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DIFFUSION MASS TRANSFER AT LOW-FREQUENCY LIQUID OSCILLATIONS IN CAPILLARIES

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Abstract. The study is devoted to the numerical modeling of diffusion mass transfer in a narrow pore channel with small low-frequency oscillations of a liquid in a laminar flow regime. This study is an elaboration of the authors' previous research. At the same time, the previously used tight constraint imposed on the channel width is weakened. The considered problem is a planar one that rather simplifies the solution without changing the qualitative parameters of the process. A solution of the dynamic problem was obtained, which formed the basis of the numerical analysis of the diffusion problem. An important feature of this solution is that for wider channels the velocity and pressure oscillations were shifted relative to each other by a certain phase angle. In addition, the oscillation frequency had a significant impact on the velocity profiles and velocity amplitudes. Contrary to the previously considered simplified model, it was found that the oscillation frequency rise at a constant pressure gradient leads to decreasing of the velocity and, accordingly, to a certain decreasing of the mass transfer intensity. At the same time, as the channel width increases, the influence of the frequency change becomes more noticeable. The calculations performed have shown that for the problem resolved in this study some parameters could be considered as independent variables during the oscillation process of the liquid columns. Increasing of the pressure gradient for a constant frequency leads to an increase of the mass transfer intensity. Expressions obtained show that increasing of the pressure gradient leads to increasing of the amplitude for the velocity oscillations. In addition, revision of the mathematical model has shown that increasing of the frequency for a constant pressure gradient leads to decreasing of velocity and, consequently, to decreasing of the mass transfer rate. Moreover, increasing of the channel width makes an influence of the frequency change stronger.

Keywords: capillary, liquid, diffusion, mass transfer, vibrations.

Introduction. The processes of interaction between the surface of a solid phase and a liquid medium in capillary porous and granular materials are very difficult to study, since they are affected by a large number of physical and chemical factors and take place in a small scale volume. The existing complexity of the description for the heat and mass transfer with interfacial interaction at the phase boundary under steady conditions rises up for the study of transient processes.

Taking into account the important practical value of the mentioned processes, researchers pay attention to this field of science. For example, in [1], data set on the increase in oil recovery under the action of low-frequency oscillations has been obtained. In [2], the influence of a low-frequency acoustic field on the capillary impregnation of porous media was experimentally studied. It was found that being exposed to a frequency of 200 Hz, the terminal water saturation of the sample has been set to rise by 13%. In [3], the influence of elastic low-frequency oscillations on filtration processes in porous materials has been studied. The acceleration of capillary impregnation under the action of low-frequency oscillations has been obtained. The results of the studies related to the research subject regarding the development of physical models of low-frequency vibroacoustic effects on the permeability of porous media are given in [4]. A comprehensive review of the literature on the consideration issues was recently published in [5].

Nowadays the physical effects that determine the vibrations influence on the heat and mass transfer processes in capillary-porous materials have not yet been comprehensively elucidated, experimental data are often contradictory and mathematical models could be applied to a limited types of problems.

This study is devoted to the numerical simulation of mass transfer in a narrow pore channel with small low-frequency oscillations of a liquid for a laminar flow This study is a development of the studies presented in [6].

In [6], a similar problem was considered, however, its formulation takes into account that the maximum channel width was limited by the condition $2\pi f h^2 / \nu < 1$ (f is the frequency, Hz; ν is the kinematic viscosity, m^2/s; h is the channel width, m), which corresponds to very narrow channels. This condition made it possible to neglect the transient term in the equation of motion as well as significantly simplify the solution of the problem.

In the present paper this limitation is removed. Further, the solution of the dynamic problem is obtained. This solution could serve as a basis for the numerical analysis of the diffusion problem. An important feature of this solution is a consideration of the phase shift between velocity and pressure oscillations for wider channels

Methods. Let’s assume, according to paper [6], that the pore channel is a long thin rectangle, which length is L and width is $2h$. In addition, we assume a laminar flow regime, the channels are straight with smooth walls and the problem is a planar one. We neglect the capillary end effects. Mass transfer is provided through the cross sections of the porous channel, when the substance being dissolved on a certain area of the upper wall is absorbed on the opposite side of the channel. Let’s assume there are no chemical reactions within computational domain. Therefore, the geometrical parameters of the domain do not vary. For these assumptions the problem is reduced to solution of the diffusion equation written as follows:

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} = D\left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2}\right), \qquad (1)$$

where x, y - cartesian coordinates; t - time, c - mass concentration; u - velocity; D - diffusion coefficient.

