УДК 528.21 DOI: https://doi.org/10.24028/gzh.v44i1.253714

On the construction of gravimetric geoid model on the Lviv region area

F. Zablotskyi[1], V. Maksymchuk[2], B. Dzhuman[1], 2022

[1]Department of Higher Geodesy and Astronomy, Lviv Polytechnic National University, Lviv, Ukraine

[2]Carpathian Branch of S.I. Subbotin Institute of Geophysics of the National Academy of Sciences of Ukraine, Lviv, Ukraine

Received 19 October 2021

Baltic Height System 1977, currently used in Ukraine, the starting point of which is the zero of the Kronstadttide-gauge, isobsoletedue to the great distance from the zero-point of the reference height and the difficulty of adapting satellite methods of geodesy. For the successful modernization of the height system of Ukraine, it is necessary to integrate it into the United European Leveling Network (UELN). For the full functioning of any modern height system, namely to determine the gravity-depend heights by satellite methods, it is necessary to operate with a high-precision geoid model. Therefore, an important task is construction of a high-precision regional model of the geoid on the territory of our state. The rear many methods of constructing a model of the regional Earth's gravitational field, including the geoid model, each of which has its advantages and disadvantages. The purpose of this article is to test the STHA-method for calculating the model of the regional gravitational field, in particular the gravimetric model of the geoid, on the territory of Lviv region and to assess its accuracy. Free air gravity anomalies Δg from WGM2012 provided by the International Gravimetric Bureau (BGI) were used as initial data. The gravimetric STHA-model of the geoid was calculated with in the procedure «Remove-Compute-Restore» up to 8 degrees/order. To assess the accuracy of the model, it was compared with 213 points of GNSS leveling, as well as with the model EGM2008 up to 360 degrees/order. There are always differences between geometric and gravimetric geoid models duet or and ommeasurement errors, in consistencies in datums, different geodynamic effects etc. Respectively the parameters of the transition between gravimetric and geometric models of the geoid on the territory of Lviv region were also found. The proposed method can be used to build a high-precision model of the geoid for the entire territory of Ukraine with its subsequent coordination with the model of the European geoid EGG2015.

Key words: height system, geoid model, gravity anomalies, GNSS-leveling.

Introduction. Baltic Height System 1977, currently used in Ukraine, the starting point of which is the zero of the Kronstadt tide-gauge, is obsolete due to the great distance from the zero-point of the reference height and the difficulty of adapting satellite methods of geodesy. Therefore, it needs to be modernized by integrating into the United European Leveling Network (UELN). One of the main stages of this integration is constructing a high-precision model of the geoid for Ukraine, which should be consistent with the European geoidEGG2015 [Denker, 2015].

During the construction of height networks, traditional levelling methods combined with the gravimetric survey remain the most accurate today. This is due to the lack of sufficiently accurate geoid/quasi-geoid models, which entails high economic costs. Satellite methods, particularly GNSS methods, allow obtaining the geodetic height of a point (the height above the ellipsoid) in the static mode with an accuracy ≈2—3 cm. However, model geodetic heights are qualitatively different from orthometric (natural) and normal (close to natural) heights. They, in general, characterize the concept of «height» but do not have a physical meaning and therefore are not used by themselves.

In turn, the accuracy of global models of the geoid surface (the height of the geoid above the ellipsoid) is 8—15 cm. Therefore, a very important task is to find methods for constructing as accurate as possible geoid models, which will allow obtaining orthometric heights (point heights above the geoid) or normal heights (point heights above the quasi-geoid) using only satellite methods, which is a cost-effective solution.

There are many methods for constructing an Earth's regional gravitational field model, including the geoid model. Among the main methods, each of which has advantages and disadvantages, we can distinguish the method of least square collocation [Moritz, 1975], fast Fourier transform [Sideris, 2005], radial basis functions [Marchenko, 1998], and spherical functions with fractional indices [Haines, 1985; De Santis, Torta, 1997].

The purpose of this work is the approbation of the STHA-method (variety of spherical functions with fractional indices) [Dzhuman, 2017; Sumaruket al., 2019a, b] for construction of the gravimetric model of geoid on the Lviv region area, as well as its comparison with the geometric model (constructed using GNSS levelling data) on this territory.

