Dispersion and scatter of elastic waves in a pre-stressed and fractured geological medium

© B. Maslov, 2010

Institute of Mechanics, National Academy of Sciences of Ukraine, Kiev, Ukraine
maslov@inmech.kiev.ua

It is known [Crampin, Peacock, 2008] shear-waves propagating in anisotropic rocks split into two approximately orthogonal polarisations that travel at different velocities. Such seismic birefringence aligned azimuthally is widely observed in almost all igneous, metamorphic, and sedimentary rocks in the Earth's crust in almost all geological and tectonic regimes. The causes and interpretation of shear wave splitting in the Earth's crust are still not clearly understood [Maslov et al., 2006]. Although, in principle, shear-wave splitting is simple in concept and easy to interpret in terms of systems of anisotropic symmetry, in practice there are subtle differences from isotropic propagation that make it easy to make errors in interpretation.

So the solution of such problems is of interest not only revealing of prominent features of wave movement in the non-heterogeneous media, as that: dispersions, attenuations, but also and revealing of the reasons causing these effects. For long-wave processes the geo structure may be considered as some homogeneous media with the overall elastic properties [Maslov et al., 2001]. However in such approach it is impossible to reveal dependence of speed elastic waves from frequency that is observed in experiments on frequency of an order of several thousand hertz. To investigate the phenomena of a dispersion and attenuation in micro non-uniform media one uses the so-called dynamic effective characteristics defined from the equations of movement for representative volume [Maslov, 1982].

Here the problem of the overall dynamic characteristics determination for random geo structure with initial stress strain state in components is considered. The basic equations of incremental theory of elasticity are resulted in [Maslov, 2008]. Basic of them are the equations of balance of an initial state

$$\sigma^R_{ma,a} = 0 \,. \tag{1}$$

The equations of elastic motion for actual state

$$\sigma^A_{ma,a} = \rho\frac{\partial^2 u_m}{\partial t^2}\,,\ \ \sigma^A_{ma} = F_{mb}s^{ba}\,,$$
$$s^{ba} = \partial W / \partial e_{ab}\,. \tag{2}$$

Boundary and initial conditions

$$\begin{aligned} &\sigma^A_{ma}n^A_a = P_m\,,\ \ \mathbf{x}\in S\,,\\ &u_m(\mathbf{x},0) = f_m(\mathbf{x},0)\,,\ \ \mathbf{x}\in S\,,\\ &u_m(\mathbf{x},t) = \psi_m(\mathbf{x})\,,\ \ t=0\,,\\ &\frac{\partial u_m}{\partial t} = \psi^1_m(\mathbf{x})\,,\ \ t=0\,. \end{aligned} \tag{3}$$

Constitutive law for increments [Crampin, Peacock, 2008]

$$\begin{aligned} &\sigma^A_{im} = L^{ijab}H_{ab}\,,\\ &L^{iamb} = F_{ik}F_{mn}\frac{\partial^2 W}{\partial e_{ak}\partial e_{nb}} + \delta_{im}\frac{\partial W}{\partial e_{ab}}\,,\\ &e_{ab} = \frac{1}{2}\left(F_{ma}F_{mb} - \delta_{ab}\right),\\ &F_{ma} = \delta_{ma} + H_{ma}\,,\ \ H_{ma} = u_{m,a}\,. \end{aligned} \tag{4}$$

The frequently used in nonlinear geodynamics plastic potential is

$$\begin{aligned} &W = \frac{1}{2}\lambda p_1 + \mu p_2 + 3\gamma\frac{n}{1+n}p^{1+1/n}\,,\\ &p_1 = e_{mm}\,,\\ &p_2 = e_{mn}e_{nm}\,,\\ &p = \frac{2}{3}\left(I_1^2 - 3I_2\right)^{1/2} = \left(\frac{2}{3}e'\cdot e'\right)^{1/2}. \end{aligned} \tag{5}$$