The boundary conditions are the following:

for $\varsigma = 0$
$c = c_L$ – for positive velocity and
$\partial c / \partial \varsigma = 0$ – for negative velocity;

for $\varsigma = 1$
$\partial c / \partial \varsigma = 0$ – for positive velocity and (2)
$c = c_N$ – for negative velocity;

for $n = -1$
$\partial c / \partial n = 0, \;\; c = 0 \quad (\bar{\varsigma}_{*} \le \varsigma \le \bar{\varsigma}_{**}), \quad (\bar{\varsigma}_{***} \le \varsigma \le \bar{\varsigma}_{****});$

for $n=1$
$\partial c/\partial n=0,\;\; c=c_0 \quad (\varsigma_* \le \varsigma \le \varsigma_{**})$.

Here c_L, c_N are the concentrations of components on opposite cross sections of the channel, $\varsigma = x/L,\;\; n = y/h$.

In this problem, as in [6], the soluble substance enters the channel at $y=h$ on the section of the channel wall $(\varsigma_* \le \varsigma \le \varsigma_{**})$ and leaves at two symmetrically located sections $(\bar{\varsigma}_* \le \varsigma \le \bar{\varsigma}_{**})$, $(\bar{\varsigma}_{***} \le \varsigma \le \bar{\varsigma}_{****})$ on the opposite wall, at $y=-h$ (Fig. 1).

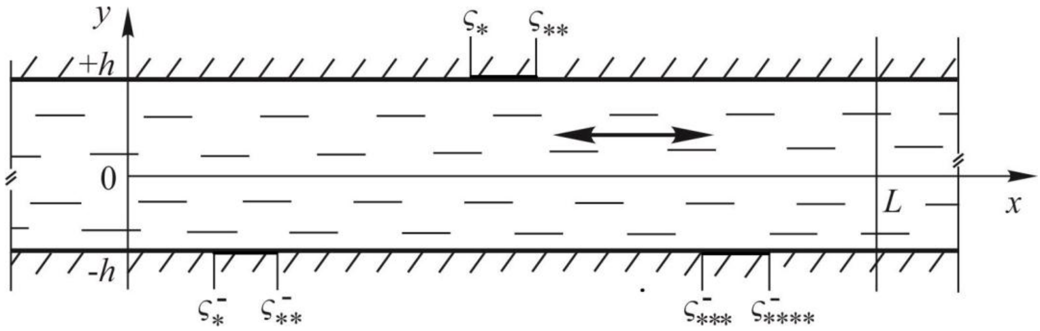

Figure 1 – Scheme of the computational domain.

Conditions for the inlet and outlet cross sections of the capillary provide a conservation of mass within this domain. Contrary to [6], where the transient term was neglected in equation of motion, which led to a simplified dependence of the velocity on the pressure gradient, authors have expanded the formulation of the problem within present study.

Since the problem is considered within framework of the boundary layer theory, the fluid pressure across the channel is constant [7]. In this case, the transient Navier-Stokes equations in the narrow channel approximation model are as follows

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\rho \partial x} + \nu \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right), \tag{3}$$

where p – pressure, ρ – density of a liquid.

The solution of equation (3) could be written as follows

$$p = P_s \cdot Sin(2\pi f t), \qquad u = U_s \cdot Sin(2\pi f t) + U_c \cdot Cos(2\pi f t)\ . \tag{4}$$

Substituting relations (5) into equation (3) as well as involving boundary conditions:

$$(\partial u/\partial y) = 0, \text{ at } y=0 \text{ and}$$

$$u = 0, \quad \text{at } y = \pm h, \tag{5}$$

the following expressions for the velocities could be obtained

$$U_s=\left\{\begin{array}{l}\dfrac{[\exp(\chi)+\exp(-\chi)]Cos(\chi)}{Zn}[\exp(\chi n)-\exp(-\chi n)]Sin(\chi n)-\\ -\dfrac{[\exp(\chi)-\exp(-\chi)]Sin(\chi)}{Zn}[\exp(\chi n)+\exp(-\chi n)]Cos(\chi n)\end{array}\right\}\frac{1}{(2\pi f)\rho}\frac{dp}{dx}, \tag{6}$$

