf{n}(x)}, \ x \in \partial\Omega_1. \qquad (5)$$

Geometrical information about the inclusion Ω_1 will be given in the form of *N* pairs of points with coordinates $\left(x_{1n}^1, x_{2n}^1\right)$ and $\left(x_{1n}^2, x_{2n}^2\right)$ (*n*=1,…,*N*) and $\partial\Omega_1$ will be modeled by *N* linear segments Γ_n, which will be set as follows:

$$\left(x_{1n}, x_{2n}\right) \in \Gamma_n, \text{ if } x_{1n} = x_{1n}^1 \varphi_1(\eta) + x_{1n}^2 \varphi_2(\eta),$$

$$x_{2n} = x_{2n}^1 \varphi_1(\eta) + x_{2n}^2 \varphi_2(\eta),$$

where $\left(x_{1n}^1, x_{2n}^1\right)$ and $\left(x_{1n}^2, x_{2n}^2\right)$ are the coordinates of the extreme points of the segment Γ_n, $\varphi_1(\eta) = 0.5(\eta - 1)\eta$, $\varphi_2(\eta) = 0.5(\eta + 1)\eta$, η is a one-dimensional coordinate, which changes from-1 to

1 when the point $\left(x_{1n}, x_{2n}\right)$ moves from $\left(x_{1n}^1, x_{2n}^1\right)$ to $\left(x_{1n}^2, x_{2n}^2\right)$ along the segment Γ_n. Since the closed broken line which simulates $\partial\Omega_1$ is continuous, we will require that

$\left(x_{1n}^2 = x_{1(n+1)}^1\right)$, $\left(x_{2n}^2 = x_{2(n+1)}^1\right)$ when $n<N$ and

$\left(x_{1n}^2 = x_{11}^1\right)$, $\left(x_{2n}^2 = x_{21}^1\right)$ when $n=N$.

Finding unknown values x_{1n}^1, x_{2n}^1, x_{1n}^2, x_{2n}^2 will be carried out in stages. First, we will write down the algorithm for solving the direct problem of the potential theory, then we will consider them known, and then we will build a method for recognizing the physical and geometric parameters of the inclusion Ω_1.

Algorithm for solving the direct problem of electric prospecting with direct current using electric profiling

Let us find the solutions of the problem (1)–(3), (5) ($\partial\Omega_b = \varnothing$) using indirect boundary element method (BEM) [Brebbia, et al., 2012], near-boundary element method (NBEM) [Zhuravchak, 2019), Zhuravchak, Zabrodska, 2021] and partially-boundary element method (PBEM) [Zhuravchak, Zabrodska, 2021]. Note that among the listed methods, NBEM is the most accurate. However, it takes more time than BEM, while the partially-boundary element method achieves higher accuracy than BEM in a shorter time than NBEM. To construct the solution, we will use a special fundamental solution for the half-plane (Green's function) of the Laplace equation (1), which automatically satisfies the zero boundary condition (3):

$$\mathbf{E}_{0h}(r) = \mathbf{E}_0(r) + \mathbf{E}_0(r'),$$

the fundamental solution for the plane in the inclusion:

$$\mathbf{E}_s(r) = \mathbf{E}_s(x,\xi) = -\frac{1}{2\pi\sigma_s}\ln|r/r_0|,$$

and their normal derivatives:

$$\mathbf{F}_{0h}(r) = \mathbf{F}_0(r) + \mathbf{F}_0(r'),$$

$$\mathbf{F}_s(r) = \mathbf{F}_s(x,\xi) = \sigma_s \sum_{i=1}^{2} \frac{n_i\left(x_i - \xi_i\right)}{2\pi r^2}.$$

Here ξ_1, ξ_2 is a coordinate system that coincides with x_1, x_2,

$$r = \sqrt{(x_1-\xi_1)^2 + (x_2-\xi_2)^2},$$

$$r' = \sqrt{(x_1-\xi_1)^2 + (x_2+\xi_2)^2},$$

constant r_0 is used to improve the accuracy of calculations.

