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MODELING THE GEOID AND POLAR MOTION IN GEOLOGICAL TIME

The main objective of our research is to: 1) conduct a correlation analysis of the relationship between geoid heights and topographic heights in the modern era using calculated moving correlation coefficients (MCC); 2) extrapolate the obtained correlation model to past geological epochs and determine the paleogeoid using known surface heights derived from paleoDEM continental reconstruction models [Scotese and Wright, 2018]; 3) perform calculations of changes in “True Polar Wander” (TPW) based on the obtained paleogeoid height data sets resulting from the movement of lithospheric plates. Methodology. To investigate the correlation between geoid heights and lithospheric surface heights, data for 1×1º trapezoids from the EGM2008 model, topographic heights from ETOPO1, and paleoDEM paleoreconstruction models were used. The center of the moving window was shifted by 1º in both latitude and longitude within grids of 3×3º and 9×9º, reflecting the global nature of the correlation and mitigating local variations. By extrapolating the modern correlation model to past geological epochs, we investigate the dynamic paleogeographic evolution and its impact on the geoid structure. To study the dynamics of changes in the Earth’s lithospheric shape, paleogeoid heights, and pole position, the concept of approximating their surfaces with a semi-parameterized biaxial ellipsoid was used. Results. Based on the calculated MCC values, a map of the correlation between geoid heights and topographic heights for the modern era was constructed. We conducted a detailed correlation analysis for different epochs – 200, 400, and 540 million years ago, as well as for intervals from the modern era to 540 million years ago, in 5 million-year steps, using paleogeoid models. This analysis was used to hypothesize about the secular movement of the Earth’s rotational poles and the associated dynamics of the lithosphere. Scientific novelty. The modeling of paleogeoid heights was performed for further assessment of the Earth’s pole displacement. We also discuss the impact of gravitational and rotational forces on the internal structure of the Earth, from the lithosphere to the inner core, suggesting cyclic geodynamic instability manifested as secular variations in the Earth’s shape and gravitational field. Our conclusions indicate a subtle understanding of the relationship between tectonic activity and paleogeoid anomalies, suggesting minimal direct influence of lithospheric plate movements on geoid height changes, but significant indirect influence through mantle convection over geological time. Practical significance. This study not only provides deeper insight into the historical configuration of the Earth’s geoid and continents but also enhances our understanding of the dynamic processes shaping the current and future geodynamic evolution of the planet.

Keywords: geoid heights, lithospheric surface heights, moving correlation coefficients, continental paleoreconstruction, paleogeoid, TPW, geodynamics.

Introduction

One of the most important problems in geodynamics is the study of the movement of lithospheric plates. By knowing the exact parameters of plate movement (coordinates of the rotation pole and angular velocity), it is possible to solve key geodynamic and geodetic tasks: constructing and refining the Earth’s coordinate system, studying tidal phenomena in oceans and the Earth’s crust, and understanding the gravitational field and rotational features (changes in rotation speed and axis) of the Earth. When considering key geodynamic problems, the shape of our planet is of significant interest as its surface is inextricably linked to the structure of the lithosphere and geodynamic processes, with the history of the Earth’s origin and evolution. Modeling the transformation of the Earth’s shape over geological time can more broadly reveal geodynamic processes, as these issues are related to the importance of building a global model of Earth’s dynamics. According to plate tectonics, the primary factor in the formation of the global Earth’s surface relief is the interaction of horizontally moving lithospheric plates. According to this concept, the parameters and orientation of the lithosphere’s surface shape should change. According to this concept, the parameters and orientation of the lithosphere's surface shape should change.Besides, it can be assumed that the kinematic movements of lithospheric plates introduce certain changes to the shape of the geoid and, consequently, alter the orientation of the Earth's rotation axis. The Earth's rotation is the most crucial factor determining the parameters of the planet's equilibrium shape. The characteristics of rotational motion provide information about the Earth's internal structure, and variations in the rotational regime (rotation speed and pole movement) are a real source of energy for tectogenesis [Tyapkin & Dovbnich, 2009; Tserklevych et al., 2022].

It is known that on time scales over 1000 years, most of the Earth reacts like a viscous fluid. If we track the geographic location (e.g., latitude) of a point on the solid surface, the following two dynamic processes can lead to significant deviations of this point relative to the vector of the Earth's mean angular momentum: plate tectonics (i. e., mantle convection) and true polar wander (TPW). Changes in the position of the rotation axis should lead to a significant restructuring of the Earth's stress state and, as a result, leave an imprint on tectonic processes.

