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# CALCULATION OF THE MAXIMUM VELOCITY OF GRAVITY FLOW IN THE POND-CLARIFIER WITH HIGHER AQUATIC PLANTS

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**Abstract.** The analysis of the possible maximum fluid flow rates when using higher aquatic plants for clarification of recycled water in the pond-clarifier of the tailing pond has carried out. The study has been performed on the basis of a mathematical model of a plane slow stationary gravity flow of a viscous fluid in two parallel layers. The results of the study made it possible to determine the fluid velocity through a layer of higher aquatic plants floating on a free surface. The maximum possible velocity depending on the layer porosity has been determined. This value is necessary to determine the rational parameters of the process of clarifying technical recycled water from particles of the given hydraulic size, taking into account the pond-clarifier geometric dimensions. It is shown that the velocity in the layer with higher aquatic plants has been determined by the ratio of two parameters of this layer - porosity and dimensionless resistance coefficient. It has been shown that the maximum velocity value coefficient in the layer with plants floating on free surface depends only on porosity of this layer and does not depend on its resistance coefficient.

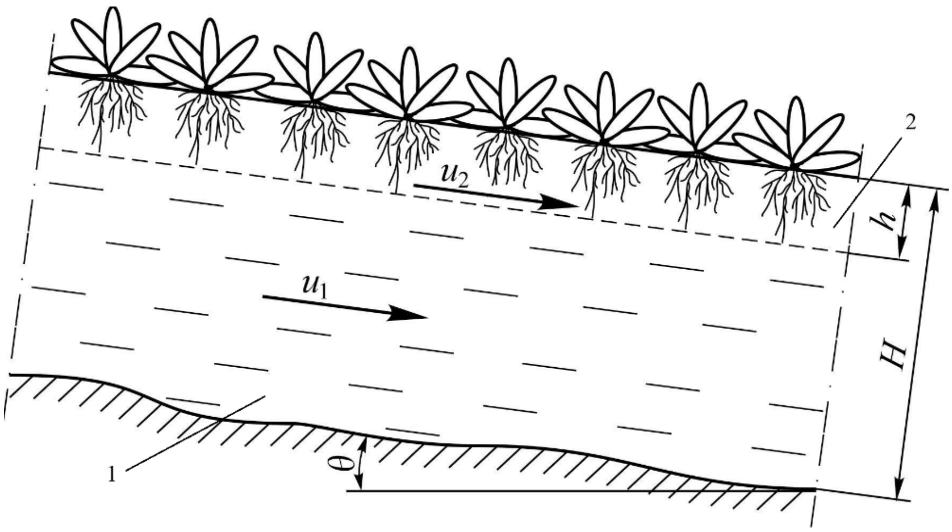
**Introduction.** The Kryvbas iron ore plants provide approximately $ 3 billion of revenue to the budget of Ukraine due to the export of ore and provide work to 60 thousand miners [1 - 4]. The stability and profitability of functioning of such city-forming and socially significant enterprises ensure the sustainable development of industrial regions, acting as one of the main components of the country's export potential [4]. One of the main factors that can lead to the shutdown of such enterprises is the failure to provide the required purification level by the existing recycled water clarification systems [5 - 7]. Operating from the middle of the last century, the iron ore tailing pond system suggests using a protective layer of water in the pond-clarifier to prevent dusting of beaches. The effectiveness of such clarification process is ensured by the large geometric dimensions of the pond-clarifier and low velocity of the water flow in it. However, it should be noted that during the operation of the dykes, the waste storage facilities of the Kryvbas iron ore mills have been increased many times over, and today they have reached such heights that the geometric dimensions of the pond-clarifiers no longer provide effective clarification of the circulating water [1]. In the circumstances, practically the only way to restore the effective operation of existing pond-clarifiers, without changing the storage system for enrichment waste, is to use higher aquatic plants floating on the free surface of water bodies. The roots of the plants are a dense natural network that can trap and bind dusty, clay and chalk particles fractions, preventing them from falling into the process. Similar purposes experience is known, where the root system of aqueous hyacinths is up to 0.5 m long. It is located in the surface layer and is completely immersed in water [8]. Water hyacinth (Fig. 1) or Eichhornia (lat. Eichhornia crassipes) is a tropical fast-growing plant, capable of doubling green mass in 2 weeks and rising one meter above the surface of the water.