Data fromWGM2012. As input data to calculate the gravimetric model of the geoid on the Lviv region area, we used free-air gravity anomalies Δg from WGM2012 [Bonvalot et al., 2012]. WGM2012 is the first implementation of the project, which involves obtaining Bouguer anomalies and free air anomalies, placed on a high-resolution grid and maps of anomalies on a global scale. This project is implemented by the International Gravimetric Bureau (BGI) in cooperation with such international organizations as the International Association of Geodesy (IAG), the Geological Survey Commission (CGMW), the International Union of Geodesy and Geophysics (IUGG), UNESCO, the International Union of Geological Sciences (IUGS) and with other scientific institutions.

Free-air gravity anomalies WGM2012 are calculated from available global gravitational models of the Earth EGM2008 and DTU10, also include topographic corrections with resolution 1′—1′ from model ETOPO1, which take into account the contribution of the surface masses (topography, atmosphere, oceans, inland seas, lakes, shelf glaciers, and ice caps), which is shown in Fig. 1. The map of gravity anomalies from WGM 2012 is shown in Fig. 2.

To obtain input data, namely free-air gravity anomalies from WGM2012, for the region of study (latitude B is [48°, 51°], longitude L is [22°, 26°]), we sent a request to BGI. Thus, 11011 values of gravity anomalies Δg were obtained with the resolution of 2′—2′, shown in Fig. 3.

Calculation of the gravimetric geoid

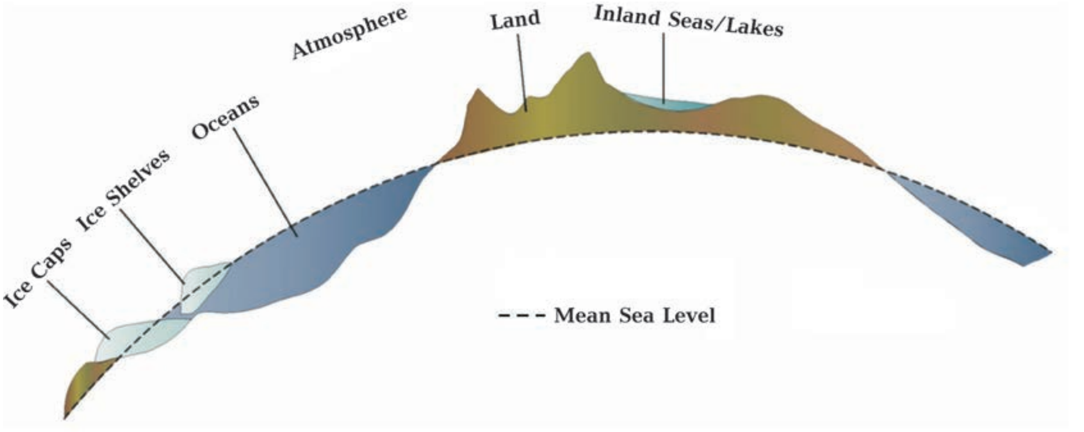

Fig. 1. Surface masses which are taken into account when calculating gravity anomalies from WGM2012 [Bonvalot et al., 2012].

