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MATHEMATICAL MODEL OF SUCROSE SOLUTION NUCLEATE BOILING

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МАТЕМАТИЧНА МОДЕЛЬ БУЛЬБАШКОВОГО КИПІННЯ РОЗЧИНУ ЦУКРОЗИ

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МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ПУЗЫРЬКОВОГО КИПЕНИЯ РАСТВОРА САХАРОЗЫ

Abstract. A problem of nucleate boiling of an aqueous solution of sucrose moving in a vertical cylindrical channel is considered. The problem is based on the one-dimensional steady-state equations of heat and mass motion and transfer for the multiphase multicomponent flow written down in view of phase transition on the surfaces of the channel and in volume of the solution. Kinetics of nucleation on the pipe wall is based on semi-empirical dependences between frequency of bubbles formation, number of nucleation sites, structure of surface and the surface overheating. Two stages of the boiling process are considered. In subcooled liquid, bubbles formed on a wall are, at achieving critical radius, picked up by a flow and brought to the cold layers where they are collapsed emitting heat into solution. Temperature of the solution increases due to action of two factors – collapse of bubbles and liberation of the heat flow from the walls. In the overheated liquid, at presence of plenty of growing bubbles, a condition is possible when heat coming from the wall is equal to heat released by the bubbles in the liquid. Thermal balance is occurred in the solution, and the solution temperature does not increase in the upstream part of the flow.

A proposed mathematical model provides basic characteristics of the solution boiling in the channel and estimates their impact on the boiling dynamics.

Keywords: solution, nucleate boiling, critical radius, heat transfer.

Introduction

Process of liquid boiling has been in focus of various researches for a long time, and, as a result, a great number of works are devoted to this phenomenon, for example [1-4].

Reason for such great attention is, in terms of theory, a variety of physical processes which impact on each other, and in terms of practice it is intensification of heat and mass transfer which can be associated with essential heat pickup, stabilization of temperature and accumulation of energy. Process of boiling can be divided into many stages or physical effects, each of them represents significant interest for study.

The stages can include: preliminary heating of a liquid to some critical local thermal spike; nucleation and bubble growth both in the liquid volume and on the liquid surface; bubble detachment from a surface; heat transfer and its dynamics in the bubbles in overheated or subcooled liquids (depending on into which medium the bubbles gets); interaction of bubbles with each other and with carrying liquids (two-phase flows); change of flow conditions, etc. All these issues have already been discussed in numerous works and still are and will be a subject of future researches related to new ideas and new designs of the power devices.

Boiling of solutions consists of the same elementary effects, and each of the effects is complicated by existence of various components in the solution and more complex interaction between them. A lot of works [1, 2, 5-7] are devoted to the solution boiling, however, they mainly concern researches of indirect integrated quantities. In this article, a problem of nucleate boiling is considered which is characterized by formation of bubbles (nucleation) on the rough wall which are first detached and then rise upstream in the overheated liquid. As amount of such bubbles is huge, a theory of two-phase flow should be laid in the heart of this problem consideration. So, the proposed mathematical model is based on kinetics of the bubble formation on the rough surfaces [3, 4], theory of multiphase medium [8, 9] and specific behavior of sugar solutions characteristics of which are given in [10-12].

1. FORMULATION OF THE PROBLEM AND BASIC EQUATIONS

It is assumed that solution consists of two components: water and sucrose, and bubbles consist only of water vapor and they are formed on the surface of a cylindrical pipe into which the heat flow is fed.

Basic equations for heat and mass motion and exchange in view of power and thermal interaction of phases can be taken from [8]. By averaging parameters of flowing over the area of the cylindrical channel, and with the help of some simplifications it is possible to receive the following system of the one-dimensional equations:

$$S \left(u_p \frac{dn_p}{dx} + n_p \frac{du_p}{dx} \right) + 2R_T n_p^w v_p^w = 0, \quad (1)$$

$$\rho_r S \left(\alpha_r \frac{du_r}{dx} + u_r \frac{d\alpha_r}{dx} \right) + 2\rho_r R_T \alpha_r^w v_r^w = -n_p j_p S, \quad (2)$$

$$n_p u_p S \frac{dM_p}{dx} + 2R_T n_p^w v_p^w (M_p^w - M_p) = n_p j_p S, \quad (3)$$