Step 1. In the BEM, we divide the boundary $\partial\Omega_1$ into boundary elements Γ_{sv} so that $\cup_{v=1}^{V}\Gamma_{sv} = \partial\Omega_1$, $\Gamma_{sv} \cap \Gamma_{sq} = \varnothing$, $v \neq q$, $v,q = \overline{1,V}$. When using NBEM and PBEM, we introduce external near-boundary domains $G_s = B_s / \Omega_s$, where $B_s \subset \mathbf{R}_1^2$, $\Omega_s \subset B_s$, $\partial B_s \cap \partial\Omega_s = \varnothing$. In NBEM we divide each near-boundary domain G_s into elements G_{sv} so that each boundary element Γ_{sv} corresponds to two near-boundary elements: G_{sv}: $G_{sv} \cap \partial\Omega_1 = \Gamma_{sv}$, $G_{sv} \cap G_{sq} = \varnothing$, $v \neq q$, $v,q = \overline{1,V}$, $\cup_{v=1}^{V} G_{sv} = G_s$. We enter partially-boundary elements as follows. In each near-boundary domain G_s we introduce curves G_{sv}^{+}, G_{sv}^{-} so that the beginning of G_{sv}^{+} is the beginning of Γ_{sv} and the beginning of G_{sv}^{-} is the end of Γ_{sv}. The union $G_{sv}^{-} \cup \Gamma_{sv} \cup G_{sv}^{+} = G_{sv}^{\Gamma}$ is called a partially-boundary element [Zhuravchak, Zabrodska, 2021]. We introduce fictitious sources of unknown intensity $g_{sv}^{\gamma}(\xi)$ on each of the discrete elements $\gamma_{sv} = [\Gamma_{sv}, G_{sv}, G_{sv}^{\Gamma}]$.

Step 2. We approximate the intensities $g_{sv}^{\gamma}(\xi)$ of unknown sources by constants d_{sv}^{γ} and move from differential equations (1), (2) to their integral representations, that is, we write down the potentials and their derivatives along the normal in the form:

$$u_0^{\gamma}(x) = \sum_{v=1}^{V} d_{0v}^{\gamma} \int_{\gamma_{0v}} \mathbf{E}_{0h}(x,\xi)\, d\gamma_{0v}(\xi) + $$
$$+\mathbf{E}_{0h}(x,\xi_A) g_0(\xi_A) + \mathbf{E}_{0h}(x,\xi_B) g_0(\xi_B),$$

$$u_1^{\gamma}(x) = \sum_{v=1}^{V} d_{1v}^{\gamma} \int_{\gamma_{1v}} \mathbf{E}_1(x,\xi)\, d\gamma_{1v}(\xi) + C_1, \tag{6}$$

$$q_0^{\gamma}(x) = -\sigma_0 \frac{\partial u_0^{\gamma}(x)}{\partial \mathbf{n}} = -0.5\chi_{0v}^{\gamma} d_{0v}^{\gamma} +$$
$$+\sum_{v=1}^{V} d_{0v}^{\gamma} \int_{\gamma_{0v}} \mathbf{F}_{0h}(x,\xi)\, d\gamma_{0v}(\xi) +$$
$$+\mathbf{F}_{0h}(x,\xi_A) g_0(\xi_A) + \mathbf{F}_{0h}(x,\xi_B) g_0(\xi_B).$$

$$q_1^{\gamma}(x) = -\sigma_1 \frac{\partial u_1^{\gamma}(x)}{\partial \mathbf{n}} = -0.5\chi_{1v}^{\gamma} d_{1v}^{\gamma} +$$
$$+\sum_{v=1}^{V} d_{1v}^{\gamma} \int_{\gamma_{1v}} \mathbf{F}_1(x,\xi)\, d\gamma_{1v}(\xi), \tag{7}$$

where $\chi_{sv}^{\gamma}(x) = 1$, $x \in \Gamma_{sv}$, $\chi_{sv}^{\gamma}(x) = 0$, $x \notin \Gamma_{sv}$.