$$U_c=\left\{\begin{array}{l}1-\dfrac{[\exp(\chi)+\exp(-\chi)]Cos(\chi)}{Zn}[\exp(\chi n)+\exp(-\chi n)]Cos(\chi n)-\\ -\dfrac{[\exp(\chi)-\exp(-\chi)]Sin(\chi)}{Zn}[\exp(\chi n)-\exp(-\chi n)]Sin(\chi n)\end{array}\right\}\frac{1}{(2\pi f)\rho}\frac{dp}{dx}, \tag{7}$$

where $\chi=\left(\dfrac{2\pi f h^2}{2\nu}\right)^{1/2}$, $Zn=[\exp(\chi)-\exp(-\chi)]^2 Sin^2(\chi)+[\exp(\chi)+\exp(-\chi)]^2 Cos^2(\chi)$.

Expressions (4), (6), (7) reveal important features of the relationship between velocities and pressure during oscillations: the change in velocity over time does not coincide in phase with the pressure gradient; the oscillation frequency has a significant effect on the velocity profiles; the velocity amplitudes themselves depend on the oscillation frequencies. The latter follows from the fact that if by amplitude we understand, for example, the expression $1/(2\pi f\rho)\,dp/dx$, then at constant gradients, the oscillation amplitude decreases with increasing frequency. This indicates that maintaining the required amplitude with increasing frequency should lead to a proportional increase in the pressure drop.

In order to proof the influence of vibrations on the mass transfer in pores, we will set the vibration frequencies and the values $(1/\rho)dp/dx$, because the very concept of velocity amplitude becomes dependent on the oscillation frequency.

Figure 2 shows the velocity profiles U_s and U_c for a channel with h = 2·10^{-4}m.

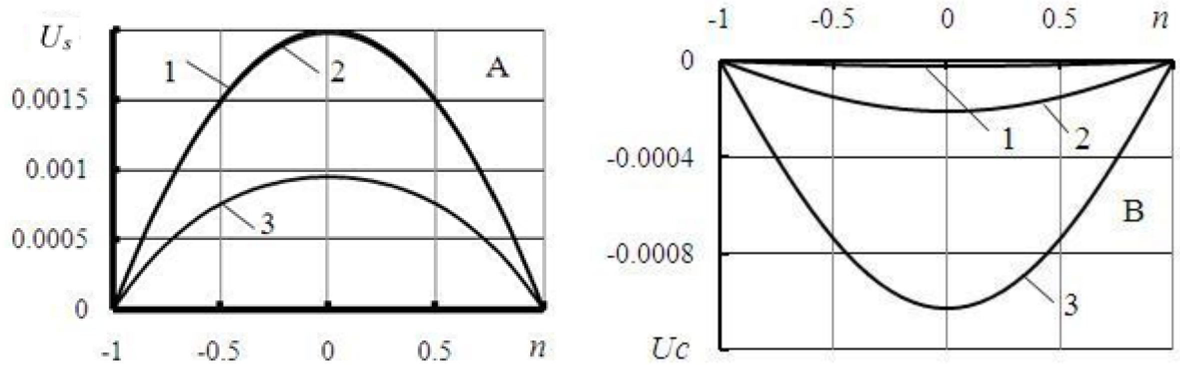

Figure 2 – Velocity profiles for a channel with h = 2·10^{-4} m for $(1/\rho)dp/dx$ = - 0.1
1 – f = 0.1 Hz, 2 – f = 1 Hz, 3 – f = 10 Hz.

It can be seen from the figure that, at a low frequency f = 0.1 Hz, the maximum value of U_c is much less in absolute value than the maximum value of U_s, which practically coincides with the Poiseuille profile. As the frequency increases, the speed

U_c increases and U_s decreases. At a frequency f = 10 Hz, these velocity components become approximately equal to each other in absolute value. This also shows that the total modulus of velocity $|u| = \sqrt{U_s{}^2 + U_c{}^2}$ decreases with increasing frequency.

Results and discussion. Diffusion problem calculati.ons were carried out for the following cases:

$\frac{1}{\rho}\frac{dp}{dz}$ = -0.1; -1.0, f = 1 Hz; 10 Hz, L = 0.1 m, $h = 2{\cdot}10^{-5}$ m; $2{\cdot}10^{-4}$ m; $2{\cdot}10^{-3}$ m.