In this case

$$\lambda_{abcd} = 2\mu K_{abcd} + 3æJ_{abcd} + \frac{3}{2}E_{abcd},$$

$$E_{abcd} = \frac{2}{3}\hat{e}_{ab}\hat{e}_{cd}, \ \hat{e}_{ab} = e'_{ab} / p,$$

$$l_{abmn} = F_{mk}\lambda^{abkn}, \ s_{ab} = l_{abmn}H_{mn},$$

$$I_{abcd} = \frac{1}{2}(\delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc}),$$

$$J_{abcd} = \frac{1}{3}\delta_{ab}\delta_{cd}, \ K_{abcd} = I_{abcd} - J_{abcd}, \qquad (6)$$

I_1, I_2 — main invariants of finite deformation tensor [Maslov, 1982; Crampin, Peacock, 2008].

The state of homogeneous preliminary compression is considered in detail. Then, having restricted to a case of small gradients of initial displacements, the incremental elasticity tensor L_{abcd} is obtained. Substituting in (4) we gain the equations of motion which under the form presented here are like the Lame linear theory of elasticity equations. Fourier transformation led to equation for fluctuations of displacement, with circular frequency dependency. Random geo structures in which casual fields are statistically homogeneous and ergodic concerning not deformed reference representative volume are considered. On the boundary of elementary macro volume the fluctuations of displacements are equal to zero. So this case provides a perfect analogy of approach to problems of the linear theory of elasticity and incremental theories. As to the solution in incremental theories we are interested in averaged gradients of displacements in components while in the linear theory of micro non-uniform geological media the problem consists in determination of averaged on components deformations (the symmetrized gradients). Green's function of the equations in long-wave approach looks like, similar to a case of the linear theory. Constants are expressed in speeds of longitudinal and shear waves.

Substituting the solution of a statistical dynamic problem in the macroscopic equations of motion, we consider in a detail a case of a plane harmonic wave. From the dispersive equation it is found phase and group speeds of waves. Attenuation factors are obtained as functions of constituent properties, frequency and initial stress state. The account of preliminary plastic deformation leads essential variations of speed values. Concrete examples of calculation of dependence of phase speed of a shear wave from frequency in a case when components are in a state of long term initial plastic deformation are considered. Natural state of media as reference example is been modeled too. As result, influence of an initial plastic state has the same order, as influence of frequency of a wave spreading.

Azimuthally-aligned shear-wave splitting is widely observed so the splitting may be used as diagnostic of some form of seismic anisotropy and stressaligned fluid-saturated micro cracks as the cause of azimuthally-aligned shear-wave splitting [Maslov, 1982; Maslov et al., 2001]. Shear-wave splitting is modeled here by dependence of incremental elasticity (4) from stressed state. And contra versus the splitting is some kind of diagnostic for some form of seismic anisotropy. The next idea is possibility to investigate stress-aligned fluid-saturated micro cracks [Maslov et al., 2001] as the cause of azimuthally-aligned shear-wave splitting.

As we outlined earlier [Maslov et al., 2006] for the analysis of a long term motion of geo structure it is useful the continuum damage concept. The micro cracks distributed in regular intervals or casually in a material, may be offered as the variable of a degree degradation of elastic properties. The measure continuum damage may be considered as formal reduction of the area of cross-section section of the sample. Then it is possible to enter effective stress and the destruction moment to identify with achievement damage values. Damage accumulation is stochastic process by the nature therefore even at performance of qualitative, well controllable field experiments, the big statistical variability of data is observed. We use further a hypothesis of equivalence of elastic energy of a material in initial state and the damaged material. If the free from stress configuration of elementary volume in a point has passed in the new form described by a field of plastic deformations $e^T_{ab}(\mathbf{x},t)$ then constitutive law (4) may be rewritten in form