$$\begin{aligned} \alpha_r \rho_r u_r S \frac{du_r}{dx} + 2R_T \rho_r \alpha_r^w v_r^w (u_r^w - u_r) = -S \frac{dp}{dx} + \\ + 2R_T \tau^w - n_p j_p S (u_p - u_r) + n_p f_p S - g \rho_r S, \end{aligned} \quad (4)$$

$$\alpha_p \rho_p u_p S \frac{du_p}{dx} + 2R_T \rho_p \alpha_p^w v_p^w (u_p^w - u_p) - n_p f_p S - g \rho_p S, \tag{5}$$

$$\begin{aligned} \alpha_r \rho_r u_r S \frac{dc_w}{dx} + 2R_T \rho_r (1 - c_w) \alpha_r^w v_r^w = \\ = -n_p (1 - c_w) j_p S, \end{aligned} \tag{6}$$

$$\begin{aligned} \alpha_r \rho_r u_r S \frac{dh_r}{dx} + 2R_T \rho_r (h_r^w - h_r) \alpha_r^w v_r^w = \\ = 2R_T (q_r^w - q_p^w) - n_p q_p S - j_p (h_r^a - h_r) S, \end{aligned} \tag{7}$$

where c_r is factor of a thermal capacity of a solution, c_s is mass concentration of sucrose in a solution, c_w is mass concentration of water in a solution, c_μ is factor of resistance of bubble, f is departure frequency, g is acceleration of free fallings, g_a is mass flow on unit of a surface, H is heat of vapourization, h_r is enthalpy of a solution, h_{max} is the maximal height of mirroughness, n_w is density of nucleation sites, n_{max} is the maximal density of nucleation sites, n_p is concentration bubbles, M is mass, p is pressure, p_a is atmospheric pressure, q is density of a heat flux, R_T is radius of a pipe, T is temperature, u is longitudinal speed, v is crosssection speed, x, r is system of coordinates, α is volumetric fraction; α^* is factor heat-transfer, λ is factor of thermal conductivity, ρ is density, ζ is factor of friction; σ is factor of a superficial tension, τ is flow shear, Subscripts a is surface of bubble; p is vapour, bubble; r is solution; S is parameters of saturation; w is wall; $S = R_T^2$,

$$\begin{aligned} (\alpha_r + \alpha_p = 1), \quad M_p = \frac{4}{3} \pi \rho_p r_p^3, \quad M_p^w = \frac{4}{3} \pi \rho_p^w r_p^{w3}, \quad \tau^w = 1/8 \zeta \rho_r u_r^2, \quad h_r = c_r T_r, \\ f_\mu = 1/2 c_\mu \rho_r (u_r - u_p)^2. \end{aligned}$$

In this system, first equation is an equation of preservation of bubble quantity, the second and the third are equations of conservation of the solution and bubble mass; (4) and (5) are equations of the solution and bubble motion, and the last is an equation of conservation of energy for the solution. The problem is considered with the following simplifications: bubble motion in the solution is assumed as equilibrium, vapour temperature is equal to temperature of saturation, i.e. $T_p = T_s$, and pressure in a bubble is according to the Laplas equation $p_p = p + 2 \sigma / r_p$. According to [10], temperature of the sugar solution boiling is equal to $T_s = T_{so} + K_{pp} + B$, where T_{so} is temperature of the water saturation, K and B are factors which are dependent on sucrose concentration in the solution. Besides, according to the equilibrium model, it is also assumed that the heat flow going to the bubbles is used only for phase change, and then $j_p = 4 \pi r_p^2 g_p^a$, where $g_p^a = \omega_p q_p^a / H$, $q_p^a = \alpha_p^* (T_s - T_r)$.

In multicomponent solutions, heat flow and flows of components to the disperse phase should be correlated. To some extent, coefficient ω_p plays this role in the mass flow, but in terms of simplicity it is assumed that this coefficient is equal to water concentration in the solution when g_p^a is directed to a bubble and to 1 when it is directed from the bubble.