Figure 1 – Water hyacinth or eichornia (lat. Eichhornia crassipes)

For the successful application of this technology, it is necessary to select and explain the parameters of the surface layer with plants, to calculate the corresponding water flow rates, and to justify the parameters of the hyacinths planting, providing the most effective clarification of the water. At the same time, it is important for developers to know the maximum velocity, where water can flow through a layer with floating plants. It will allow to determine performance of the clarification system of the recycled water, method of plant attachment, as well as efficiency of solid particles deposition by the plant roots trapping. Thus, the selection of the density of filling the layer with plants has been carried out taking into account the maximum possible flow rate in the layer, which determines high efficiency of the circulating water clarification.

There are two types of pressureless flows with plants in the stream, which are used to capture particulate matter and impurities and to clean the recycled water of mining mills using biotechnology (Fig. 2, 3) [8 - 12]. For the flow in the channel, experimental methods for calculating the parameters are considered in the case when hyacinths are planted at the bottom of the channel [9]. On the basis of field and industrial experiments with the so-called biological sites, empirical dependencies have been proposed that determine the amount of precipitation accumulated per unit area depending on the content of suspended solids in the feed water. The choice of hydraulic process parameters has not been justified. For the fluid flow in the clarifying pond, the idea of using plants has been stopped at the discussion stage without developing methods for calculating hydraulic parameters [10]. Therefore, it should be noted that none of the known calculation methods is assumed to determine rational parameters ensuring high efficiency of the clarification process. The purpose of this article is to establish the dependence of the maximum possible flow rate (in a layer with plants floating on the free surface of a pressureless fluid flow) on the layer porosity.

**Methods.** For the flows under consideration, we assume that the entire flow vertically is divided into two layers (Fig. 2, 3) [9, 11, 12]: a free pressureless fluid flow (Area 1) and a fluid flow in a porous layer formed by the roots of floating on the liquid surface higher aquatic plants (Area 2). We assume that the boundary between these layers goes along the lower marking of the plant root systems, the length of which is considered the same on the first approximation. Plants floating on the free surface occupy part of the liquid volume in Area 2. This is taken into account by introducing into the corresponding equation the dimensionless value of the liquid layer with plants porosity. Such value expresses unoccupied by plants root systems liquid layer volume fraction. It is proposed to take into account influence of the plants on the hydraulic characteristics of the flow by introducing dimensional value of the resistance coefficient into the corresponding equation. Outside the fluid layer with floating plants, that is, in Area 1, the porosity and resistance coefficient values are unit and zero, respectively.




1 – non-pressure fluid flow, 2 – fluid flow in a porous layer; $u_1$, $u_2$ – velocity in layers, m/s; $h$ – layer thickness with hyacinths, m; $H$ – layer height from the bottom of the channel to the layer with hyacinths, m; θ – channel bottom slope to the horizon, degrees

Figure 2 – Scheme of a non-pressure flow in a fluid channel with plants floating on a free surface [12]

This method of taking into account the influence of the root systems of plants floating on a free surface, on the flow field is based on the analogy of the considered flow with the processes of liquid filtration through a porous medium [13], as well as on the widely known principles of modeling two-phase media [14]. Let us assume that at the boundary between the Areas the conditions are of equal pressure, velocity and their first derivatives with respect to the longitudinal coordinate are valid, and on the free surface the pressure and velocity of the liquid are set.

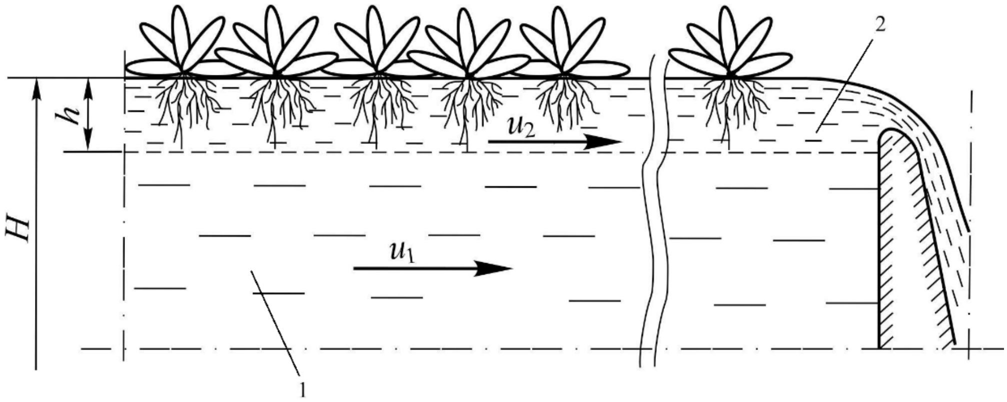



Figure 3 – Scheme of non-pressure fluid flow with plants floating on a free surface in clarifying-pond of the tailing pond [12]: 1 – non-pressure fluid flow, 2 – fluid flow in a porous layer