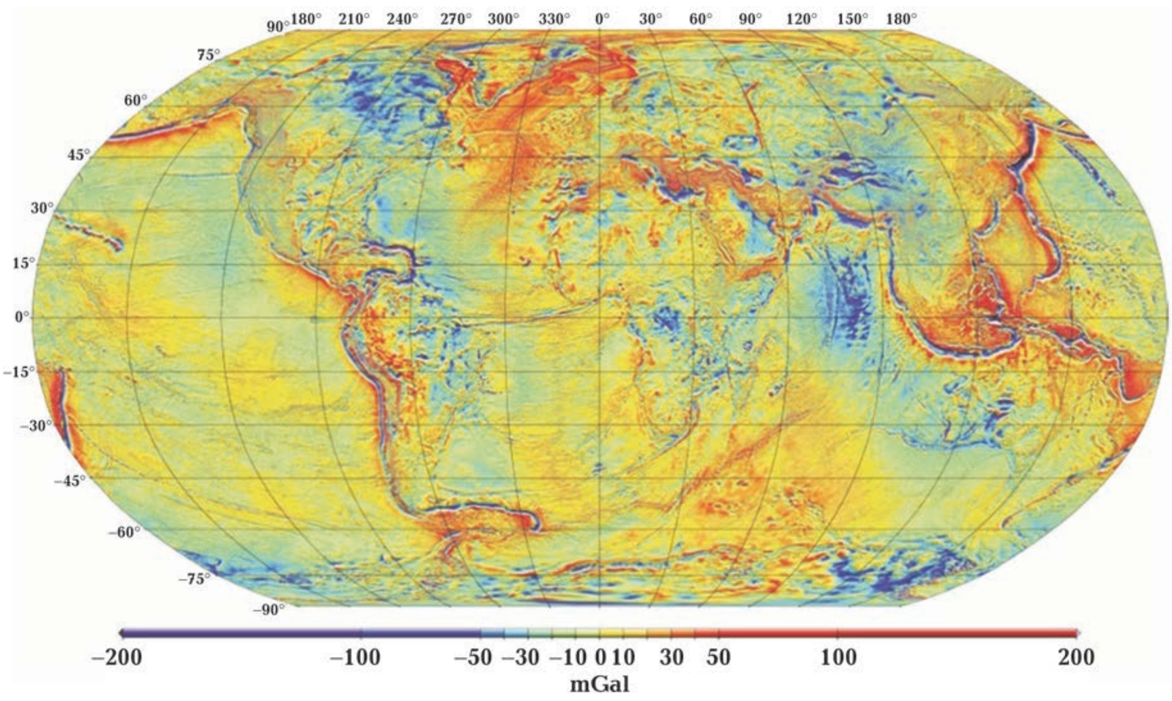

Fig. 2. Map of gravity anomalies from WGM2012 [Bonvalot et al., 2012].

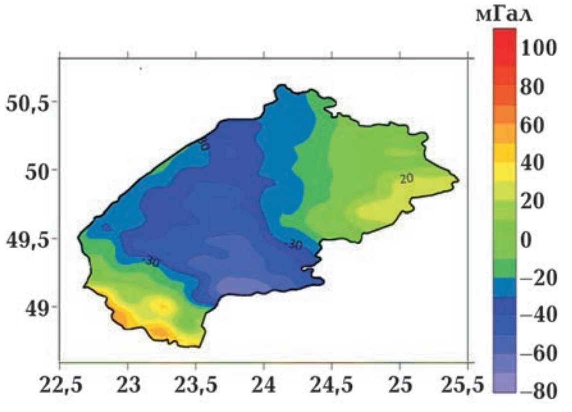

Fig. 3. Gravity anomalies Δg from WGM2012 on the Lviv region area.

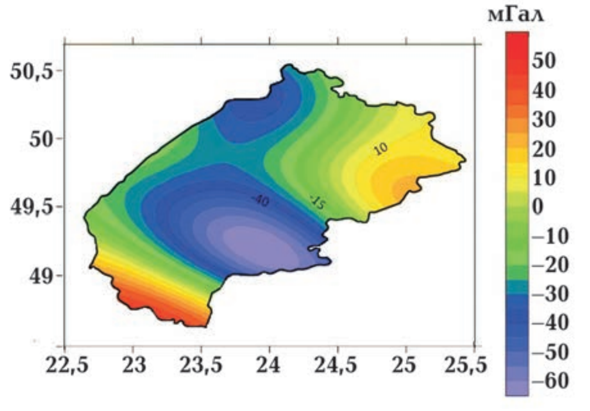

Fig. 4. Gravity anomalies calculated according to the EGM2008 model up to 360 degrees/order.

model for the Lviv region area. According to the obtained gravity anomalies Δ*g*, the gravimetric geoidmodel on the Lviv region area is calculated within the «Remove-Compute-Restore» procedure up to 8 degrees/order. The article [Marchenko, Lukyanchenko, 2018] shows the expediency of constructing global geoid models according to WGM2012 up to 600 degrees/order, which corresponds to the 8th degree/order of STHA-functions for the region of study. As the systematic component, we used the geoid model calculated according to the global gravitational model EGM2008 up to 360 degree/order using the resource ICGEM [Pavlis et al., 2008; Ince et al., 2019]. Fig. 4 shows the gravity anomalies of the EGM2008 model up to 360 degrees/order, and Fig. 5 shows the differences between the gravity anomalies from WGM2012 and these anomalies.