The idea that polar wander (PW) occurs has been debated in the scientific community since the late 18th century. In [Lambeck, 1980], it is noted that since Kelvin's time in 1863 until the work published by Gold in 1955 [Gold, 1955], several qualified physicists believed that polar wander was not only possible but even inevitable if the long-term rheology of the planet was non-elastic. Geologists, paleontologists, and paleoclimatologists assumed that the Earth's equator must have been located far from its current position in the distant geological past. In the early 1950s, paleomagnetologists provided quantitative evidence that the geographic latitudes of individual continents indeed changed over time, indicating that the instantaneous geographic pole or the rotation pole moved relative to most continents. The path along which the pole moved in geological history was called "apparent polar wander path" (APWP) because it was unclear whether the pole or the continent moved [Courtillot, 2007]. It soon became clear that the continents (more precisely, tectonic plates) moved relative to each other and that a significant part of APW was actually caused by these relative movements. The question was whether there remained a portion of polar wander that would characterize the "Earth's rotation mode as a whole" and was not accounted for by plate tectonics. This portion is what is usually referred to as "true polar wander" (TPW) [Courtillot, 2007]. It is important to note the distinction between continental drift and true polar wander. If any true polar wander occurred, it would appear as a rigid rotation of the lithosphere relative to a fixed rotation axis. Such dynamic processes might have occurred on Mars, where, as known, plate tectonics is known to be absent. TPW can result from centrifugal forces acting on anomalous masses distributed on or within the Earth. For example, an excessive mass would cause the Earth to slowly deform and change its rotation axis, forcing the mass to move toward the equator without shifting relative to the solid Earth. In an absolute astronomical reference frame, the direction of the rotation axis is fixed; positive masses drift toward the equator (and negative ones in the opposite direction). Therefore, the relative position of the rotation axis changes relative to the mass distribution.The change in the rotation axis causes the Earth to deform, and the change in shape forces the axis to move further until it asymptotically approaches a new equilibrium. Thus, the geodynamic evolution of the Earth's shape can be considered a consequence of the redistribution of its mass in a dynamically changing force field. The transition processes from one state of equilibrium to another occur on different time and spatial scales in the planetary geodeformation field with changing geoid characteristics. For the current geological epoch, true polar wander (TPW) is relatively small, but simple theoretical considerations suggest that it could have been larger in other epochs. Therefore, modeling TPW is of considerable interest and has made significant progress in the last decade. A review of current TPW research can be found in the works of [Courtillot, 2007; Adhikari et al., 2018].

On geological time scales, the movement of global plates is reconstructed using geomagnetic reversals recorded on mid-ocean ridges and hotspot tracks. By combining these with paleomagnetic data from continents, the "true" position of the rotation axis can be determined [Courtillot & Besse, 2004], as it coincides with the Earth's magnetic dipole. This "true polar wander" (TPW) can be geodynamically modeled as the rotation axis follows the axis of maximum inertia, which can be determined from the second-order geoid in the expansion of heights by spherical functions [Adhikari et al., 2018]. Thus, the characteristics of the planet's inhomogeneous mass distribution, which can extend to the core, and their evolutionary redistribution due to mantle convection, directly affecting different spatial wavelengths of geoid heights, are also important information for determining TPW.

For the non-hydrostatic modern geoid, three large anomalies predominantly dominate: a high anomal gravitational potential area in the equatorial part of the Pacific Ocean; another stretching from Greenland through Africa to the southwestern part of the Indian Ocean; and a semi-continuous low area running from Hudson Bay through Siberia to India and further to Antarctica. None of these three high-amplitude (over 60 m) and long-wavelength anomalies does not correspond to the modern placement of tectonic plates [Chase & Sprowl, 1983, Clement et al., 1983]. However, if the modern geoid is mapped to the position of continents and tectonic plates 125 million years ago (reconstructed relative to hotspots), then a significant correlation appears. The modern geoid minimum corresponds to the positions of subduction zones that surrounded the Pacific Ocean 125 million years ago. The geoid maximum, which is now centered in Africa, completely located within the ancient Pangea, and the equatorial Pacific maximum overlaps the locations of spreading centers preserved in central Pacific magnetic anomalies [Ricard, 1993]. Thus, the most likely cause of the large long-wavelength features of the geoid's amplitudes is convection of lower mantle, which is, however, does not reflect a close correlation with plate movements. Moreover, TPW is a significant geodynamic process that, in terms of continental movement, may even dominate over plate tectonics during certain periods of Earth's history.

Objective

To deepen the understanding of the mechanisms of dynamic processes in which gravity plays a key role, it is essential to establish a correlation between the geoid and the physical surface of the lithosphere. By comparing the modern geoid map with the topographic surface, we arrive at the widely known conclusion about the lack of a connection between geoid heights and tectonic structures [Stacey, & Davis, 2008; Tserklevych et al., 2022]. Long-wavelength heights of geoid exhibit a completely independent distribution even concerning the largest tectonic structures of the lithosphere: continental protrusions and oceanic depressions. Therefore, it can be hypothesized that the planetary geoid forms are isostatically compensated or predominantly formed by deeply situated heterogeneous masses, and hence, the movement of lithospheric plates likely has little effect on the height of the geoid on a planetary scale. To verify whether this hypothesis has a factual basis, we decided to calculate the moving correlation coefficients (MCC) between geoid heights and lithospheric surface heights for the modern era. Assuming that the statistical model of the correlation for the modern era between smoothed topographic heights and the geoid remains unchanged, it is possible to construct geoids for determined past geological epochs. Based on the resulting data sets of paleogeoid heights and by approximating them with a semi-parameterized ellipsoid, we can determine the positions of changes in the "True Polar Wander" (TPW) resulting from the movement of lithospheric plates over certain geological time intervals.

Methodology

Determining the Correlation between Geoid Heights and the Physical Surface of the Earth

To explore the planetary features of the correlation between geoid heights and lithospheric surface topography, we use moving correlation coefficients (MCC). We take into account [Tserklevych, 2013] that the radius of the moving window should be large enough for the resulting estimate of the moving correlation coefficient or variance to be justified, and simultaneously small enough to provide a local estimate. This moving radius is empirically determined by modeling different options and is chosen to cover an area on the planet's surface of 3 and 9-degree trapezoids which corresponds to an area of 1000×1000 km.

The correlation coefficient characterizes the strength of the relationship between two data sets. If the Earth's relief heights are denoted by H and the geoid heights by h, then we obtain the following expression for the moving correlation coefficient (MCC):

$$r_{hH} = \frac{K_{hH}}{D_{0h} D_{0H}}, \tag{1}$$

where

$$K_{hH} = \frac{1}{n-1} \sum_{i=1}^{n} (h_i - \bar{h})\left(H_i - \bar{H}\right), \tag{2}$$

$$D_{0h} = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (h_i - \bar{h})^2}, D_{0H} = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (H_i - \bar{H})^2}. \tag{3}$$

To investigate the correlation between geoid heights and topographic heights, data for 1×1º trapezoids were used. The center of the moving window was shifted by 1º in both latitude and longitude, reflecting the global nature of the correlation and mitigating its local manifestations.

