

SUCCESSIVE APPROXIMATION METHOD APPLYING TO OBTAIN THE TRANSIENT ELASTIC MEDIUM DEFORMATIONS

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ВИКОРИСТАННЯ МЕТОДУ ПОСЛІДОВНОЇ АПРОКСИМАЦІЇ ДЛЯ РОЗРАХУНКУ ДЕФОРМАЦІЙ У ПРУЖНОМУ СЕРЕДОВИЩІ

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ИСПОЛЬЗОВАНИЕ МЕТОДА ПОСЛЕДОВАТЕЛЬНОЙ АППРОКСИМАЦИИ ДЛЯ РАСЧЕТА ДЕФОРМАЦИЙ В УПРУГОЙ СРЕДЕ

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Annotation. In the first part of the article, the well-known sequential approximation method developed at the Institute of Geotechnical Mechanics named by N. Poliakov of National Academy of Sciences of Ukraine was used. This method was used to solve the problem of unsteady deformation of an elastic medium under the influence of variable internal pressure in wells. The essence of the method is that the resultant function, according to the results of its numerical research and given in tabular form, can be presented in an analytical form – a product of functions. This method has shown its high efficiency in solving various problems. The objective function, the maximum normal tensile stresses, is represented as the product of power functions, each of which depends on only one parameter. The pattern of changes in maximum tensile stresses from the main mining-technical factors affecting this process is established. Such factors are: the value of the internal pressure of the working agent before discharge, the inner diameter of the well, the speed of the elastic wave in the agent, the decay time of the working agent from the well. The decay time of the working agent is determined by the speed of the elastic wave in the working agent. Thus, it was found that the magnitude of tensile stresses is directly proportional to the pressure of the working agent and the radius of the well and inversely proportional to the speed of the elastic wave in the coal rock mass and the time of pressure relief. Thus, a complex dynamic problem is solved in a relatively simple way. In the second part of the article, a dimension theory method was used to solve this problem. According to the theory, the unknown value of radial stresses on the inner radius of the cylindrical cavity of a well can be represented in the form of the product of power functions. This fully confirms the dependence of the stress change established above. Solving a difficult problem by the method of successive approximation, we can obtain the dependence for any number of initial parameters in the mathematical model as well as the degree of influence of each parameter on the process under study.

Keywords: sequential approximation, stresses, dimension.

Introduction. In practice, the class of power functions [1] has become widespread for representing functional dependencies. The main advantages of such a presentation are the ability to assess the degree of influence of parameters on the function itself and determine the compliance of the physical content of the resulting dependencies with the properties of the original process function. In connection with the noted features, we will look for the representation of the original function as a product of power functions. Using the method of successive approximation [1, 2], all numerical solutions were analyzed on the basis of the integral Fur'ye conversion, as well as solutions for the ratio of the external diameter of a hollow cylinder b to internal a equal to $10 > b/a > 1.1$, using model A .N. Krylov, for which the author of the work has developed the conditions for its use for the relation $\infty \geq b/a > 1.1$ [3].

For the case of plane deformation, the calculation of radial σ_r and tangential σ_θ stresses can be carried out using the formulas given in [4].

The main parameters influencing the process of transient deformation of an elastic medium include the inner radius of the well r_0 , the internal pressure p_0 of the working agent, the elastic wave velocity in the massif v_p , and the internal pressure in the working agent release time t_c .

All further calculations and transformations are reduced to the maximum value of tensile radial stresses in the first half-wave of stress variation at the site of inertial forces (Figure 1) [5].