Taking into account the above features and assumptions, on the basis of the Navier-Stokes equations and the law of mass conservation, the following dependences for calculating the flow parameters in the channel have been obtained [11 - 13]:

$$p_2 = \rho gh(1-\eta_2)\cos\theta + p_A, \quad p_1 = \rho gH(1-\eta_1)\cos\theta + p_A + \rho gh, \tag{1}$$

$$u_1 = \frac{\Lambda_1}{\Lambda}\left[1+\frac{\lambda}{2}+\frac{\lambda}{2\Lambda_1}\eta_1\right]\eta_1\frac{Q}{H}, u_2 = \frac{\Lambda_2}{\Lambda}\left[\left(\frac{\lambda}{2}-1\right)\left(e^{\lambda\eta_2}+e^{2\lambda}e^{-2\lambda\eta_2}\right)-\frac{\varepsilon}{\Lambda_2}\right]\frac{Q}{H}, \tag{2}$$

$$\Lambda = \frac{\Lambda_1}{2}\left[1+\left(1+\frac{2}{3\Lambda_1}\right)\frac{\lambda}{2}\right]+\frac{h}{H}\Lambda_2\left[\frac{e^{2\lambda}+2e^{\lambda}-3}{\lambda}\left(\frac{\lambda}{2}-1\right)-\frac{\varepsilon}{\Lambda_2}\right],$$

$$\lambda = \frac{kH^2}{\nu},$$

$$\Lambda_1 = \left[2\left(1+e^{2\lambda}\right)-\left(1-e^{2\lambda}\right)\frac{h\lambda}{H}\right]\Lambda_2,$$

$$\Lambda_2 = \frac{1}{\left(1-e^{2\lambda}\right)\frac{h\lambda}{H}-\left(1+e^{2\lambda}\right)}, \ \eta_1 = \frac{y}{H}, \ \eta_2 = \frac{y-H}{h},$$

where $p_1$, $p_2$ – fluid pressure in layers, Pa; $p_A$ – atmospheric pressure, Pa; $\rho$ –density, kg/m $^3$; $u_1$, $u_2$ – velocities in layers, m/s; $g$ – acceleration of a freely falling body, m/s $^2$; $k$ – coefficient of resistance of the layer with hyacinths, 1/s; $\theta$ – channel bottom slope to the horizon, degrees; $\varepsilon$ – is the porosity of the layer with hyacinths; $h$ – is the thickness of the layer with hyacinths, m; $Q$ – is the volumetric flow rate of the liquid through the channel of unit width, $m^2/s$; $H$ – layer height from the bottom of the channel to the layer with hyacinths, m; $\nu$ – kinematic coefficient of viscosity, $m^2/s$; $\Lambda$, $\Lambda_1$, $\Lambda_2$ – are dimensionless complexes; $y$ – current coordinate along the height of the layer, m; $\eta_1$, $\eta_2$ – dimensionless current coordinates; $\lambda$ – the dimensionless coefficient of the layer resistance.

Formulas (1) and (2) allow us to study the distribution of velocities along the height of the flow layers, as well as to determine the height, where the velocity in the layer is a maximum. However, these dependences do not allow us to characterize the flow velocity through the layer as a whole, for which it is necessary to determine the average velocities in each of the flow layers [11, 12]:

$$U_1 = \frac{1}{H}\int_0^1 u_1 d\eta_1 \, , \; U_2 = \frac{1}{h}\int_0^1 u_2 d\eta_2 \, ;$$

$$U_1 = \frac{\Lambda_1}{2\Lambda}\left[1 + \left(1 + \frac{2}{3\Lambda_1}\right)\frac{\lambda}{2}\right]\frac{Q}{H^2}, \quad U_2 = \frac{\Lambda_2}{\lambda\Lambda}\frac{1}{\frac{h}{\lambda H}}\left[\frac{e^{2\lambda} + 2e^{\lambda} - 3}{\lambda}\left(\frac{\lambda}{2} - 1\right) - \frac{\varepsilon}{\Lambda_2}\right]\frac{Q}{H^2}. \quad (3)$$

where $U_1$, $U_2$ – average velocities in the flow layers, m/s.