Note that except for $\partial\Omega_1$ (6) exactly satisfies (1), (2) in Ω. This fact frees us from constructing a grid in Ω.

Step 3. To find the intensities of unknown sources, we will use the collocation technique, that is, we will satisfy the ideal contact conditions in the middle of each boundary element. Substituting (6), (7) into (5) and adding the condition of equality to zero in $\mathbf{R}_1^2$ the sum of all sources at infinity, we obtain a system of linear algebraic equations (SLAE) for finding unknown values d_{sv}^{γ} and C_1:

$$\sum_{v=1}^{V} d_{0v}^{\gamma} \int_{\gamma_{0v}} \mathbf{E}_{0h}(x^w,\xi)\,d\gamma_{0v}(\xi) - \sum_{v=1}^{V} d_{1v}^{\gamma} \int_{\gamma_{1v}} \mathbf{E}_1(x^w,\xi)\,d\gamma_{1v}(\xi) - C_1 = -\mathbf{E}_{0h}(x,\xi_A)g_0(\xi_A) - \mathbf{E}_{0h}(x,\xi_B)g_0(\xi_B),\quad x^w \in \partial\Omega_1, \tag{8}$$

$$-0.5\chi_{0w}^{\gamma} d_{0w}^{\gamma} + \sum_{v=1}^{V} d_{0v}^{\gamma} \int_{\gamma_{0v}} \mathbf{F}_{0h}\left(x^w,\xi\right)d\gamma_{0v}(\xi) + 0.5\chi_{1w}^{\gamma} d_{1w}^{\gamma} - \sum_{v=1}^{V} d_{1v}^{\gamma} \int_{\gamma_{1v}} \mathbf{F}_1\left(x^w,\xi\right)d\gamma_{1v}(\xi) = -\mathbf{F}_{0h}(x,\xi_A)g_0(\xi_A) - \mathbf{F}_{0h}(x,\xi_B)g_0(\xi_B),\quad x^w \in \partial\Omega_1, \tag{9}$$

$$\sum_{v=1}^{V} d_{1v}^{\gamma} \int_{\gamma_{1v}} d\gamma_{1v}(\xi) + C_1 = 0. \tag{10}$$

Step 4. After finding the unknown values d_{sv}^{γ} and C_1 as solutions of SLAE (8)–(10), we calculate the desired potential in the medium and inclusion and the flow through their boundaries using formulas (6), (7), since the medium and inclusion are now considered as completely independent domains.

Solving the inverse problem of electrical prospecting with direct current using electrical profiling

Geometric information about $\partial\Omega_1$ is stored in the form of *N* quadruplets of numbers x_{1n}^1, x_{2n}^1, x_{1n}^2, x_{2n}^2. If we now take into account that they are rather difficult to include in integral representations (6), (7), as well as the fact that we will use iterative procedures to find them, then it is advisable to reduce the number of unknown values at the first stages of recognition. To do this, we will introduce additional dependencies between x_{1n}^1, x_{2n}^1, x_{1n}^2, x_{2n}^2 and limit ourselves to the case of *N*=4 for the inclusion.

We organize the iterative recognition algorithm as follows.