The system of the equations (1-7) includes parameters on a wall of the channel which are determined by kinetic conception of boiling on the rough surfaces [3, 4]. The conception is based on the following ratios which coordinate processes in the two-phase flows and boiling itself on the surface:

$$\alpha_p^w v_p^w = -\omega_w \frac{4}{3} \pi r_{potr}^3 n_w f, \quad (8)$$

$$h = 2 \frac{\sigma \xi_h}{p_p (\exp \gamma - 1)}, \quad (9)$$

where $\gamma = \frac{H \rho_p}{p_p (1 + T_s / (T_r^w - T_s))}$; ξ_h is a geometrical parameter; h is a height of the roughness which participates in bubble formation at the given temperature difference. In accordance with [7], coefficient ω_w is assumed to be equal to mass concentration of water in the solution. Between the roughness and density of the nucleation sites there is a ratio which is established in the works [13, 14]:

$$\bar{n} = \frac{\exp(-3.125 \bar{h})}{b \bar{h}^\nu}, \quad (10)$$

where $\bar{n} = n_w / n_{\max}$ is a relative density of nucleation sites; $\bar{h} = h / h_{\max}$ is a relative height of micro roughness; b and ν are experimental factors.

Various methods are presented in [3, 4] for calculating frequency of the bubble formation and radius of the bubble detachment. Let's use the following dependences:

$$f = 0.56 \sqrt{\frac{g(\rho_r - \rho_p)}{2 \rho_r r_{potr}}}, \quad (11)$$

$$r_{potr} = \frac{[0.09 \sigma / (g(\rho_r - \rho_p))]^{1/2}}{(1 + 2u_r^\kappa)}, \quad (12)$$

$$\text{where } \kappa = \begin{cases} 1.5 & u_r < 0.305 \\ 0.33 & u_r > 0.305 \end{cases}.$$

The wall temperature is determined in the following way. A total heat flow q_T can

be divided into two flows: q_p^w which is used for vaporization and q_r^w which is used for heating the liquid. It is assumed that temperature of a vapour bubble on the wall is equal to temperature of saturation, and temperature of the liquid surrounding a bubble on the wall is the same as temperature of the wall. In this case one more component participates in the vaporization – it is a heat flow formed by surrounding liquid which is equal to:

$$q_p^w = \chi c_r \rho_r \alpha_r^w v_r^w (T_r^w - T_s). \quad (13)$$

As all these heat flows together are used for phase transition it is possible to write down that:

$$q_p^w + \chi c_r \rho_r \alpha_r^w v_r^w (T_r^w - T_s) = H \rho_r \alpha_r^w v_r^w, \quad (14)$$

where χ is a factor showing those share of the bubble surface which is in the liquid (in terms of simplicity, it is equal to 1).

By determining the heat flow q_r^w with the heat-transfer coefficient:

$$q_r^w = \alpha_w^* (T_r^w - T_r), \quad (15)$$

the following formula can be written for the liquid overheat on the wall:

$$T_t^w = \frac{q_T + \alpha_w^* T_r - (H + c_r T_s) \rho_r \alpha_r^w v_r^w}{\alpha_w^* - c_r \rho_r \alpha_r^w v_r^w}. \quad (16)$$

Dependence given in [15] is used for the coefficient of heat-transfer on the wall:

$$\frac{2\alpha_w^* R_T}{\lambda_r} = 2.6 \text{Re}_{rp}^{0.39} \text{Pr}_r^{0.33}. \quad (17)$$

where Pr is Prandtl numer, $\text{Re}_{rp} = 2R_T \left(\frac{\alpha_r \rho_r u_r}{\mu_r} + \frac{\alpha_p \rho_p u_p}{\mu_p} \right)$.

Friction stress was determined by a simple dependence [16]:

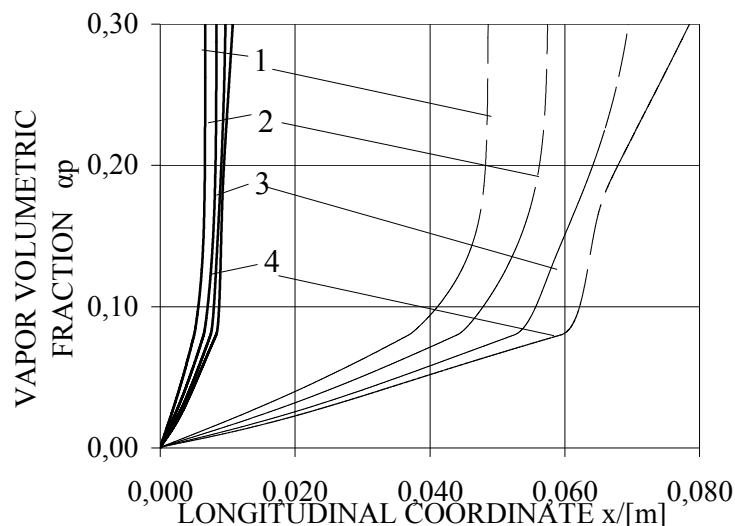
$$\tau_r^w = (1 - \alpha_p)^{-1.42} \tau_0^w, \quad (18)$$