$$\sigma_{ij} = L^{ijab}\left(e_{ab} - e^T_{ab}\right). \qquad (7)$$

Such a model of incremental elastic behavior of the geological media does not assume change of elasticity owing to occurrence of a field of deformations of transformation. Thus, formally some element of a source is supposed free from initial stress if deformations are equal in it to the deformations of transformation mentioned earlier (7). The general models of cracks or the destructions of ruptures represented as a surface of rupture of a field of dislocations also in a limit can be described as field distribution $e^T_{ab}(\mathbf{x},t)$ in a narrow zone. If to put a crack of a zone of transformation aspiring to zero corresponding components will aspire to infinity so that there is the rupture of dislocations equivalent to the phenomenon of destruction. Thus deformations we present here through Dirac δ-function. Therefore the solution of a problem with some kind of damage degradation also can be presented through Green dynamic function [Maslov, 1982].

The crack density [Maslov et al., 2006] used for calculation is approximately equal to one hundredth of the percentage of shear-wave velocity anisotropy in aligned cracks in a reference initial medium. Fluid-saturated micro cracks model suggested to evaluate viscous effects dispersion, scatter and splitting of elastic waves in pre stressed and fractured geological medium.

References

Crampin S., Peacock S. A review of the current understanding of seismic shear-wave splitting in the Earth's crust and common fallacies in interpretation // Wave Motion. — 2008. — **45**, № 6. — P. 675—722.

Maslov B. P. Overall dynamic characteristics of composite materials with initial stress // Prikl. Mech. — 1982. — **18**. — № 6. — P. 75—80 (in Russian).

Maslov B. P. Thermal-stress concentration near inclusions in visco-elastic random composites // J. Engineering Mathematics. — 2008. — **61**. — P. 339—355.

Maslov B. P., Prodaivoda G. T., Vyzhva S. A. A new mathematical modeling method of rupture processes in Lithosphere // Geoinformatics. — 2006. — № 3. — P. 53—61 (in Russian).

Maslov B. P., Prodayvoda G. T., Vyzhva S. A. Mathematical modeling of elastic wave velocity anisotropy in a cracked geological medium // Geophys. J. — 2001. — **20**, № 2. — P. 191—212 (in Russian).

Estimating the stresses within the lithosphere: parameter check with applications to the African Plate

© ***S. Medvedev*[1] *and "The African Plate" Working group*[1,2]*, 2010***

[1]Physics of Geological Processes, Oslo University, Oslo, Norway
sergeim@fys.uio.no
[2]Center for Geodynamics, NGU, Trondheim, Norway

Several mechanisms control the state of stress within plates on Earth. The list is rather long, but well known and includes ridge push, mantle drag, stresses invoked by lateral variations of lithospheric density structure and subduction processes. We attempt to quantify the influence of these mechanisms and to construct a reliable model to understand modern and palaeo-stresses using the African plate (TAP) as an example.

Constructing the base model lithosphere of TAP we follow [Steinberger et al., 2001]. Combining data on topography, age of ocean floor and global model for crust structure, CRUST2 [Bassin et al., 2000], we compute the gravitational potential energy (GPE) for the entire TAP. GPE, proportional to the double integration of the density profile through thickness of the model lithosphere, describes the forces rising from lateral density heterogeneities within lithosphere. In particular, GPE of the base model accounts for push from the mid-oceanic ridges surrounding TAP and stresses rising from the crustal thickness changes.

The finite-element based suite ProShell was utilized to calculate stresses using the real, non-planar geometry of TAP. The modeled results are tested and iterated to match the observed stress pattern recorded or derived from observations. We combined several studies to complete set of observational data. That includes non-seismic data from WSM [Heidbach et al., 2008], compilation of the field observation [Bird et al., 2006], and integrated inversion of focal mechanism data [Delvaux, Barth, 2010]. Fig. 1 presents the distribution of data on stress regimes and orientation of most compressive mean stress. We adopted several numerical characteristics describing proximity of model results and observations: 1) the average misfit angle is the mean difference in orientations; 2) the angle fitting factor is the percentage of the number of observa-