To investigate the correlation between geoid heights and lithospheric surface heights, data for 1×1º trapezoids from the EGM2008 model [Pavlis et al., 2008] and topographic heights from ETOPO1 [ETOPO1 Global Relief Model], as well as paleoDEM paleoreconstruction models [Scotese, 2017; Scotese and Wright, 2018], were used. Moving the sliding center was moved by 1º in latitude and longitude, which conveys the global nature of the correlation and eliminates its local manifestations.

Let's consider the obtained values of the KCC from the standpoint of isostasy. Analyzing Fig. 1 and the obtained digital values of the CCC for a more detailed study of their distribution on the Earth's surface, we found a significant asymmetry in the isostatic compensation of the southern and northern hemispheres. The figure shows a global map of the correlation coefficients between topography heights and geoid heights, reflecting the spatial variability of the correlation in different regions of the Earth. The correlation coefficients range from −0.9 (blue regions) to 1.0 (red regions), as shown in the scale on the right. Color description:

− Red zones (high positive values) indicate a significant direct correlation, i.e., in these regions, the topography and geoid heights change in a coordinated manner.

− Blue zones (high negative values) show a significant inverse correlation, indicating opposite changes in topography and geoid elevations.

− Green and yellow zones indicate a weak or moderate correlation, where the consistency between topography and geoid heights is less pronounced.

The map of correlation coefficients between topography and geoid heights shows regions with high positive and negative correlation values, which are associated with tectonic features and lithospheric structure.

Peaks of direct correlation (red zones):

– The Andes (South America) – this region shows a significant positive correlation, indicating a consistency between the elevations of the topography and the geoid. This can be explained by high mountain ranges that have a corresponding gravitational anomaly that affects the shape of the geoid.

– Himalayas (South Asia) – the Himalayas also show a significant positive correlation, where the high elevation of mountain peaks is consistent with corresponding changes in geoid elevations.

– Ethiopian highlands and the East African Rift – in this region, where rifting and volcanic processes are active, the correlation between topography and geoid is also high, indicating that they are interconnected.

Peaks of negative correlation (blue areas):

– Pacific trenches – regions such as the Mariana Trench and other deep water areas in the Pacific Ocean show a strong negative correlation. This means that the low elevations of the topography (deep trenches) have an opposite gravitational response reflected in the geoid.

– North Atlantic (region near Iceland) – this region also shows a negative correlation, possibly due to the influence of the Mid-Atlantic Ridge, where active tectonic processes create relief features that do not correspond to the geoid heights.

– Indian Ocean (near the Seychelles and Maldives) – in some parts of the Indian Ocean, areas of negative correlation can be seen, which may be due to the peculiarities of underwater topography and gravitational anomalies associated with riftogenesis and oceanic ridges.

The map of correlation coefficients between topography and geoid heights for Antarctica shows specific features of the correlation indices, which is explained by the unique geophysical conditions of this continent.

Antarctica shows a predominantly inverse correlation between topography and geoid elevations (blue and green zones), which may be due to the presence of a thick ice sheet that overlays the underlying topography and causes gravitational anomalies. The ice sheet that covers the continent affects the mass distribution, which is different from solid mountain ranges like the Himalayas or the Andes. The low temperatures, high ice thickness, and subglacial topography cause gravitational changes that do not fully correspond to the topography of the solid ground beneath the glacier.

In addition, there are areas with a weak direct correlation at the edges of the Antarctic continent, usually located closer to the ocean, where the ice is thinner or absent. This may indicate some consistency between the relief and the geoid in these regions, possibly due to a more stable underlying geologic structure.

In summary, the correlation pattern for Antarctica indicates a complex interaction between the ice sheet, subglacial relief, and geodynamic processes that have a significant impact on the gravitational field. Antarctica is a special case in the global correlation picture because the ice sheet significantly modifies the gravitational field, creating gravitational anomalies that differ from continental mountain systems.

This distribution of forward and inverse correlations indicates a complex interaction between tectonic structures, gravitational anomalies, and the Earth's topography. High positive correlations are usually associated with mountainous areas, while high negative correlation values often correspond to oceanic trenches and tectonically active zones where the elevations of the geoid and the topography are not consistent. These data can be used to better understand the geophysical processes that shape the Earth's surface and gravitational field.

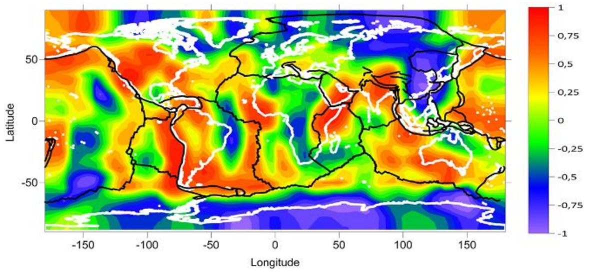

Fig. 1. MCC between topography heights and the geoid for the modern era with smoothing of 9×9º.

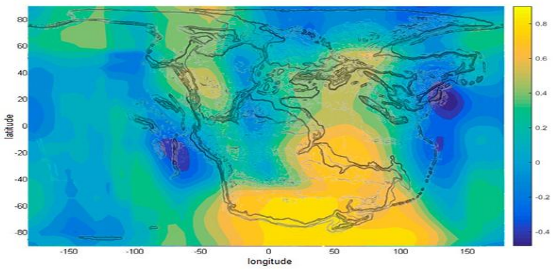

Fig. 2. MCC between topography heights and the modern geoid for 200 million years ago.