The analytical expression for the expansion of differences δΔ*g* into a series of STHA-

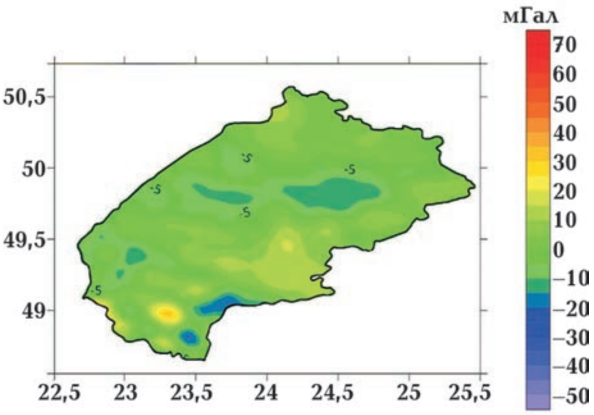

Fig. 5. Differences between gravity anomalies according to the EGM2008 model up to 360 degrees/order and according to WGM2012.

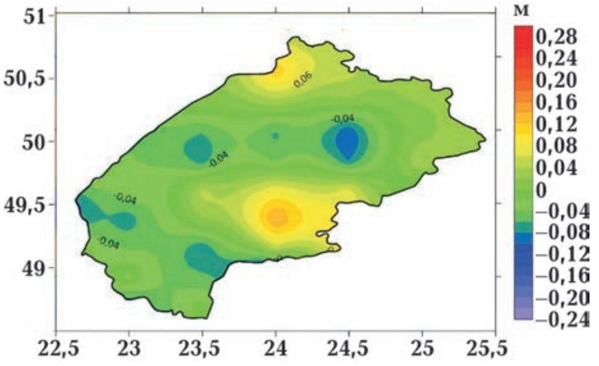

Fig. 6. Model of residual values of the gravimetric geoid heights.

functions is the following [Dzhuman, 2017]:

$$\delta\Delta g(\theta,\lambda)=\frac{GM}{R^2}\sum_{k=1}^{k_{\max}}\sum_{m=0}^{k}(n_k-1)\times$$
$$\times\left\{\bar{C}_{km}\cos\left(2\pi m\frac{\lambda-\lambda_{\min}}{\lambda_{\max}-\lambda_{\min}}\right)+\right.$$
$$\left.+\bar{S}_{km}\sin\left(2\pi m\frac{\lambda-\lambda_{\min}}{\lambda_{\max}-\lambda_{\min}}\right)\right\}\bar{P}_{km}(\theta), \qquad (1)$$

where GM isgravitational constant, γ isthe normal value of the free-fall acceleration, $\bar{C}_{km}$ and $\bar{S}_{km}$ areunknown model coefficients, found by the least-squares method.

According to the obtained coefficients $\bar{C}_{km}$ and $\bar{S}_{km}$, we calculated the model values of the residual heights of the geoid δN_m using the following formula [Dzhuman, 2017]:

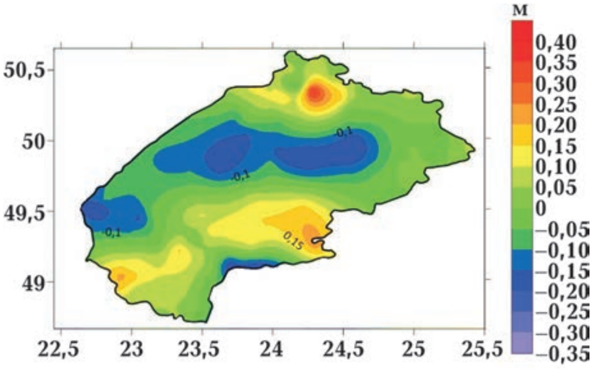

Fig. 7. Differences between the calculated model and the geoid heights from GNSS leveling.