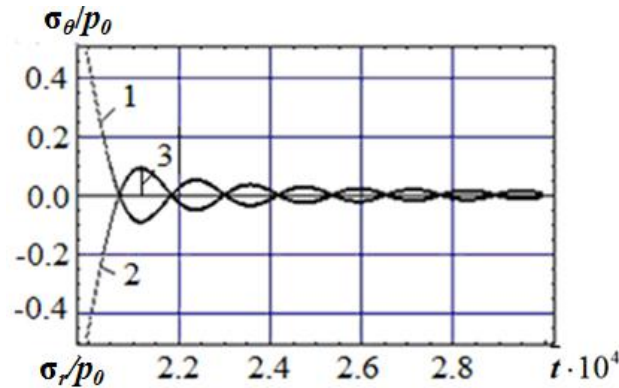


Figure 1 – Change of radial and tangential stresses in time in dimensionless form at the section of inertial force action when the internal pressure is released from the well: 1 – tangential stress; 2 – radial stress; 3 – maximum tensile stresses in the first half-wave

Problem definition and solution method. Let us present the solution of this problem in the form of simple engineering formulas convenient for calculating stresses or displacements of an elastic medium in the vicinity of wells during short-term unloading using the method of successive approximation of a variable [1, 2].

The essence of the method is that the resultant function, according to the results of its numerical research and given in tabular form, can be presented in an analytical form – a product of functions, each of which depends only on one variable.

Let in general, the form of the solution is represented as a function $F(x) = F(x_1, x_2, \dots, x_n)$ defined and continuous in a closed area D . In a neighborhood of a point $X_0 = (x_1^0, x_2^0, \dots, x_n^0) \in D$ function $F(x)$ can be represented as a dependence given in [1].

$$F(X) \approx \varphi(x_1, \dots, x_n) = c \cdot \prod_{i=1}^n g_i(x_i), \quad (1)$$

where $g_i(x_i)$ – approximation functions for f_1, f_2, \dots, f_n specified in tabular form; c – proportionality coefficient.

Let's select reference point $M = M(x_1^0, x_2^0, \dots, x_n^0)$, $M \in D$, where $x_1^0, x_2^0, \dots, x_n^0$ – average values of the studied parameters in a given range of their changes.

These functions $g_i(x_i)$ are formed as follows: we fix the average values of the studied parameters and vary only one of them in the range of its change, then establish the type of function and the exponent of the degree of its influence α_i :

$$f_1(x_1) = F(x_1, x_2^0, \dots, x_n^0); f_2(x_2) = F(x_1^0, x_2, \dots, x_n^0); f_n(x_n) = F(x_1^0, x_2^0, \dots, x_n).$$

Repeating this approximation procedure successively for each parameter studied (x_1, x_2, \dots, x_n) , we obtain the desired representation in the form:

$$F(x_1, x_2, \dots, x_n) \approx \varphi(x_1, x_2, \dots, x_n) = c_n \cdot g_1(x_1) \cdot g_2(x_2), \dots, \cdot g_n(x_n). \quad (2)$$

The total proportionality coefficient for all parameters is determined by the formula:

$$c_n = F(x_1^0, x_2^0, \dots, x_n^0) / g_1(x_1^0)^{\alpha_1} \cdot g_2(x_2^0)^{\alpha_2} \cdot \dots \cdot g_n(x_n^0)^{\alpha_n}. \quad (3)$$

Results and discussion. As a function, which we are looking for, we take the value of the maximum tensile stresses in the first half-wave of its change, referred to the maximum amplitude of the internal pressure p_0 before the relief.

$$\sigma_r / p_0 = \sigma(t_c^{\alpha_1} \cdot v_p^{\alpha_2} \cdot r_0^{\alpha_3}), \quad (4)$$

where $\alpha_1, \alpha_2, \alpha_3$ – exponents of the functions of the parameters studied.

The domain of definition is defined by the intervals of change of all the main parameters: the interval of change of the inner radius of the cylindrical cavity $r_0 = (0.05 - 0.5)$ m; elastic wave velocity $v_p = (500 - 2500)$ m/s; time of pressure relief $t_c = (0.001 - 0.5)$ s.

Reference point $M = M(x_1^0, x_2^0, \dots, x_n^0)$, $M \in D$, is set by average values of the parameters: pressure release time $t_c^0 = 0.01$ s elastic wave velocity $v_p^0 = 600$ m/s, borehole radius $r_0^0 = 0.1$ m.