Figures 3–5 show curves of dimensionless diffusion fluxes $q_P = \frac{D}{D_*}\int\limits_{\varsigma_*}^{\varsigma_{**}} \left.\frac{\partial c}{\partial n}\right|_{n=1} d\varsigma$ and $q_u = \frac{D}{D_*}\left(\int\limits_{\bar{\varsigma}_*}^{\bar{\varsigma}_{**}} \left.\frac{\partial c}{\partial n}\right|_{n=-1} d\varsigma + \int\limits_{\bar{\varsigma}_{***}}^{\bar{\varsigma}_{****}} \left.\frac{\partial c}{\partial n}\right|_{n=-1} d\varsigma\right)$ for these cases as well (D_* is the diffusion scale factor equals to $1{\cdot}10^{-9}$ m^2/s).

Figure 3 shows the changes in dimensionless flows from dimensionless time $\tau = t/T$ ($T=h^2/D_*$ is the time scale) for a very narrow channel with $h=2{\cdot}10^{-5}$ m. It can be seen from the figures that the flows q_P diffusing into the channel mass approach to the value of the diffusing mass from the channel q_u.. A change in the pressure gradient by an order of magnitude slightly increases the rate of approach of these flows to each other, while a change in the frequency within the scales of the given figures practically does not change the curves, i.e. they practically merge. This is the case of a channel with a small gap, which was considered in [4], when the oscillation frequency does not affect the mass transfer. This is due to the fact that, as the calculations of velocity profiles (6), (7) show, with a decrease in the channel width, the effect of frequency on the velocity profiles U_s and U_c decreases. As a result, for the considered frequencies, the velocity profile is still quite close to the Poiseuille profile, i.e. the speed U_s corresponds to the Poiseuille speed, and the speed U_c is practically equal to zero.

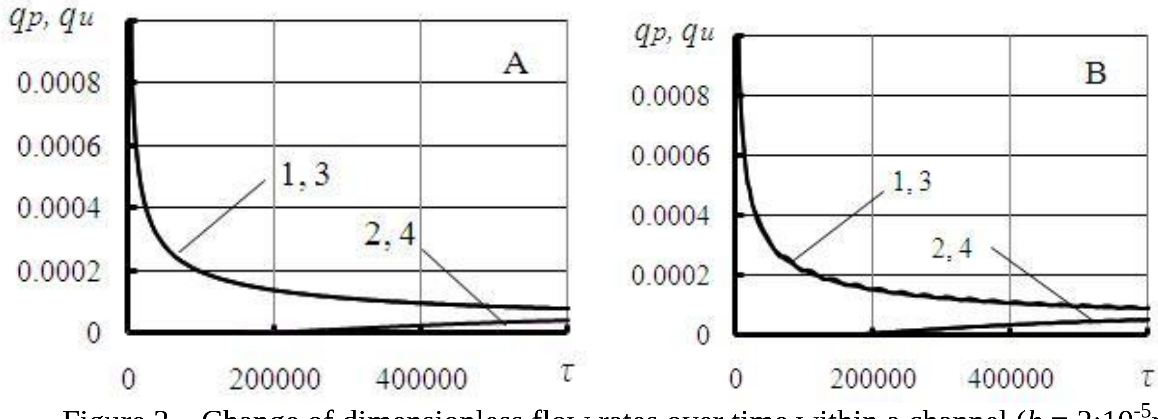

Figure 3 – Change of dimensionless flow rates over time within a channel ($h = 2{\cdot}10^{-5}$m.)
A – $(1/\rho)dp/dx$ = -0.1; B – $(1/\rho)dp/dx$ = -1. Curves 1, 2 – f = 1 Hz; 3, 4 – f = 10 Hz.

A slightly different picture could be observed in a wider channel. Figure 4 shows curves of the inflow and outflow mass for the case $h = 2{\cdot}10^{-4}$m.