When comparing topographic heights in past epochs with the modern geoid, the obtained MCC values changed significantly. Illustrations of the MCC calculations are presented in Figs. 2, 3.

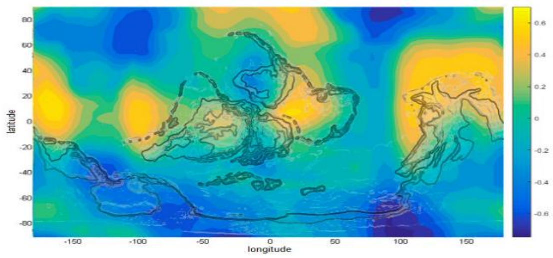

Fig. 3. MCC between topography heights and the modern geoid for 400 million years ago.

Thus, the apparent lack of correlation between geoid heights and the structure of the lithosphere is likely due to the fact that they are an integrated result of the statistical effect on the Earth's gravitational field of mass heterogeneities that differ in scale, depth, and sign. The integral effect of heterogeneous masses on the gravitational field leads to the appearance of a complex interference pattern, which is reflected on geoid maps. Therefore, studying the nature of the relationship between geoid undulations and lithospheric tectonic structures should be conducted with this integration in mind.

The most clear and reliable relationships between geoid heights and topography can be obtained using so-called residual geoids, which can be calculated by gradually excluding from the heights of the (true) geoid the heights associated with mass heterogeneities at different levels that make up the planet (e. g., sequentially excluding the influence of crust, lithosphere, and mantle heterogeneities). However, this procedure of separating the gravitational field is quite uncertain and complex in terms of data collection and interpretation and therefore was not used here. Instead, we performed averaging with different sliding radius values, which can characterize the impact of lithospheric plate relief and mantle heterogeneities on the correlation relationship with some degree of certainty and presented the results for a slip radius covering a 9-degree trapezoid.

An analysis of geoid height correlation maps with topography in a global scale reveals a large spread of MCC values on the planet's surface. However, in general, it can be noted that most continental uplifts are characterized by an inverse form of the correlation relationship between residual geoid heights and relief. For oceans, both inverse and direct relationships are equally characteristic. The largest MCC values encompass the periphery of continents and oceans, i. e., places with the greatest contrast in relief heights and the structure of the Earth's crust.

Modeling Geoid Heights (Paleogeoids) in Geological Time

Assuming that the statistical correlation model between topography heights and the geoid for the modern era remains unchanged, it is possible to construct geoids for determined past geological epochs. This assumption is based on the understanding that the planetary geoid, i.e., the long-wave amplitudes of the geoid's heights, are mainly caused by deep mass inhomogeneities [Stacey, & Davis, 2008; Tserklevych et al., 2022].

The basic algorithm for such constructions is the proposed model of the covariation relationship between the geoid heights and the topography of the lithospheric surface in the modern era. The basis of this approach is the relation for determining the paleogeoid heights through the geoid heights determined by the gravity field model

$$\left(h_{палеогеоїд}\right)_i = (h_{сучасн.геоїд})_i + k(H_{вис.PaleoDem})_i,$$

$$k = \frac{\sum_{j=1}^{n} h_j H_j}{\sum_{j=1}^{n} \left(H_j^2\right)} \qquad (4)$$

In the formula, k – the covariance coefficient is calculated from sets of averaged data in 1-degree trapezoids of geoid heights h_j and lithospheric surface topography heights H_j in the modern era, n –is the number of trapezoids that fall within the slip envelope.

To perform the calculations, we used known topography heights taken from the databases of paleoreconstruction models [Scotese and Wright, 2018]. For example, for the geological epochs of 200, 400, and 540 million years ago, Figs. 4−15 present images of model paleoreconstructions of continents and corresponding paleogeoids, as well as cumulative curves of topography and paleogeoid heights (mapping schemes were built in the "Surfer" software using sets of relevant digital data). Looking at the paleogeoid map schemes shown in Figs. 5 and 6, and 9, 10, and 13, 14, can notice small changes in elevations due to different averaging of data for 3×3º and 9×9º trapezoids.

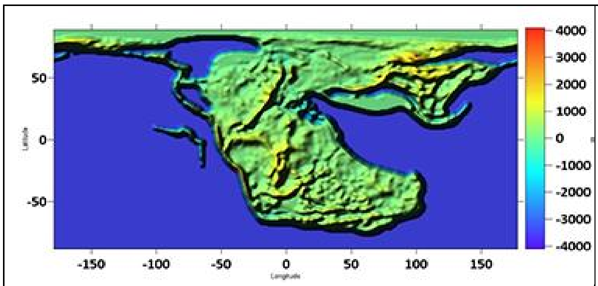

Fig. 4. Topography 200 Ma.

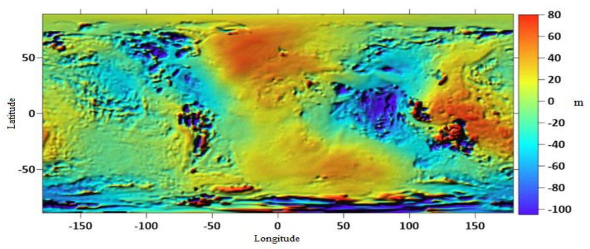

Fig. 5. Paleogeoid 200 Ma. Trapezoid size 3×3º.