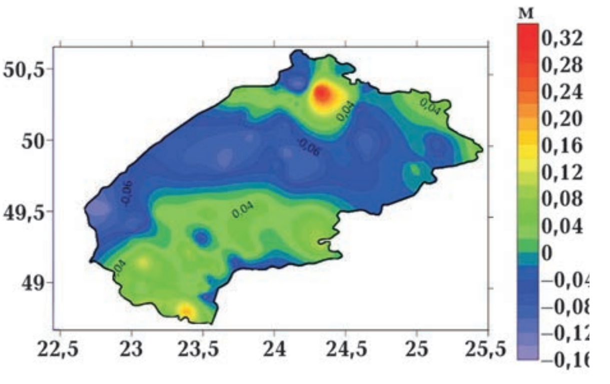

Fig. 8.Differences between the heights from the EGM2008 model up to 360 degree/order and the geoid heights from GNSS levelling.

$$\delta N_m =$$
$$=\frac{GM}{\gamma R}\sum_{k=2}^{8}\sum_{m=0}^{k}\left\{\bar{C}_{km}\cos\left(2\pi m\frac{\lambda-\lambda_{\min}}{\lambda_{\max}-\lambda_{\min}}\right)+\right.$$
$$\left.+\bar{S}_{km}\sin\left(2\pi m\frac{\lambda-\lambda_{\min}}{\lambda_{\max}-\lambda_{\min}}\right)\right\}\bar{P}_{km}(\theta), \qquad (2)$$

and also built a model of residual values, shown in Fig. 6. The main characteristics of this model are as follows: standard deviation is 0.084 m, the minimum value is –0.239 m, the maximum value is +0.289 m, the average value is –0.004 m.

To assess the accuracy of the obtained model, we compared it with geoid heights from 213 points of GNSS levelling. The model EGM2008 up to 360 degrees/order we also compared with the same geoid heights. The differences between the calculated model and the geoid heights from GNSS levelling

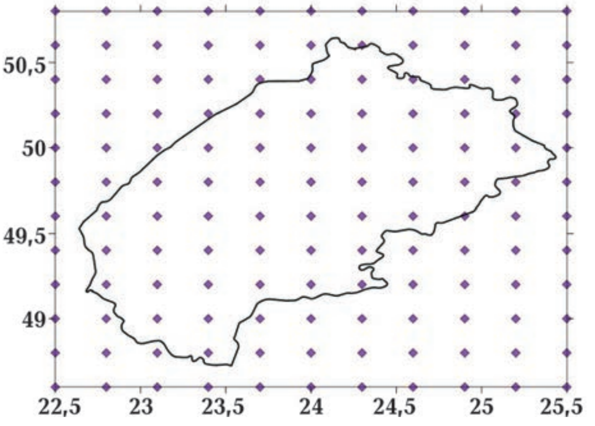

Fig. 9. Scheme of points for finding the parameters of the transition between geometric and gravimetric models of geoids.

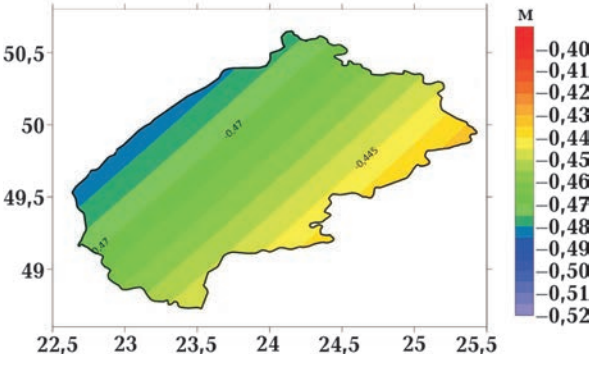

Fig. 10. Distribution of the ΔN values on the study area.

are shown in Fig. 7. The differences between the heights from the EGM2008 model up to 360 degrees/order and the geoid heights from GNSS levelling are shown in Fig. 8.

In both cases, a systematic component was traced between the surfaces of the geometric and gravimetric geoids. In the first case, it is –0.449 m and in the second one, it is –0.454 m. Fig. 7 and 8 show the differences of geoid heights with the systematic component. The standard deviation in the first case is 0.071 m, and in the second one, it is 0.124 m. This indicates that the obtained model significantly improved the values of geoid heights compared to the EGM2008 model up to 360 degree/order, which was used as a systematic component.

Relationship between gravimetric and geometric geoid models. There are always differences between geometric and gravimetric geoid models caused by random measurement errors, inconsistency of datums, various geodynamic effects, etc. Differences between gravimetric and geometric geoid models ΔN can be found mostly empirically according to the algorithms given in [Kotsakis, Sideris, 1999]. The general formula for ΔN has the form:

$$\Delta N = a_i^T x + v_i\,, \tag{3}$$

where a_i is a vector of known coefficients, x is a vector of unknown parameters, and v_i is a random noise vector.