Step 1. According to the nature of the change in apparent resistivity $\rho^{\gamma} = \frac{k_u}{I}\left|u_0^{\gamma}(x_M) - u_0^{\gamma}(x_N)\right|$ we determine the sign and initial approximations for $\Delta\sigma$, $(x_{10,}x_{20})$ – the center of mass of the inclusion, modeled by a rectangle with sides $2l_1$, $2l_2$ or a rhombus with the same diagonals of length $2l_0$. We determine exactly x_{10} by the extremum of the curve ρ^{γ} and x_{20} and l_0 (or l_1, l_2) – approximately by that curve integrated within the limits between its inflection points. Here

$$k_u = 2\pi\left(\frac{1}{\ln r_{AM}} - \frac{1}{\ln r_{AN}} - \frac{1}{\ln r_{BM}} + \frac{1}{\ln r_{BN}}\right)^{-1}$$

is the coefficient of device ABMN,

$$r_{CD} = \sqrt{\left(x_1^C - x_1^D\right)^2 + \left(x_2^C - x_2^D\right)^2}\,..$$

The apparent resistivity ρ^* of a homogeneous half-plane is equal to unity at each point.

Step 2. We put $\sigma_1 = \sigma_0 + \Delta\sigma$, considering that σ_0 is known.

Step 3. We organize the first cascade of iterations to clarify the location of local heterogeneity and its approximate dimensions.

1. We model $\partial\Omega_1$ with a rectangle or a rhombus with the coordinates of the vertices:

$x_1^1 = x_{10} - l_1, x_2^1 = x_{20} - l_2$, $x_1^2 = x_{10} + l_1, x_2^2 = x_{20} - l_2$,

$x_1^3 = x_{10} + l_1, x_2^3 = x_{20} + l_2$, $x_1^4 = x_{10} - l_1, x_2^4 = x_{20} + l_2$,

or $x_1^1 = x_{10} - l_0, x_2^1 = x_{20}$, $x_1^2 = x_{10}, x_2^2 = x_{20} - l_0$,

$$x_1^3 = x_{10} + l_0, x_2^3 = x_{20},\ x_1^4 = x_{10}, x_2^4 = x_{20} + l_0. \tag{11}$$

2. For the selected σ_1 according to the algorithm of solving of the direct problem, described above, we calculate the potential $u_0^{\gamma}(x)$ by formula (6) for $x \in \partial\Omega_b$.

3. Minimize the functional

$$I^f = \int_{\partial\Omega_b} |u_b(x) - u_0^{\gamma}(x)|\,d\partial\Omega_b(x), \tag{12}$$

allowing variation only $x_{20,}l_0$ (or l_1, l_2).

4. We fix $x_{20,}^f l_0^f$ (or l_1^f, l_2^f), which correspond to the found minimum of the functional (12), and refine the electrical conductivity using minimization (12), denote it by σ_1^f.

5. As a result, using formulas similar to (11), we will find the specified coordinates of the vertices of a rectangle or rhombus (x_1^{nf}, x_2^{nf}), *n*=1,...,4.

Step 4. We organize the second cascade of iterations to clarify the shape and orientation of inclusion in the space.

1. We will rotate the rectangle or rhombus found in step 3 around its center of mass, simultaneously scaling along the axes. To do this, we enter three new parameters φ_0, s_1, s_2 and calculate the new coordinates of the vertices of the rectangle or rhombus:

$$x_1^{nr} = (x_1^{nf} - x_{10})\cos\varphi_0 - (x_2^{nf} - x_{20}^f)\sin\varphi_0 + x_{10},$$
$$x_2^{nr} = (x_1^{nf} - x_{10})\sin\varphi_0 + (x_2^{nf} - x_{20}^f)\cos\varphi_0 + x_{20}^f,$$
$$x_1^{nc} = x_1^{nr} s_1 + (1 - s_1)x_{10},$$
$$x_2^{nc} = x_2^{nr} s_2 + (1 - s_2)x_{20}^f. \quad (13)$$

2. We minimize the functional (12) by variation φ_0, s_1, s_2 and fix $\varphi_0^f, s_1^f, s_2^f$, which correspond to the found minimum.

3. For constants $x_{10}, x_{20,}^f l_0^f$ (or l_1^f, l_2^f), $\varphi_0^f, s_1^f, s_2^f$, we specify the electrical conductivity σ_1^f using minimization (12), denote it by σ_1^{f2}.