where τ_0^w is friction of solution with no bubbles. All necessary thermophysical parameters of the solution were taken from works [10-12], and parameters for calculating interphase interaction were taken from [8].

2. DISCUSSION OF RESULTS

The above mentioned equations were used in calculations for a cylindrical pipe with a diameter 10 cm and two values of the heat flow ($q_T=5 \cdot 10^3$ and 10^4 W/m²).

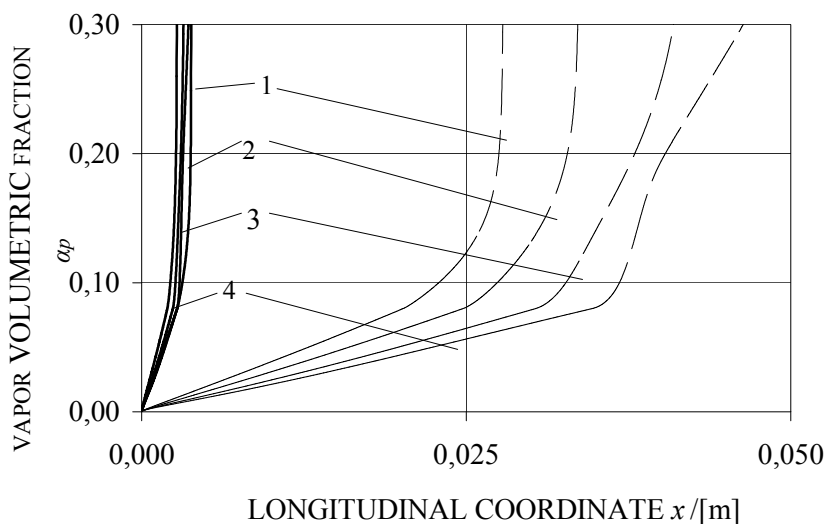
Values of relative pressure over the solution surface were assumed as equal to $p/p_a = 0.6$ and 0.8 , and values of sucrose concentration in the solution were 0.0 ; 0.2 ; 0.4 and 0.6 . Initial temperature of the solution was equal to temperature of the solution boiling, i.e. $T_{ro} = T_s$ in view of depression. Curves of volume fractions of the bubbles occurred on the surface of the pipe during boiling are shown on Fig. 1 and Fig. 2.



1 - $c_s = 0.0$; 2 - 0.2 ; 3 - 0.4 ; 4 - 0.6

Continuous curves - $p/p_a = 0.6$; dotted - $p/p_a = 0.8$

Figure 1 – Change of vapor volumetric fraction in the boiling zone at $q_T = 5\ 000\ \text{W/m}^2$



1 - $c_s = 0.0$; 2 - 0.2 ; 3 - 0.4 ; 4 - 0.6

Figure 2 - Change of vapor volumetric fraction along the boiling zone at $q_T = 10\ 000\ \text{BT/m}^2$ Continuous curves - $p/p_a = 0.6$; dotted - $p/p_a = 0.8$.

Comparison between these Figures shows, as expected, that the greater is a heat flow the more intensive is the boiling, i.e. the faster volume fraction of bubbles increases. One more specificity: when c_s increases the volume fraction of bubbles also increases, however, calculation of relative consumption of water (ratio of water consumption to its initial charge) shows that within the clerical error they are

approximately the same. Calculated values of the bubble speeds show that despite of the greater volume of bubbles at great concentrations of sucrose, speed of the vapor phase is less because of higher level of the solution viscosity.