Changing a parameter t_c with a certain step, we calculate the sequence of function σ_r / p_0 values. For the sequence $(t_c, \sigma_r / p_0)$ we will find approximation function $g_1(t_c)$ in the form: $g_1(t_c) = c_1 t_c^{\alpha_1}$, where: $c_1 = 0.0002018$; $\alpha_1 = -0.96$.

Graphical representation of the implementation of these stages is shown in Fig. 2.

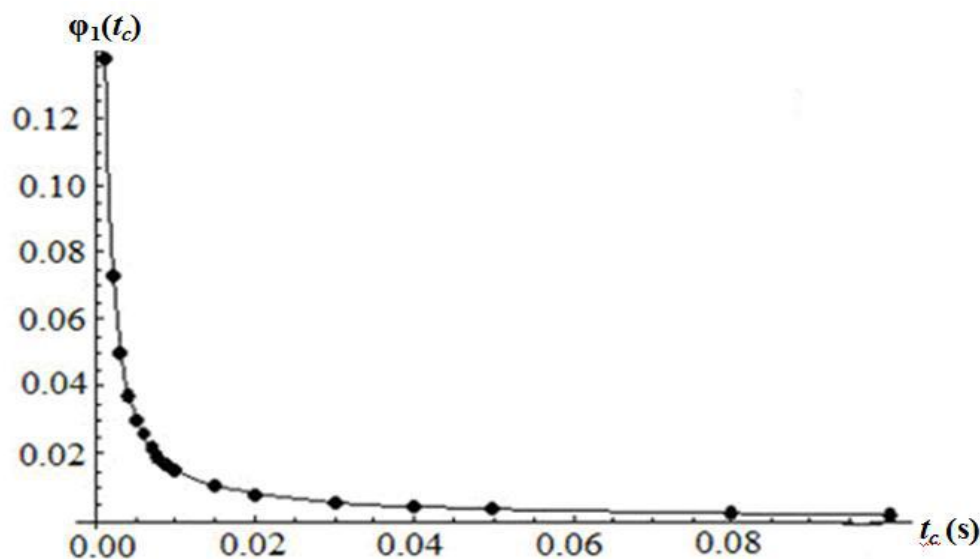


Figure 2 – Function $\varphi_1(t_c)$ in tabular form and its approximation function

Thus, the dimensionless tensile stresses load release time t_c will have the form:

$$\sigma_r/p_0(t_c, v_p^0, r_0^0) \approx \varphi_1(t_c) = c_1 \cdot t_c^{\alpha_1}. \quad (5)$$

Formula (5) can already be used in calculations to determine tensile stresses depending on the different release times (for fixed values of the parameters at the reference points v_p^0 and r_0^0).

Changing the next parameter v_p , with a certain step, we calculate the sequence of function values σ_r/p_0 . For sequence $(v_p, \sigma_r/p_0)$ we will find approximation function $g_2(v_p)$ in the form:

$$g_2(v_p) = c_2 \cdot v_p^{\alpha_2},$$

where $c_2 = 0.09132$; $\alpha_2 = -0.957$.

Graphical representation of the implementation of these stages is presented in Fig. 3.

Thus, the dimensionless tensile stresses from the time of pressure drop t_c , and the elastic wave velocity v_p will have the form

$$\sigma_r/p_0(t_c^0, v_p, r_0^0) \approx \varphi_2(t_c, v_p) = c_2 \cdot t_c^{\alpha_1} \cdot v_p^{\alpha_2}. \quad (6)$$

Formula (6) can be used for particular calculations of stresses with a fixed value of the inner radius r_0^0 and time t_c^0 .