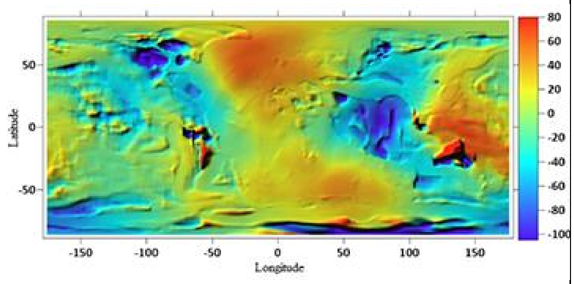

Fig. 6. Paleogeoid 200 Ma. Trapezoid size 9×9º.

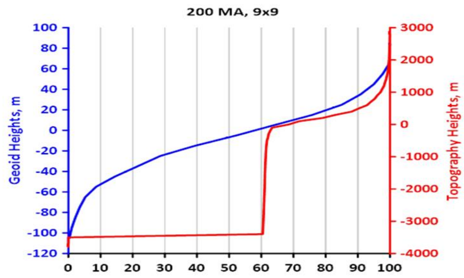

Fig. 7. Cumulative curves of topography and paleogeoid heights 200 Ma.

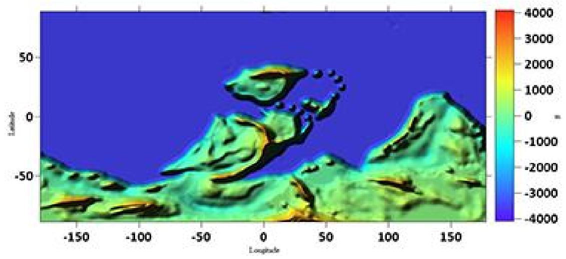

Fig. 8. Topography 400 Ma

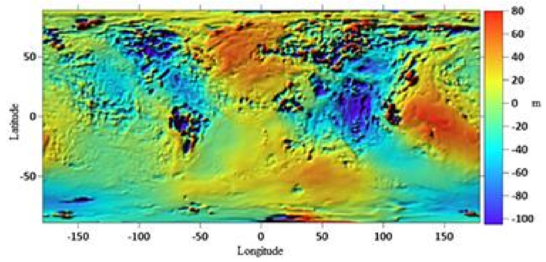

Fig. 9. Paleogeoid 400 Ma. Trapezoid size 3×3º.

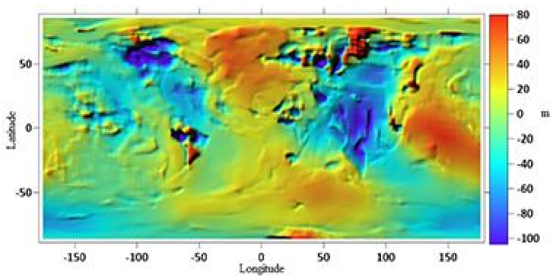

Fig. 10. Paleogeoid 400 Ma. Trapezoid size 9×9º.

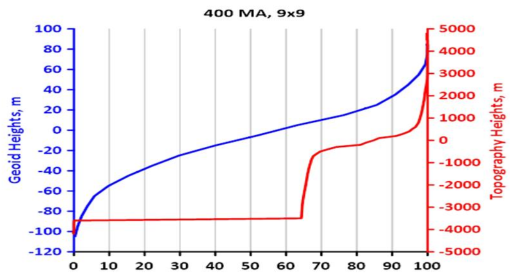

Fig. 11. Cumulative curves of topography and paleogeoid heights 400 Ma.

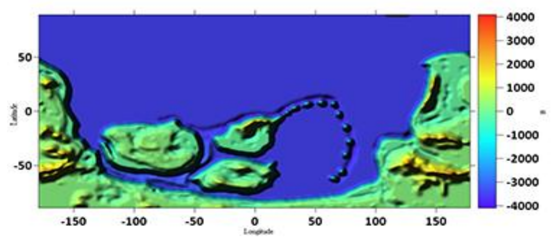

Fig. 12. Topography 540 Ma.

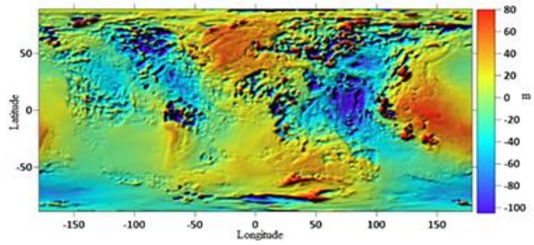

Fig. 13. Paleogeoid 540 Ma. Trapezoid size 3×3º.

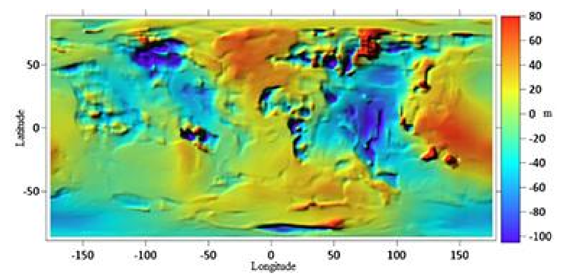

Fig. 14. Paleogeoid 540 Ma.
Trapezoid size 9×9º.

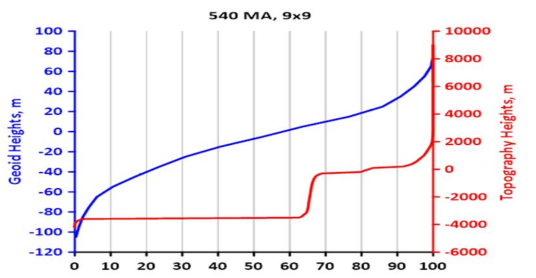

Fig. 15. Cumulative curves of topography and paleogeoid heights 540 Ma.