Step 5. The found values x_1^{nc}, x_{2n}^{nc} serve instead of variables x_1^{nf}, x_2^{nf} in formulas (13) for further refinement in the iterative process of minimization (12) at constant σ_1^{f2}. Note that it is sometimes advisable to repeat the last two steps several times.

The results

The direct and inverse problems of electrical prospecting with direct current using electrical profiling were solved by the indirect method of near-boundary elements for A=(–25.0) and B=(25.0), $g_A = -0.5$ and $g_B = 0.5$ respectively. The current strength I and the electrical conductivity σ_0 of the geological environment Ω_0 were assumed to be equal to one. The distance between the receiving electrodes was chosen as MN = =0.1AB. They are moved along the line (–25, 25) with a step of 0.1.

Having some information about the research area from previous experience, we determine the initial approximations and the possible range of parameters we need to find. The asymmetry of the graph shows that the inclusion is placed at angles to the horizontal axis other than 0 and 90 degrees. Initial approximations for $\Delta\sigma$ (deviation from the electrical conductivity of the medium) and $(x_{10,} x_{20})$ (center of mass of the inclusion) are determined by the apparent resistivity graph (Fig. 1).

Its convexity shows that the electrical conductivity of the inclusion is lower than the electrical conductivity of the medium. The apparent resistance is the inverse value of the electrical conductivity, so we put $\Delta\sigma$ as negative. The concavity on the graph shows that the electrical conductivity of the inclusion is greater than the electrical conductivity of the medium, so we assume $\Delta\sigma$ is positive. The coordinate x_{10} is determined exactly by the extremum of the curve ρ^γ, and x_{20} and l_0 (or l_1, l_2) – approximately by that curve integrated within the limits between its inflection points. The horizontal size of the inclusion l_1 (or l_0) will be smaller than the distance between the minima of the curve by about 50 %. The vertical size and depth of the center of mass h will be proportional to the height of the maximum on the graph.

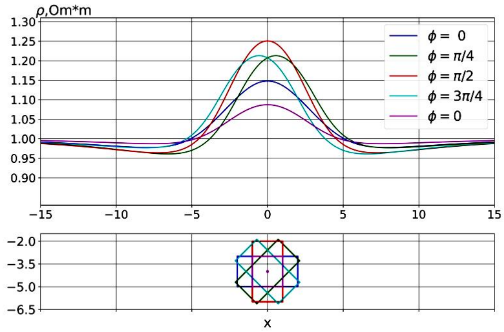


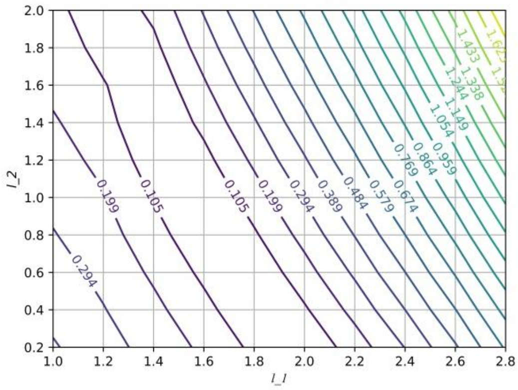
Fig. 1. Graphs of apparent resistivity for the selection of initial approximations.

First, let us solve the inverse problem for inclusion in the form of a rectangle with sides $2l_1$ =4, $2l_2$ =2, placed horizontally, that is, its longer side is parallel to the day surface. Having found the initial approximations, we calculate the values of the functional with two fixed parameters (σ_1 and h) and two variables. We find the range of values of variable parameters around which the functional is minimal. Fig. 2 presents isolines that show that there are areas where the values of the functional are the smallest, and they cover possible pairs of problem solutions: the desired values l_1 are in the range from 1 to 2.1, and l_2 – from 0.2 to 2.