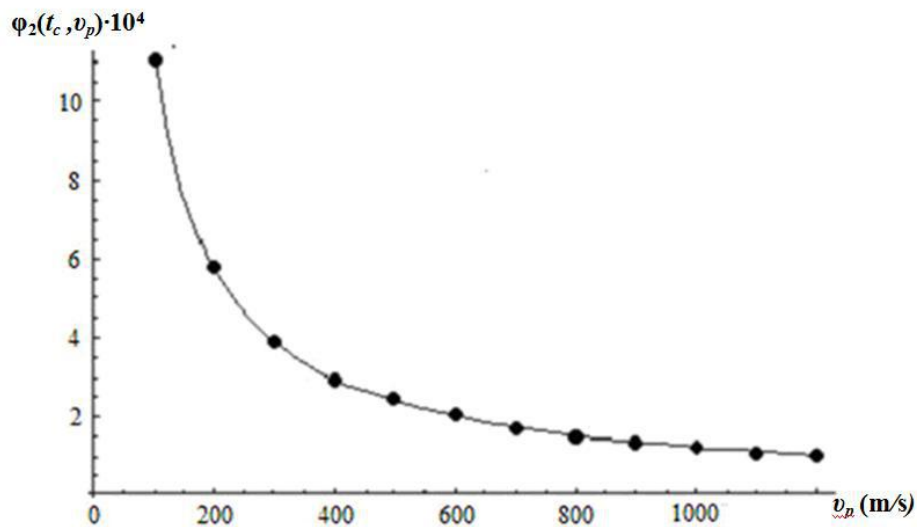


Figure 3 – Function $g_2(v_p)$ in tabular form and its approximation function

Doing so and changing the parameter r_0 with a certain step, we calculate the sequence of values of the function σ_r/p_0 . For sequence $(r_0, \sigma_r/p_0)$ we will find approximation function $g_3(r_0)$ in the form:

$$g_3(r_0) = c_3 r_0^{\alpha_3},$$

where $c_3 = 0.9172$; $\alpha_3 = 1.01$.

Graphical representation of the implementation of these stages is presented in Fig. 4.

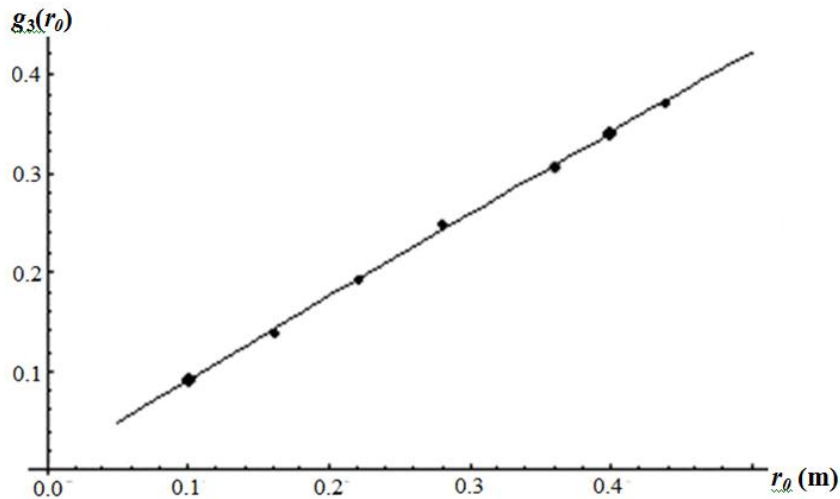


Figure 4 – Function $g_3(r_0)$ in tabular form and its approximation function

Thus, the dimensionless tensile stresses from the load release time t_c and the velocity of the elastic wave v_p and the inner radius r_0 of the well will have the form:

$$\sigma_r/p_0(t_c^0, v_p^0, r_0) \approx \varphi_3(t_c, v_p, r_0) = c_3 \cdot t_c^{\alpha_1} \cdot v_p^{\alpha_2} \cdot r_0^{\alpha_3}. \quad (7)$$

The final formula (8) for tensile radial stresses from all studied parameters is written in the form:

$$\sigma_r = 0.96 \cdot (p_0^1 \cdot r_0^{1.01} / v_p^{0.957} \cdot t_c^{0.96}). \quad (8)$$

Taking into account the proximity of all exponents of the studied parameters to a unit in expression (8), we obtain a more convenient linearized formula and regularity for calculating tensile stresses on the internal contour of the cylindrical cavity in the first half-wave of the variation of the stresses in the form:

$$\sigma_r = p_0 \cdot r_0 / v_p \cdot t_c. \quad (9)$$

Comparison of the relative error of stresses calculated by formula (9) with the exact one, obtained by the method of the integral Fur'ye conversion and the method based on the solution of A.N. Krylov [3], showed that the average relative error for the release time does not exceed 5.7 %, for the elastic wave velocity – 5.2 %, and for the inner radius – 10 %.