Approximation of Lithospheric Surface Heights and Paleogeoids with a Biaxial Ellipsoid

The solution to this problem was considered using the example of approximating the heights of the Earth's lithospheric surface with biaxial and triaxial ellipsoids in the article [Tserklevych et al., 2016].

Consider Fig. 16, which shows: P_L' – the Earth's physical surface; G – the geoid; E_L – the ellipsoid, whose parameters need to be determined; o – the center of the Earth's mass; o' – the center of the sought ellipsoid; oZ – the direction of the Earth's rotation axis; $o'Z'$ – a line parallel to the axis oZ; $o'Z''$ – the direction of the ellipsoid's minor axis E_L; κ, ς – two of the three Euler angles; N – the direction of the node line; $\theta_0, \lambda_0, \rho_0$ – the angles and distance that determine the shift of the sought ellipsoid's center from the center of mass; x_0, y_0, z_0 – values that determine the shift of the sought ellipsoid; T – a point on the Earth's physical surface; $oT' = \rho'$; $o'o = \rho_0$; $oT = \rho$; $TT'' = h$, h – height of the geoid above the ellipsoid; $o'T'' = r$; $TT' = H$, H – the height of the point T above sea level (geoid).

The size and orientation of the biaxial ellipsoid are usually determined by the condition $\sum h^2 \to \min$. This condition implies that the sum of the squares of the distances between the surfaces of the sought ellipsoid and the surface of the lithosphere or paleogeoid is minimal.

Table 1 presents the values of the seven parameters of the biaxial ellipsoid for the entire Earth and the northern and southern hemispheres. Calculations of the ellipsoid parameters were performed based on the data from the ETOPO1 digital model of the Earth's surface relief [ETOPO1 Global Relief Model]. This digital relief model is presented in the WGS84 geodetic coordinate system and covers the entire planet's surface with a resolution of one minute. For the indicated digital model, the heights were averaged within 5×5º trapezoids for further processing. As a result, 2592 values of lithospheric surface heights were obtained, which served as input data.

Table 1

Biaxial ellipsoid parameters

DEM ETOPO1	Whole Sphere		Northern Hemisphere		Southern Hemisphere	
x_0, M	-741,88	±86,04	-1170,64	±1056,31	-38,99	±13,19
y_0, M	-491,57	±57,01	-388,46	±350,52	1,70	±0,57
z_0, M	1421,79	±164,89	1868,02	±1685,58	7287,17	±2462,39
a_{xy}, M	6375117,00	±169,04	6375114,91	±437,96	6376300,87	±533,92
a_z, M	6355640,08	±247,83	6355772,53	±2294,90	6362552,67	±2798,15
$1/\alpha$	327,31	±6,32	329,59	±33,20	463,79	±80,16
κ	133°,77	±0°,02	159°,30	±0°,01	110°,03	±0°,01
ζ	2°,61	±0°,03	2°,61	±0°,01	2°,78	±0°,01

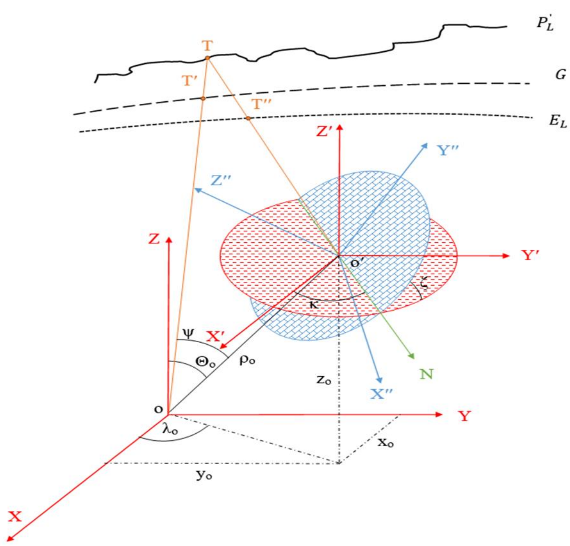

Fig. 16. Illustration of the approximation of the lithospheric surface and paleogeoid with a biaxial ellipsoid.

Fig. 17 shows the graphical representation of the displacement of the minor axis pole of the biaxial ellipsoid, which approximates the heights and depths of the surface of the paleoreconstructions of the continents. For this illustration, calculations were performed for 26 obtained ellipsoids that approximate the surface of the paleoDEM paleoreconstructions for different geological epochs every 50 million years. The maximum deviation of the minor semi-axis pole from the Earth's north pole does not exceed 2.6°, which may indicate the critical value of the geographical pole's deviation from the Earth's rotation axis for any configuration of the continents and water surface.

Parameters of the Biaxial Ellipsoid

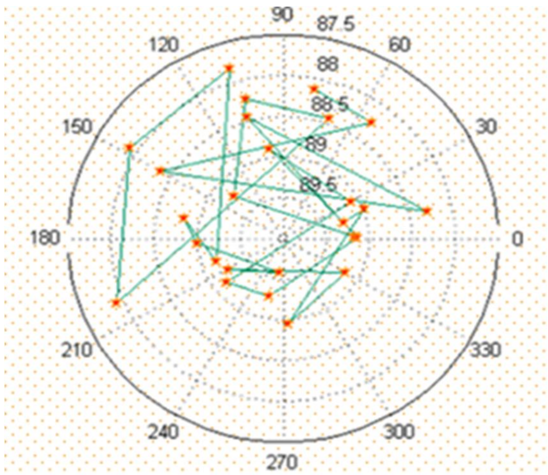

Fig. 17. Displacement of the minor axis pole of the biaxial ellipsoid approximating the heights and depths of the surface paleoreconstructions of the continents based on paleoDEM data [Scotese and Wright, 2018].