The developed formulas for calculating the fluid velocity and pressure in these two Areas in the case of flow along a rectangular channel of finite depth make it possible to obtain a solution for the case of flow in clarifying tailing pond (Fig. 3). Based on the obtained solutions, by passing to the limit with a layer ratio tending to zero $h/H \to 0$, the dependences for the average liquid flow rates in each Area have been determined, while maintaining the necessary for averaging layer thicknesses values [11, 12]:

$$U_1 = \frac{Q}{H^2}, \qquad U_2 = \frac{H}{h}\frac{\varepsilon + \frac{e^{2\lambda} + 2e^{\lambda} - 3}{\lambda\left(1 + e^{2\lambda}\right)}\left(\frac{\lambda}{2} - 1\right)}{1 + \frac{\lambda}{3}}\frac{Q}{H^2}. \qquad (4)$$

**Results and discussion.** It can be seen from formulas (4) that in the limiting case, the parameters of plants floating on the free surface of the pond-clarifier do not affect the average velocity in the lower layer. At the same time, the average velocity in the layer with floating plants directly proportionally depends on the porosity of the layer and nonlinearly on the resistance coefficient. To calculate the flow parameters in the pond-clarifier of the tailing pond, the formula for the average velocity in the layer with plants can be represented as follows:

$$U_2 = \frac{1}{\eta} \frac{\varepsilon + \frac{1}{E}\left(\frac{\lambda}{2} - 1\right)}{\left(1 + \frac{\lambda}{3}\right)} \frac{Q}{H^2}, \quad \eta = \frac{h}{H}, \quad E = \frac{\lambda\left(1 + e^{2\lambda}\right)}{e^{2\lambda} + 2e^{\lambda} - 3}. \tag{5}$$

where $E$ – dimensionless complex; $\eta$ – relative thickness of the layer with plants.

The results of a numerical study of the parameter $E$ on units $\lambda$ dependence (Fig. 4) indicate that this dependence, for $\lambda \geq 2$, can be approximated with high accuracy by the following dependence ($R^2$ = 0.9999):

$$E = \lambda. \tag{6}$$

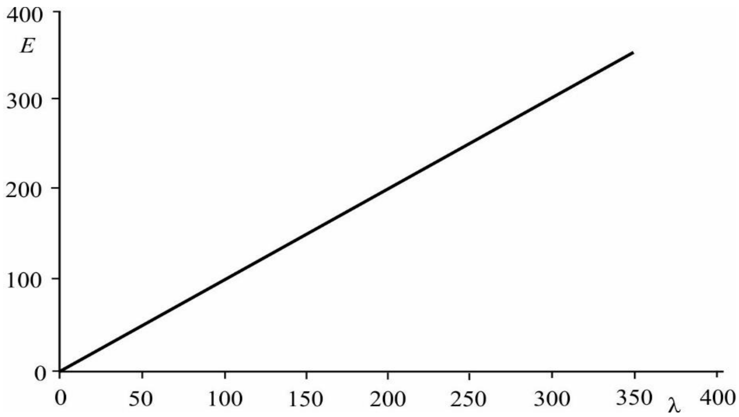



Figure 4 – Parameter $E$ dependence on the size of the dimensionless coefficient of the layer resistance

Using the approximation dependence (6), the formula for the average consumption rate in the layer with plants can be expressed in the following approximation:

$$U_2 = \frac{\mu}{\eta} \frac{Q}{H^2}, \qquad \mu = \frac{3}{2} \frac{(2\varepsilon + 1)\lambda - 2}{(3 + \lambda)\lambda},$$

where $\mu$ – velocity coefficient in the layer with plants floating on a free surface.

The first derivative of $\mu$ with respect to $\lambda$ vanishes in the roots of the following quadratic equation:

$$\lambda^2 - \frac{4}{2\varepsilon + 1}\lambda - \frac{6}{2\varepsilon + 1} = 0, \tag{7}$$

the positive root (Fig. 5) corresponds to the maximum of $\mu$, and the negative root corresponds to the minimum of the value:

$$\lambda_* = \frac{1 + \sqrt{3\varepsilon + 2.5}}{\varepsilon + 0.5}, \quad \lambda_{**} = \frac{1 - \sqrt{3\varepsilon + 2.5}}{\varepsilon + 0.5},$$

where $\lambda_{**}$, $\lambda_*$ – roots of the equation (7).