The correctness of the physical content of the obtained solution (9) is easy to confirm on the basis of the theory of dimensions. According to the theory of dimensions, the unknown value of radial stresses on the inner radius of the cylindrical cavity of a well can be represented in the form of the product of power functions [6]:

$$\sigma_r = b_1 \cdot p_0^p \cdot r_0^q \cdot v_p^r \cdot t_c^k, \quad (10)$$

where p, q, r, k – unknown exponents; b_1 – dimensionless proportionality coefficient.

Equating the dimensions on the left and right side of equation (10) we obtain:

$$M \cdot T^{-2} \cdot L^{-1} = M^p \cdot T^{2p} \cdot L^p \cdot L^q \cdot L^r \cdot T^r \cdot T^k. \quad (11)$$

We construct the following equations to determine the exponents:

$$p = 1, \quad (12)$$

$$-2 = -2p - r + k, \quad (13)$$

$$-1 = -p + q + r. \quad (14)$$

We have three equations (12, 13, 14) and four unknown exponents to be determined. From equation (12) we immediately obtain the exponent at p_0 equal to unit $p = 1$. From equation (13) it follows that the exponent at v_p and t_c equal to $r = k$. The exponent of the influence of the inner radius r_0 is taken equal to the unit $q = 1$ of the calculations given in [5].

Consequently, the remaining exponents from equation (14) with v_p and t_c will be equal to $r = k = -1$.

Thus, the general structure of formula (9) is fully confirmed by the independent theory of dimensionality, has the nature of regularities for linearly elastic media and is valid in a wide range of variation of the investigated variable parameters.

However, it should be noted that the theory of dimension does not always make it possible to find the proportionality coefficient b_1 , connecting all variables. When using the method of successive approximation, it is found in a relatively simple way with any number of parameters.

The verification of the errors of approximate representations of functions specified in an analytical form using the method of successive approximation confirmed high accuracy for engineering calculations in the vicinity of the selected point $M(x_1^m, x_2^m, \dots, x_n^m)$ and good one throughout the entire domain.

Conclusions. The inertial tensile stresses that occur in the near-wellbore zone of the coal and rock massif after the working agent pressure release from wells, are directly proportional to its pressure before release and the well radius and inversely proportional to the velocity of the elastic wave in the massif and the time of pressure drop in the working agent. The time of pressure drop in the working agent is calculated in each case separately for a specific type of working agent and parameters of technological wells.

A rational value of the well radius for initiating the destruction of the carbon-forming medium is determined from the technological point of view and from the point of view of the fracture efficiency.

The speed of the elastic wave in the investigated disturbed coal-rock massif is determined by taking into account the degree of decrease in the elastic modulus during gas saturation.

The generality and reliability of the obtained regularity is confirmed by use of fundamental provisions of continuum mechanics, known classical solutions by Fur'ye and Krylov, mine research by Institute of Geotechnical Mechanics named by N. Poliakov of National Academy of Sciences of Ukraine under hydrodynamic effects on gas-saturated coal beds. For the first time, a regularity has been established that relates the loading parameters, the elastic and deformation properties of the carbon-bearing medium, and the geometric parameters of the well.

For the first time, a regular approximation method is established for tensile stresses in the near-wellbore zone of a coal-bearing massif from the influence of inertial forces under the action of alternating internal pressure, which relates the pressure of the working agent in the well, the phase state and the time of pressure drop in the working agent, elastic and deformation parameters of the medium and geometric characteristics of well and developed on its basis a calculation method that allows you to determine the necessary values of loading parameters for coal media destruction initialization.