Based on the constructed elevation models of paleogeoids, for which sets of digital data on a 1ºx1º grid over 5 million years in the interval from the present to 540 million years ago were used, we obtained the parameters of semiparametric ellipsoids representing these paleogeoids. Table 2 presents the results of approximating the heights of the current geoid and paleogeoids for the geological epochs of 200, 400, and 540 million years ago. Based on the analysis of Graph 1 and Table 2, the results of approximating the heights of the current geoid and paleogeoids for the geological epochs of 200, 400, and 540 million years ago demonstrate the following:

The current geoid (EGM008) in the 3×3 and 9×9 models has slight differences in the coordinates of the center of mass, polar and equatorial radii. The pole displacement and pole displacement along the meridian are the same for both models.

The paleogeoid 200 million years ago (200 MA) in the 3×3º model shows a significant shift in the coordinates of the center of mass compared to the 9×9º model. There are significant differences in the polar and equatorial radii between the models. The pole displacement in the 3×3º model (1.302") is significantly larger than in the 9×9º model (0.504"). The pole displacement along the meridian in the 3×3º model (39 m) is significantly larger than in the 9×9º model (15 m).

The paleogeoid 400 million years ago (400 MA) has significant differences in the coordinates of the center of mass between the 3×3º and 9×9º models. The polar and equatorial radii differ significantly. The pole displacement in the 3×3º model (1.366") is larger than in the 9×9º model (0.716"). The pole displacement along the meridian in the 3×3º model (41 m) is larger than in the 9×9º model (21 m).

The paleogeoid 540 million years ago (540 MA) in the 3×3º model has center of mass coordinates that differ from the 9×9º model. The polar and equatorial radii are also different. The pole displacement in the 3×3º model (0.028") is significantly smaller than in the 9×9º model (0.179"). The pole displacement along the meridian in the 3×3º model (1 m) is smaller than in the 9×9º model (5 m).

In Fig. 18 shows two plots showing TPW changes along the meridian every 5 Ma until 540 Ma. The blue color shows the movement of the pole of the minor axis of the ellipsoid due to continental movement in the paleoDEM models. The maximum deviation of the pole of the minor axis from the Earth's north pole does not exceed 375 km, which may indicate the critical value of the deviation of the geographic pole from the Earth's rotation axis for any configuration of the location of continents and water surface. The results of TPW determination based on paleogeoid models are shown in red. The largest amplitude on the graph reaches approximately 29 meters. Note also that the correlation coefficient between the two data sets (pole position) shown on the graph is 0.41 with a confidence interval of 0.24 to 0.56 with a 95 % probability. These graphs demonstrate fluctuations in the amplitudes of pole motion, indicating changes in the heights of topography and paleogeodes in different geologic epochs. Changes in the amplitudes of pole motion are uneven and periodic, which may be due to the different placement of continents in the paleoDEM digital elevation model. Also, the periodic oscillations of the pole on the graph may indicate that the Earth is wobbling around its axis, which further creates pole wobble.

Changes in the amplitudes of pole motion are uneven and periodic, which may be due to the different placement of continents in the paleoDEM digital elevation model. Also, periodic pole oscillations on the graph may indicate that the Earth is wobbling on its axis, which further creates pole wobble. This phenomenon, known as "true polar wobble" (TPW), occurs as a result of mass redistribution on the Earth's surface and in its interior, which affects the position of the planet's rotation axis. TPW can be caused by the movement of lithospheric plates, gravitational and rotational forces acting on the Earth's internal structure from the lithosphere to the inner core, as well as cyclic geodynamic instability, which manifests itself as secular variations in the Earth's shape and gravitational field [Steinberger et al., 2017].

Table 2

Results of the approximation of the heights of the modern geoid and paleogeoids for the geological epochs of 200, 400, and 540 million years ago.

Geoid model	x_0, m	y_0, m	z_0, m	ρ_0	a_x, m	b, m	ζ,″	Pole displacement along meridian, m
EGM008 3×3	0.001	0.003	0.045	0.045	6378136.889	6356752.215	0.179	5
EGM008 9×9	−0.000	0.000	0.043	0.043	6378136.730	6356752.088	0.179	5
200 MA 3×3	−0.024	−0.241	0.151	0.285	6378134.241	6356749.874	1.302	39
200 MA 9×9	0.120	0.059	0.055	0.144	6378133.436	6356748.788	0.504	15
400 MA 3×3	−0.074	−0.144	0.037	0.167	6378133.640	6356748.973	1.366	41
400 MA 9×9	0.018	0.038	0.043	0.060	6378132.996	6356748.312	0.716	21
540 MA 3×3	−0.009	0.001	0.006	0.011	6378137.841	6356753.163	0.028	1
540MA 9×9	−0.005	0.178	0.154	0.236	6378134.386	6356749.735	0.179	5

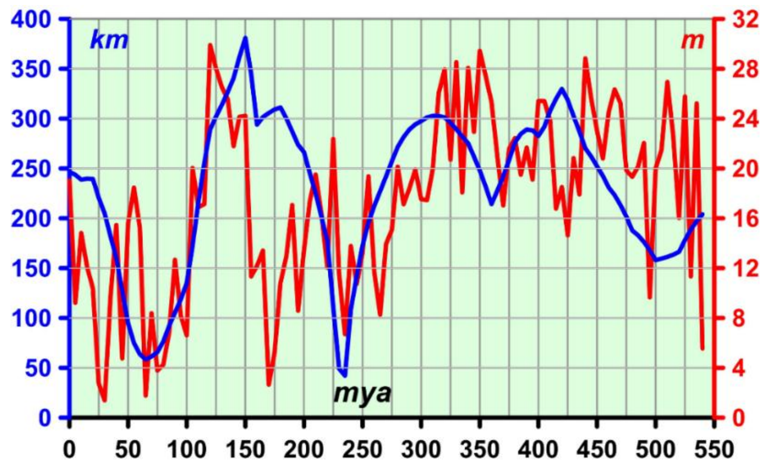

Fig. 18. Results of modeling the change in the pole position along the meridian on a geologic time scale.