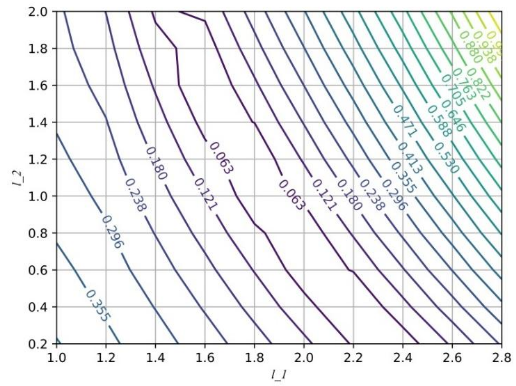
Fig. 3 shows the step-by-step selection of inclusion parameters, based on the minimization of the functional (12), from σ_1 =0.05, h=3.6 to the optimal σ_1 =0.2, h=4.

Next, we will find the solution of the inverse problem for inclusion in the form of a square with side $2l_0$ =2, parallel to the day surface. To demonstrate the refinement of the orientation of inclusion in the space, a rhombus is chosen as an initial approximation.

Fig. 4 shows the step-by-step selection of inclusion parameters, based on the minimization of the functional (12), from σ_1 =0.1, $\varphi_0 = 0$ to the optimal σ_1 =0.2, $\varphi = \pi/2$ at h=4.



a



b

Fig. 2. Graphs of isolines of functional values, calculated for two fixed parameters:

a – σ_1 =0.05, *h*= 3.6 m; b – σ_1 =0.2, *h*= 3.6 m.

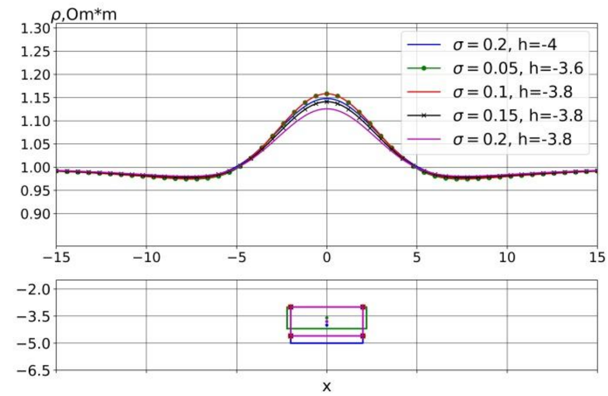


Fig. 3. Graphs of step-by-step finding of the inclusion-rectangle parameters, a rectangle is chosen as the initial approximation.

Numerical results showed that during profiling with a conventional gradient device, it is possible to approximately determine the length of the inclusion (its beginning and end) by a coordinate parallel to the day surface. This involves taking into account the distance between the inflection points of the apparent resistivity curve. Additionally, it allows forthe determination of whether the inclusion has a greater or lesser electrical conductivity compared to the main environment. At the same time, the "center of mass" of the inclusion is almost always identified by the extremum (minimum or maximum) of the curve, but in the case of two inclusions located close to each other, two inclusions may be identified as one. The change in the shape of the inclusion is slightly reflected in the curve. The distance between the "centers of mass" of two inclusions of the same conductivity placed horizontally is identified by the distance between the maxima on the apparent resistance curves. The obtained data serve as initial approximations when solving inverse problems.

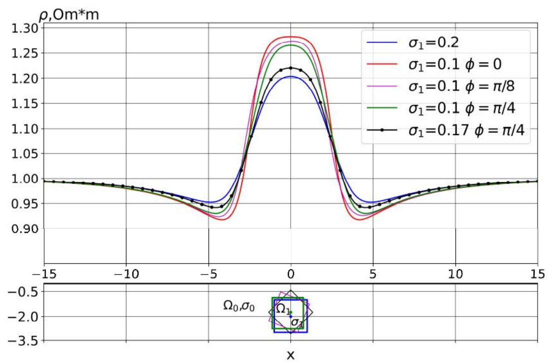


Fig. 4. Graphs of step-by-step finding of the inclusion-square parameters, a rhombus is chosen as the initial approximation.