Application of the method of successive approximation to transform a complex solution of a dynamic problem allowed us to obtain a simple formula connecting the elastic characteristics of the massif, the geometric parameters of the elastic medium and the loading parameters. Using the solution obtained, it is easy to play different situations when internal pressure is released from wells, varying parameters in the range of their changes to obtain the desired value of tensile stresses. The application of the method of successive approximation to the analysis of the solution of the problem of transient deformation of an elastic medium under the action of a variable internal load made it possible for the first time to establish the pattern of change in inertial maximum radial tensile stresses.

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Анотація. У першій частині статті був використаний відомий метод послідовної апроксимації, розроблений в Інституті геотехнічної механіки ім. М.С. Полякова НАН України. Цей метод був використаний для вирішення задачі нестационарної деформації пружного середовища під впливом змінного внутрішнього тиску в свердловинах. Сутність методу полягає в тому, що результуюча функція в табличній формі, згідно з результатами її чисельного дослідження, може бути представлена в аналітичному вигляді добутку функцій. Цільова функція, максимальні нормальні розтягуючі напруження, представляються у вигляді ступеневих функцій, кожна з яких залежить тільки від одного параметра. Встановлено закономірність зміни максимальних розтягуючих напружень від основних гірничотехнічних факторів, що впливають на цей процес. Такими факторами є: значення внутрішнього тиску робочого агента перед скиданням, внутрішній діаметр свердловини, швидкість пружної хвилі в агенті, час згасання тиску робочого агента. Час згасання тиску робочого агента визначається швидкістю пружної хвилі в робочому агенті. Було встановлено, що величина розтягуючих напружень прямо пропорційна тиску робочого агента і радіусу свердловини і обернено пропорційна швидкості пружної хвилі в масиві і часу скидання тиску. Таким чином, складна динамічна задача вирішується відносно простим способом. У другій частині статті для вирішення цієї проблеми використовувався метод теорії розмірностей. Відповідно до неї, невідоме значення радіальних напружень на внутрішньому радіусі циліндричної порожнини свердловини може бути представлено у вигляді добутку ступеневих функцій. Це повністю підтверджує залежність зміни напружень, встановлену вище. Вирішуючи складну задачу методом послідовної апроксимації, ми можемо отримати залежність для будь-якого числа вихідних параметрів в математичній моделі, а також ступінь впливу кожного параметра на досліджуваний процес.

Ключові слова: послідовна апроксимація, розтягуючі напруження, розмірності.

Аннотация. В первой части статьи был использован известный метод последовательной аппроксимации, разработанный в Институте геотехнической механики им. Н.С. Полякова НАН Украины. Этот метод был использован для решения задачи нестационарной деформации упругой среды под воздействием переменного внутреннего давления в скважинах. Суть метода состоит в том, что результирующая функция в табличной форме, согласно результатам ее численного исследования, может быть представлена в аналитической форме в виде произведения функций. Целевая функция, максимальные нормальные растягивающие напряжения, представляется как произведение степенных функций, каждая из которых зависит только от одного параметра. Установлена закономерность изменения максимальных растягивающих напряжений от основных горнотехнических факторов, влияющих на этот процесс. Такими факторами являются: значение внутреннего давления рабочего агента перед сбросом, внутренний диаметр скважины, скорость упругой волны в агенте, время затухания давления рабочего агента. Время затухания рабочего агента определяется скоростью упругой волны в рабочем агенте. Было установлено, что величина растягивающих напряжений прямо пропорциональна давлению рабочего агента и радиусу скважины и обратно пропорциональна скорости упругой волны в массиве и времени сброса давления. Таким образом, сложная динамическая задача решается относительно простым способом. Во второй части статьи для решения этой проблемы использовался метод теории размерностей. Согласно нему, неизвестное значение радиальных напряжений на внутреннем радиусе цилиндрической полости скважины может быть представлено в виде произведения степенных функций. Это полностью подтверждает зависимость изменения напряжения, установленную выше. Решая сложную задачу методом последовательной аппроксимации, можно получить зависимость для любого числа исходных параметров в математической модели, а также степень влияния каждого параметра на изучаемый процесс.

Ключевые слова: последовательная аппроксимация, растягивающие напряжения, размерности.

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