Usually, we use a model consisting of four parameters:

$$a_i^T x = x_0 + x_1 \cos\theta_i \cos\lambda_i + \\ + x_2 \cos\theta_i \sin\lambda_i + x_3 \sin\theta_i\,. \tag{4}$$

Formula (4) is analogous to the following transformation model:

$$\Delta N_i = \Delta a + \Delta X_0 \cos\theta_i \cos\lambda_i + \\ + \Delta Y_0 \cos\theta_i \sin\lambda_i + \Delta Z_0 \sin\theta_i\,, \tag{5}$$

where Δa is the difference of the major half-axis of used ellipsoids, ΔX_0, ΔY_0, ΔZ_0 is a shift of parameters between two «parallel» datums.

Let us find the parameters (5) between gravimetric and geometric models of the geoid on the Lviv region area. The geometric geoid model (model calculated exclusively from GNSSl evelling data) we take from [Zablotskyi, Dzhuman, 2021]. Since they belong to the same ellipsoid, we accept Δa=0. To find other parameters, we found the geoid height on the grid that completely covers the study area (133 points). The location of these points is shown in Fig. 9.

The following parameters were obtained using the least-squares method: ΔX_0= =–1.814 m, ΔY_0=0.996 m, ΔZ_0=0.768 m. The distribution of the ΔN values according to the calculated parameters on the study area is shown in Fig. 10.

Conclusions. 1. The STHA-method for construction gravimetric model of a geoid was tested.

2. Gravimetric STHA-model of geoid up to

8 degree/order on the Lviv region area within the «Remove-Compute-Restore» procedure was calculated using free-air gravity anomalies Δ*g* from WGM2012, obtained from the International Gravimetric Bureau. This model is compared with the geoid heights calculated from GNSS levelling. It was found that the obtained model almost twice improved the accuracy of geoid heights compared to the model EGM2008 up to 360 degree/order, which was used as a systematic component.

3. The parameters of the transition between geometric and gravimetric models of the geoid on the Lviv region area are found.

References

Bonvalot, S., Balmino, G., Briais, A., Kuhn, M., Peyrefitte, A., Vales, N., Biancale, R., Gabalda, G., Reinquin, F., & Sarrailh, M. (2012). *World Gravity Map*. Retrieved from http://www.cartographie.ird.fr/images/carte_gravimetrie_2012/WGM_Notice_2012br.pdf

De Santis, A. & Torta, J. (1997). Spherical cap harmonic analysis: a comment on its proper use for local gravity field representation. *Journal of Geodesy, 71*(9), 526—532. https://doi.org/10.1007/s001900050120.

Denker, H. (2015).A new European gravimetric (quasi) geoid EGG2015.*XXVI General Assembly of the International Union of Geodesy and Geophysics (IUGG), Earth and Environmental Sciences for Future Generations, Prague, Czech Republic, 22 June—02 July 2015*.

Dzhuman, B.B. (2017). Modeling of the Earth's gravitational field using spherical functions. *Geodesy, cartography and aerial photography*, (86), 5—10. https://doi.org/10.23939/istcgcap2017.02.005.

Haines, G. (1985). Spherical cap harmonic analysis. *Journal of Geophysical Research: Solid Earth, 90*(B3), 2583—2591. https://doi.org/10.1029/JB090iB03p02583.

Ince, E.S., Barthelmes, F., Reißland, S., Elger, K., Förste, C., Flechtner, F., & Schuh, H. (2019). ICGEM — 15 years of successful collection and distribution of global gravitational models, associated services and future plans. *Earth System Science Data, 11*(2), 647—674. http://doi.org/10.5194/essd-11-647-2019.

Kotsakis, C., & Sideris, M. (1999).On the adjustment of combined GPS/levelling/geoid networks.*Journal of Geodesy, 73*, 412—421.https://doi.org/10.1007/s001900050261.

Marchenko, A.N. (1998). Parameterization of the Earth's Gravity Field: Point and Line Singularities. Published by Lviv Astronomical and Geodetic Society, 210 p.