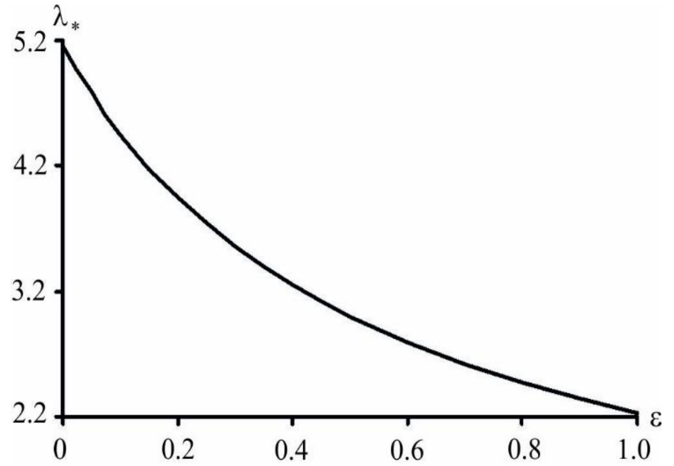



Figure 5 – The dependence of $\lambda_*$ on the porosity of the layer from the plant on the surface

When the value of $\lambda_*$ is substituted in the formula for the velocity coefficient in the layer with plants floating on the free surface, we obtain the following dependence for calculating the maximum value of μ (fig. 6):

$$\mu_* = 3\left(\frac{\varepsilon + 0.5}{1 + \sqrt{3\varepsilon + 2.5}}\right)^2 . \tag{8}$$

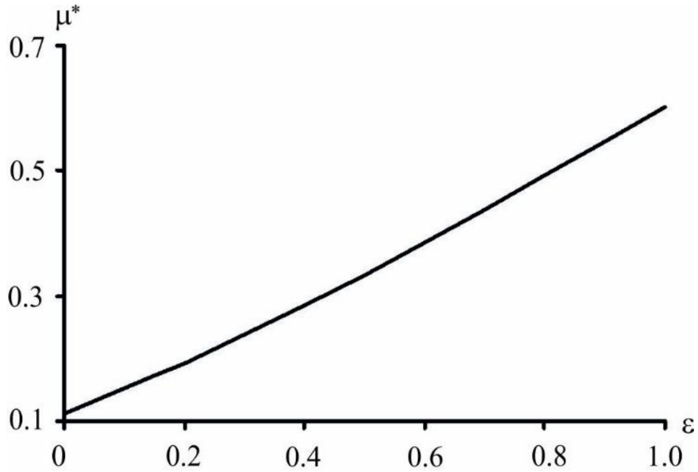



Figure 6 – The dependence of the maximum value of the velocity coefficient in the layer with plants floating on the free surface on the layer porosity

Figures 5 and 6 show that both dependences are nonlinear, but not extreme, and the maximum values of $\lambda_*$ and μ do not exceed 5.2 and 0.6, respectively. The maximum value of $\lambda_*$ is achieved with a minimum porosity of the layer with the roots of plants floating on the surface, and the maximum value of μ is realized at the maximum porosity. The minimum values of $\lambda_* = 2.2$ and $\mu = 0.11$. In this case, the minimum value of $\lambda_*$ is achieved at the maximum value of the porosity of the layer with the roots of plants floating on the surface, and the minimum value of μ is realized at the minimum values of porosity.

**Conclusions.** The above mathematical models allow to determine the velocity of a fluid through a layer of higher aquatic plants floating on a free surface, as well as the maximum possible velocity depending on the porosity of the layer. It is necessary for determining the rational parameters of the clarification process of technical circulating water from particles of a given hydraulic size, taking into account geometrical sizes of a pond-clarifier. The formulas (5) – (8) which have been obtained and shown in the article indicate that during water treatment in the pond-clarifier of the tailing pond, the velocity in the layer with higher aquatic plants is determined by the ratio of two parameters of this layer - porosity and dimensionless resistance coefficient. The ratio of these parameters has been established, where the maximum value of the velocity in the layer under consideration has been achieved. It has been shown that the value of the velocity coefficient in a layer with plants floating on a free surface corresponding to the maximum velocity depends only on the porosity of this layer and does not depend on its resistance coefficient.

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