Discussion and Conclusions

The modeling of the generalized shape of the lithosphere showed the presence of a deviation of the small semi-axis of the shape of the physical surface of the planet from the rotation axis of ~3°, which can affect the stress-strain state of the lithosphere and, accordingly, activate tectonic processes on Earth. The largest rise and fall of one ellipsoid above the other, which generally represents the Earth's lithosphere and geoid, is 950 m, and this rise leads to the emergence of tangential mass forces.

“The True Polar Wander” (TPW) can be modeled geodynamically because the spin (rotational) axis follows the axis of maximum inertia, which can be determined from geoid parameters on a geologic time scale. We used paleoDEM digital elevation models of the lithosphere, which are statistically the most reliable estimates available today, and based on our studies, we have verified that the movement of lithospheric plates does not significantly change the height of the planetary geoid. However, on the other hand, we suggest that TPW studies based on paleogeoid models, which involve lithospheric plate movements due to mantle convection, provide new insights into pole displacement in geologic time. The main conclusion of this article is that the amplitudes of such displacements can reach up to 30 m along the meridian.Thus, the obtained results not only deepen our understanding of the dynamic processes that shape the current and future geodynamic evolution of the planet but also provide valuable information for studying the historical configuration of the Earth’s geoid and continents.

Summarizing the above, it can also be stated that there are now fairly accurate data on the topography of the earth’s surface both on the continents and for the ocean and seabed. Digital elevation models paleoDEM in the sense of collected data for modeling paleoreconstructions of continents on the scale of geologic time also provide opportunities to perform modeling of paleogeoids. Therefore, it became possible to calculate the generalized shape of the physical surface and model paleoids in the form of a biaxial ellipsoid with its own axis orientation parameters.

Thus, the results obtained not only deepen our understanding of the dynamic processes that shape the present and future geodynamic evolution of the planet, but also provide valuable information for studying the historical configuration of the Earth’s geoid and continents and the geodynamically modeled polar wander of the pole.

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МОДЕЛЮВАННЯ ГЕОЇДА І РУХУ ПОЛЮСА В ГЕОЛОГІЧНОМУ ЧАСІ

Основні цілі дослідження: 1) виконання кореляційного аналізу між висотами геоїда і топографічними висотами в сучасну епоху із використанням розрахункових коефіцієнтів ковзної кореляції (ККК); 2) екстраполювання отриманої моделі кореляційного зв’язку на минулі геологічні епохи і визначення палеогеоїда за відомими висотами поверхні літосфери, отриманими із моделей палеореконструкцій континентів paleoDEM (Scotese and Wright, 2018); 3) обчислення змін руху полюса за отриманими наборами даних висот палеогеоїдів “True Polar Wander” (TPW) унаслідок переміщення літосферних плит. Методика. Для дослідження кореляційного зв’язку між висотами геоїда і висотами поверхні літосфери використано дані для трапецій розміром 1×1º за моделлю EGM2008 та топографічні висоти ETOPO1, а також моделі палеореконструкцій paleoDEM. Переміщення центра околу ковзання здійснювалось через 1º по широті та довготі в межах гріда 3×3º та 9×9º, що засвідчує глобальність кореляційної залежності й нівелює її локальні прояви. Екстраполюючи сучасну модель кореляційного зв’язку на минулі геологічні епохи, ми дослідили динамічну палеогеографічну еволюцію та її вплив на структуру геоїда. Для дослідження динаміки зміни фігури літосфери Землі та висот палегеоїдів і положення полюса використано апроксимацію їх поверхонь семипараметризованим двовісним еліпсоїдом. Результати. За обчисленими значеннями ККК побудовано карту кореляційного зв’язку між висотами геоїда і висотами топографічної поверхні для сучасної епохи. Детально розглянуто кореляційний аналіз на прикладі різних епох – 200, 400 і 540 млн років тому і на проміжку від сучасної епохи до 540 млн років з інтервалом 5 млн років із використанням моделей палеогеоїдів, щоб запропонувати гіпотезу про секулярний рух полюсів обертання Землі та пов’язану із ним динаміку літосфери. Наукова новизна. Здійснено моделювання висот палеогеоїдів для подальшого оцінювання переміщення полюса Землі. Розглянуто також вплив гравітаційних і обертальних сил на внутрішню структуру Землі, від літосфери до внутрішнього ядра, за припущення про циклічну геодинамічну нестабільність, що проявляється у вигляді вікових варіацій форми Землі та гравітаційного поля. Наші висновки підтверджують глибоке розуміння взаємозв’язку між тектонічною активністю та аномаліями палеогеоїдів, за припущення про мінімальний прямий вплив рухів літосферних плит на зміни висоти геоїдів, але істотний непрямий вплив через конвекцію мантії протягом геологічного часу. Практичне значення. Це дослідження поглиблює не лише уявлення про історичну конфігурацію геоїда і континентів Землі, але й розуміння динамічних процесів, що визначають теперішню і майбутню геодинамічну еволюцію планети.

Ключові слова: висоти геоїда, висоти поверхні літосфери, ковзні коефіцієнти кореляції, палео-реконстукція материків, палеогеоїд, TPW, геодинаміка.

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