Marchenko, A.N., & Lukyanchenko, Yu.O. (2018). Approbation of the global gravity field model constructing method with a non-diagonal normal equations matrix based on the DTU10 gravity anomalies.*Visnyk astronomichnoyi shkoly*, (14), 35—41 (in Ukrainian).

Moritz, H. (1975). *Integral formulas and Collocation*. Publisher — Ohio State University, Dept. Of Geodetic Science, 63 p.

Pavlis, N.,Kenyon, S., Factor, J., & Holmes, S. (2008). Earth Gravitational Model 2008. *SEG Technical Program Expanded Abstracts* (pp. 761—763). https://doi.org/10.1190/1.3063757.

Sideris, M. (2005). *Geoid determination by FFT techniques. International School for the Determination and Use of the Geoid*. Budapest University of Technology and Economics, 64 p.

Sumaruk, Yu.P., Yankiv-Vitkovska, L.M., & Dzhuman, B.B. (2019a). Modeling of regional magnetic field applying spherical functions: theoretical aspect. *GeofizicheskiyZhurnal, 41*(1), 180—191. https://doi.org/10.24028/gzh.0203-3100.v41i1.2019.158872 (in Ukrainian).

Sumaruk,Yu.P., Yankiv-Vitkovska, L.M., & Dzhuman, B.B. (2019b). Modeling of regional magnetic field applying spherical functions: practical aspect. *Geofizicheskiy Zhurnal, 41*(6), 165—172. https://doi.org/10.24028/gzh.0203-3100.v41i6.2019.190073 (in Ukrainian).

Zablotskyi, F.D., & Dzhuman, B.B. (2021).Construction of STHA-model of geometric geoid on the Lviv region area. *Suchasni dosiahnennia heodezychnoi nauky ta vyrobnytstva*, (2), 49—56 (in Ukrainian).

Про побудову гравіметричної моделі геоїда на територію Львівської області

Ф.Д. Заблоцький[1], В.Ю. Максимчук[2], Б.Б. Джуман[1], 2022

[1]Кафедра вищої геодезії та астрономії, Національний університет «Львівська політехніка», Львів, Україна
[2]Карпатське відділення Інституту геофізики ім. С.І. Субботіна НАН України, Львів, Україна

Діюча на сьогодні в Україні Балтійська система висот 1977 р., початковим пунктом якої є нуль Кронштадтського футштока, морально застаріла через велику віддаленість від нуль-пункту відліку висот і складність адаптації до застосування методів супутникової геодезії. Для успішної модернізації висотної системи України необхідно її інтегрувати в Об'єднану європейську нівелірну мережу (UELN). Для повноцінного функціонування будь-якої сучасної висотної системи, а саме для визначення гравітаційно залежних висот супутниковими методами, необхідно оперувати високоточною моделлю геоїда. Тому важливою задачею є побудова високоточної регіональної моделі геоїда на територію нашої держави. Існує багато методів побудови моделі регіонального гравітаційного поля Землі, зокрема моделі геоїда, кожен із яких має свої переваги та недоліки. Мета статті — апробація STHA-методу для обчислення моделі регіонального гравітаційного поля, зокрема гравіметричної моделі геоїда, на територію Львівської області та оцінювання її точності. Як вихідні використано дані щодо гравітаційних аномалій у вільному повітрі Δg із WGM2012, надані Міжнародним гравіметричним бюро (BGI). Обчислено гравіметричну STHA-модель геоїда в межах процедури «Вилучення—Обчислення—Відновлення» до 8 ступеня/порядку. Для оцінювання точності моделі виконано її порівняння з 213 пунктами GNSS-нівелювання, а також із моделлю EGM2008 до 360 ступеня/порядку. Між геометричною та гравіметричною моделями геоїда завжди є розходження, спричинені випадковими похибками вимірів, невідповідністю датумів, різними геодинамічними ефектами тощо. Відповідно, знайдено параметри переходу між гравіметричною та геометричною моделями геоїда на територію Львівської області. Запропонований метод можна використати для побудови високоточної моделі геоїда на всю територію України з подальшим її узгодженням із моделлю Європейського геоїда EGG2015.

Ключові слова: висотна система, модель геоїда, гравітаційні аномалії, GNSS-нівелювання.