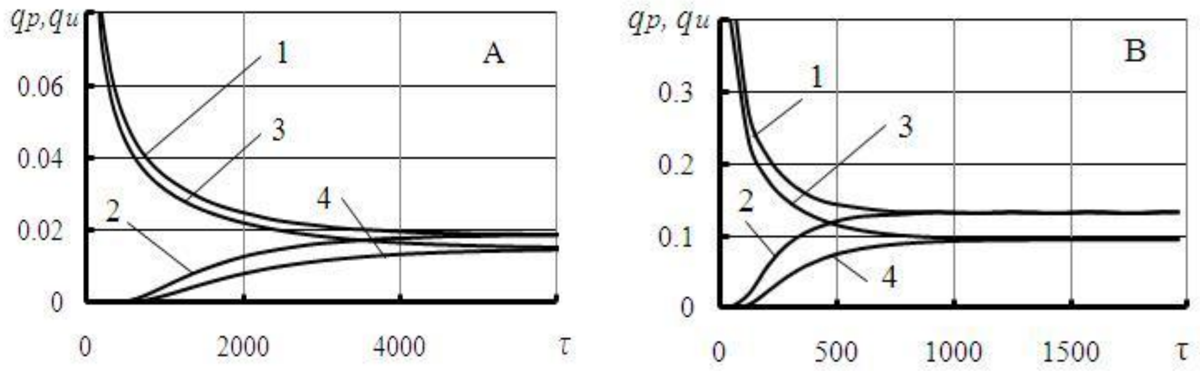

Figure 4 – Change of dimensionless flow rates over time within a channel ($h = 2{\cdot}10^{-4}$ m.)
A – $(1/\rho)dp/dx = 0.1$; B – $(1/\rho)dp/dx = 1$. Curves 1, 2 – $f = 1$ Hz; 3, 4 – $f = 10$ Hz

Figure 4 shows that increasing of the pressure gradient by an order of magnitude for a constant frequency significantly reduces an equilibrium time set up, while an increasing of the frequency for a constant gradient slightly increases this time gap, while increasing of the value $(1/\rho)dp/dx$ leads to the diffusion rates increasing. Definitely, an increasing of the pressure gradient leads to an intensification of mass transfer. A simple increase of frequency reduces the effect of the pressure drop within the channel, reduces the fluid velocity (Fig. 2), therefore this leads to some decreasing of mass transfer efficiency.

In qualitative sense the same pattern remains for a wider channel (Fig. 5). In this case, the influence of frequency on the velocity profiles appears to be stronger. Increasing the frequency significantly reduces the velocity of the liquid and, accordingly, increases the time for equilibrium to be established.

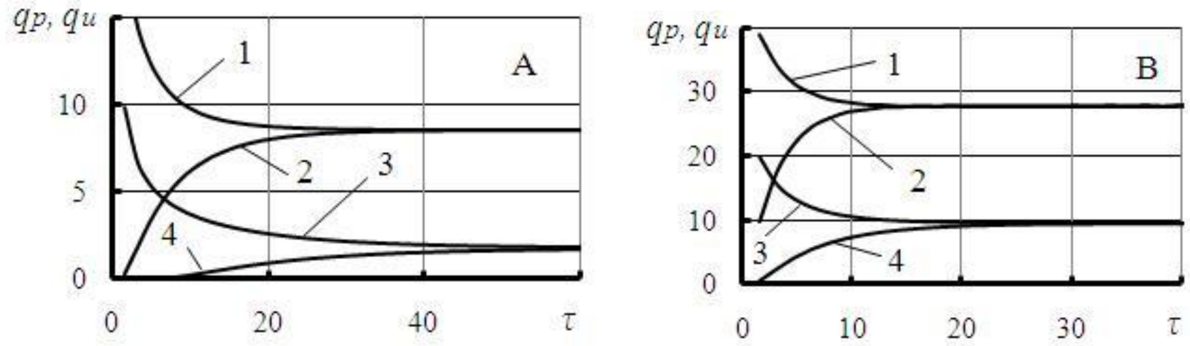

Figure 5 – Change of dimensionless flow rates over time in a channel ($h = 2{\cdot}10^{-3}$ m.)
A – $(1/\rho)dp/dx = 0.1$; B – $(1/\rho)dp/dx = 1$. Curves 1, 2 – $f = 1$ Hz; 3, 4 – $f = 10$ Hz

Figure 6 shows the distribution curves of the species concentrations within the channel along the horizontal axis for quasi steady-state flow ($t = 1600$ s).