The Figures above also demonstrate that pressure over the solution surface essentially impact on the vaporization. A group of curves at $p/p_a=0.6$ locates considerably more to the left from the second group where $p/p_a=0.8$: this is an evidence of the fact that vaporization here is more intensive. Concerning these Figures it is necessary to mention the following: the variants were calculated a little bit beyond the existing regime of nucleate boiling. As it is known from [15], starting from $\alpha_p > 0.23$ a flow regime begins to change, and nucleate boiling transfers to the ebullition regime. The Figures show that nucleate boiling exists in rather narrow limits.

For researching a boiling process, an important parameter is overheating of liquid on the wall or, more exactly, a value of $\Delta T = T_r^w - T_s$ which plays an important role for sugar solutions as even slight overheating creates an inadmissible crystal sediment on the channel surface. The Table below presents values of these parameters depending on pressure, heat flows and sucrose concentrations. Two values are presented for each case: value at the beginning of and value in the end of the calculated area. It is obvious that rate of overheating varies to a very little degree along the height of the nucleate boiling area. At the same time, at lower pressure rate of the overheat is a little bit greater than at higher pressure. And with increase of sucrose concentration rate of overheat also increases.

Table 1 – Overheating liquids

c_s	$\Delta T = T_r^w - T_s$			
	$p/p_a=0.6$		$p/p_a=0.8$	
	$q_T=5\ 000$ W/m ²	$q_T=10\ 000$ W/m ²	$q_T=5\ 000$ W/m ²	$q_T=10\ 000$ W/m ²
0.0	0.874-0.881	0.956-0.961	0.659-0.665	0.718-0.721
0.2	1.056-1.061	1.159-1.162	0.803-0.807	0.878-0.880
0.4	1.256-1.259	1.383-1.385	0.962-0.965	1.056-1.058
0.6	1.472-1.472	1.631-1.631	1.132-1.134	1.249-1.251

In the conclusion it should be noted that the proposed model is just an attempt to unite different aspects of the nucleate boiling process. Taking into account complexity and great variety of the process parameters, significant scatter of experimental data and those dependences which were taken from [3, 4], as well as current development of the theory it is possible to speak only about the qualitative laws determined by modeling representations.

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Анотація. Розглянуто задачу про бульбашкове кипіння водного розчину сахарози, що рухається у вертикальному циліндричному каналі. Завдання засноване на одновимірних стаціонарних рівняннях руху та тепломасообміну багатофазного багатокомпонентного середовища, записаних з урахуванням фазового переходу, на поверхні каналу та в об'ємі розчину. В основу кінетики пароутворення на стінці покладені напівемпіричні залежності, що зв'язують частоту утворення бульбашок, кількість центрів пароутворення зі структурою поверхні і з її перегрівом. Отримано, що в не догрітій рідині бульбашки, які утворюються на стінці, потрапляють в холодні шари та схлопуються, підвищуючи температуру розчину. У перегрітій рідині можливе існування режиму теплового балансу, коли все тепло, яке надходить від стінки, йде на утворення та зріст бульбашок в об'ємі рідини. В цьому випадку температура розчину майже не збільшується вгору по потоку. Розроблена математична модель дає можливість отримати основні характеристики процесу кипіння розчину в каналі і оцінити вплив того чи іншого параметра на розвиток процесу.

Ключові слова: розчин, бульбашкове кипіння, критичний радіус, теплообмін.

Аннотация. Рассмотрена задача о пузырьковом кипении водного раствора сахарозы, движущегося в вертикальном цилиндрическом канале. Задача основана на одномерных стационарных уравнениях движения и тепломассообмена многофазной многокомпонентной среды, записанных с учетом фазового перехода, на поверхности канала и в объеме раствора. В основу кинетики парообразования на стенке положены полуэмпирические зависимости, связывающие частоту образования пузырьков, количество центров парообразования со структурой поверхности и с ее перегревом. Получено, что в недогретой жидкости, образующиеся на стенке пузыри попадают в холодные слои и схлопываются, повышая температуру раствора. В перегретой жидкости возможно существование режима теплового баланса, когда все тепло, поступающее от стенки, идет на образование и рост пузырьков в объеме жидкости. В этом случае температура раствора практически не увеличивается вверх по потоку. Разработанная математическая модель дает возможность получить основные характеристики процесса кипения раствора в канале и оценить влияние того или иного параметра на развитие процесса.

Ключевые слова: раствор, пузырьковое кипение, критический радиус, теплообмен.

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