It should be noted that two inclusions placed one under the other (vertically) are identified as one in the profiling method. To distinguish them, as well as to determine the depth of their occurrence, it is necessary to use vertical electrical sensing.

The software implementation of the proposed approaches was carried out in the modern, powerful Python programming language, as it is freely distributed and has a large number of additional libraries, including NumPy, SciPy, Matplotlib, etc. Their use significantly speeds up and facilitates the writing of programs that allow you to visually control the processes of forming a geometric research domain, a discrete model, and the computing process in general. Based on the proposed approach, automated computer modules were created in Python for solving direct and inverse problems of the theory of electrical exploration with direct current.

In further research, we plan to expand the proposed algorithm for solving the inverse problem using the profiling method for two, three, or more inclusions placed horizontally with the same and different electrical conductivity, as well as develop an algorithm for recognizing two inclusions placed one under the other by the method of vertical electrical sensing.

Originality

The method of boundary integral equations is shown to be effective for constructing numerical solutions of direct and inverse problems of potential theory in a piecewise homogeneous half-plane, using indirect methods of boundary, near-boundary, and partial-boundary elements as variants.

Practical significance

The proposed approach for solving the inverse problem of electrical exploration with direct current is effective because it allows fora step-by-step, “cascade” recognition of the shape, size, orientation, and electrical conductivity of the inclusion. We follow the principle of not using all the details of the model and not attempting to recognize parameters with little effect on the result, especially with imprecise initial approximations.

Conclusions

1. It is possible to analytically solve a direct problem, and, accordingly, to give methods of interpretation only for sources of disturbances in the form of simple geometric models (sphere, cylinder, ledge, etc.). Approximation of real geological objects by such models for a number of cases is conditional because geological objects of ideal shape are rare. However, even depth estimation plays a significant role in geology. In more complex cases, the problem is solved by numerical methods, which must be highly accurate, reliable, and fast. NBEM has proven itself well in solving direct and inverse problems of electric exploration with direct current for inclusions of non-canonical form.

2. For the interpretation and geological explanation of anomalies, it is necessary to study in detail the physical characteristics of the rocks, the patterns of their change both in horizontal directions and with depth. Anomalous physical characteristics of geological objects should be greater the deeper they lie. The efficiency of electric prospecting increases if the physical characteristics of the investigated geological object are significantly different from the physical characteristics of the host rocks.

3. According to the principle of superposition of fields, the effects caused by various geological factors are added. Total anomalies of the first derivative of the potential are determined by the deep structure of the earth's crust and its different strengths, the relief of the surface of the crystalline foundation and its petrographic composition, the heterogeneity of the structure of layers of sedimentary rocks and the presence of certain structures and minerals within them. Theoretically, there is a functional dependence between geological factors and anomalies of the potential field, but in practice, only a correlation dependence is most often established. The main method of geological interpretation of exploration data is the comparison of maps and graphs of the potential field with geological maps. A correlation can be observed between potential field anomalies and known geological anomalies, which indicates the identity of these geological formations and the identified source of the field disturbance. If there is no such connection, the field is caused by deeper and unknown geological formations. The accuracy of the geological interpretation of potential field anomalies depends on the degree of consideration of the noted features.

4. In case the physical characteristics and shape of the objects are unknown, the mathematical solution of the inverse problem of electrical reconnaissance is ambiguous and the quantitative interpretation gives several options. To increase the reliability of the interpretation, it is worth applying a complex of various geophysical methods of analysis and technologies. This helps to obtain the most reliable data on the geological structure of the research area. In turn, the creation of geoinformative systems will make it possible to fully apply all known technologies to effectively interpret geophysical data. The software makes it possible to increase the degree of automation of the measurement process with the direct formation of a working project, ensuring the promptness of obtaining information about the effective and quantitative characteristics of the studied environment.