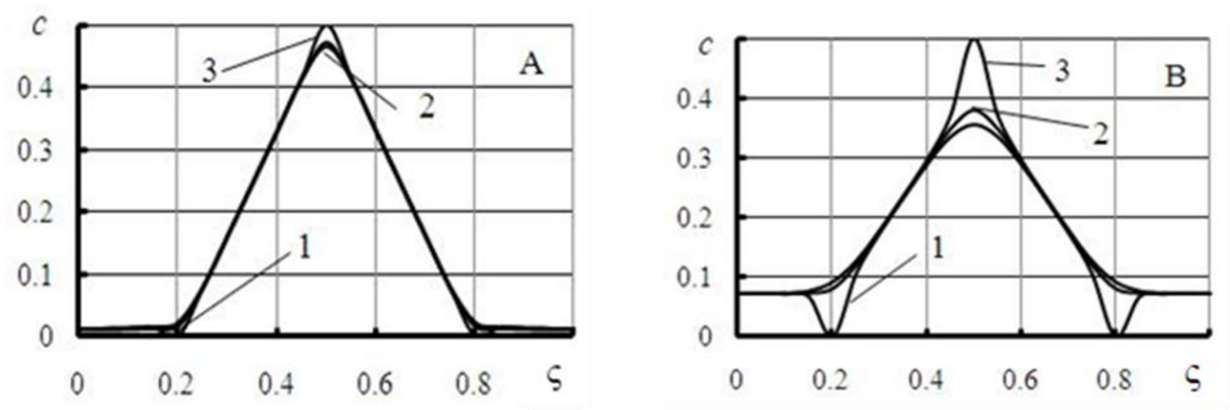

Figure 6 – Distribution of concentrations along the channel (h = 2·10^{-3}m.)

A – $\frac{1}{\rho}\frac{dp}{dx}$ = - 0.1, f = 10 Hz; B – $\frac{1}{\rho}\frac{dp}{dx}$ = - 1, f = 10 Hz. Curves: 1 – n = -1; 2 – n = 0; 3 – n = 1.

This figure shows the concentration values: c = 0 and c = 0.5 that were set up as boundary conditions for the sections of the channel boundaries. In the vicinity of these boundary sections the curves are slightly distinguished while the rest of the figure's area shows that they nearly coincide.

Conclusions. The calculations performed have shown that for the problem resolved in this study the parameter $(1/\rho)dp/dx$ and frequency f could be considered as independent variables during the oscillation process of the liquid columns. Increasing of the pressure gradient for a constant frequency leads to an increase of the mass transfer intensity. This conclusion completely confirms the results obtained in [6]. Expressions (6) and (7) show that increasing of the pressure gradient leads to increasing of the amplitude for the velocity oscillations. In addition to the results of the paper [6], revision of the mathematical model has shown that increasing of the frequency for a constant pressure gradient leads to decreasing of velocity and, consequently, as figures show, to some decreasing of the mass transfer. Moreover, increasing of the channel width makes an influence of the frequency change stronger.

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ДИФУЗІЙНИЙ МАСООБМІН ПРИ НИЗЬКОЧАСТОТНИХ КОЛИВАННЯХ РІДИНИ В КАПІЛЯРАХ

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Анотація. Робота присвячена чисельному моделюванню дифузійного масообміну у вузькому каналі при невеликих низькочастотних коливаннях рідини у ламінарному режимі течії та є розвитком попередніх досліджень авторів. При цьому знято жорстке обмеження, що використовується раніше, на ширину каналу. Розглянута задача є плоскою, що трохи спрощує рішення, не змінюючи при цьому якісних характеристик процесу. Отримано рішення динамічного завдання, яке лягло в основу чисельного аналізу дифузійного завдання. Важливою деталлю такого рішення є те, що для ширших каналів коливання швидкості та тиску зсунуті відносно один одного на деякий фазовий кут. Крім того, частота коливань істотно впливає на профілі та амплітуди швидкостей. На відміну від раніше розглянутої спрощеної моделі отримано, що збільшення частоти коливань при постійному градієнті тиску призводить до зменшення швидкості і, відповідно, до деякого зменшення інтенсивності масопереносу. При цьому зі зростанням ширини каналу вплив зміни частоти стає більш помітним. Проведені розрахунки показали, що для розв’язаної в цьому дослідженні задачі деякі параметри можна розглядати як незалежні змінні під час процесу коливань стовпів рідини. Збільшення градієнта тиску при постійній частоті призводить до збільшення інтенсивності масообміну. Отримані вирази показують, що збільшення градієнта тиску призводить до збільшення амплітуди коливань швидкості. Крім того, перегляд математичної моделі показав, що збільшення частоти для постійного градієнта тиску призводить до зменшення швидкості і, як наслідок, до зменшення швидкості масообміну. Крім того, збільшення ширини каналу посилює вплив зміни частоти.

Ключові слова: капіляр, дифузія, масообмін, рідина, коливання

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