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Любов ЖУРАВЧАК[1], Наталія ЗАБРОДСЬКА[2]

[1] Національний університет "Львівська політехніка", вул. С. Бандери, 12, Львів, 79013, Україна, тел. +38(032)2582578, e-mail: liubov.m.zhuravchak@lpnu.ua, https://orcid.org/ 0000-0002-1444-5882

[2] Карпатське відділення Інституту геофізики ім. С. І. Субботіна НАН України, вул. Наукова, 3-б, Львів, 79060, Україна, тел. +38(032)2648563, e-mail: nataliya.zabrodska@gmail.com, https://orcid.org/0009-0001-8850-6721

АЛГОРИТМ ВИЗНАЧЕННЯ ПАРАМЕТРІВ ВКЛЮЧЕННЯ ПРИ РОЗВ'ЯЗУВАННІ ОБЕРНЕНИХ ЗАДАЧ ГЕОЕЛЕКТРОРОЗВІДКИ МЕТОДОМ ПРОФІЛЮВАННЯ

Мета. З використанням непрямих методів граничних, приграничних та частково-граничних елементів побудувати алгоритм розпізнавання фізичних та геометричних параметрів включення за даними потенціального поля. Методика. Розв'язано пряму та обернену двовимірні задачі теорії потенціалу стосовно геофізики під час моделювання земної кори неоднорідною півплощиною, складеною із вміщувального середовища та включень, які перебувають в ідеальному контакті. Для побудови інтегрального подання розв'язку прямої задачі використано спеціальний фундаментальний розв'язок для півплощини (функцію Гріна) рівняння Лапласа, який автоматично задовольняє нульову крайову умову другого роду на денній поверхні, та фундаментальний розв'язок для включення. Для визначення інтенсивностей невідомих джерел, уведених у граничних, приграничних чи частково-граничних елементах, використано колокаційну методику, тобто умови ідеального контакту задовольняються у середині кожного граничного елемента. Після розв'язання отриманої системи лінійних алгебраїчних рівнянь знайдено шуканий потенціал у середовищі та включенні й потік через їхні межі, враховуючи, що середовище і включення розглянуто як цілком незалежні області. Результати. Обчислювальний експеримент виконано для задачі електророзвідки постійним штучним полем методом опору, зокрема, електропрофілюванням. Увагу зосереджено на фізичній та геометричній інтерпретації даних. За зміною позірного опору визначено початкові наближення для електропровідності включення, його центра мас, орієнтації та розмірів. Для розв'язання оберненої задачі організовано два каскади ітерацій: перший для уточнення місцезнаходження локальної неоднорідності та її приблизних розмірів, другий – для уточнення її форми та орієнтації в просторі. Здійснено мінімізацію функціонала, розглянутого на ділянці межі, де задано надлишок крайових умов. Наукова новизна. Обґрунтовано ефективність непрямих методів граничних, приграничних та частково-граничних елементів (як варіантів методу граничних інтегральних рівнянь) для побудови числових розв'язків прямої та оберненої задач теорії потенціалу в кусково-однорідній півплощині. Практична значущість. Ефективність запропонованого підходу до розв'язування оберненої задачі електророзвідки постійним струмом зумовлена тим, що вдалося реалізувати поетапне, "каскадне" розпізнавання форми, розмірів, орієнтації та електропровідності включення, керуючись принципом: у разі доволі неточних початкових наближень не використовувати усі тонкощі моделі й не виконувати розпізнавання параметрів, що мало впливають на результат.

Ключові слова: математичне моделювання; теорія потенціалу; пряма задача; обернена задача; непрямий метод приграничних елементів; частково-граничні елементи; кусково-однорідне середовище; електричне профілювання.

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