UDC 622.831: 622.537.87 DOI: https://doi.org/10.15407/geotm2022.162.126

INFLUENCE OF GAS-FILLED FRACTURES ON THE ELECTRICAL CONDUCTIVITY OF COAL AT DIRECT CURRENT

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Abstract. The article is devoted to the theoretical study of the influence of fractures on the static electrical conductivity of coal in order to develop and justify the non-destructive method for determining the fracturing of coal and other rocks that conduct electric current. The inability of the effective medium model to describe the behavior of fractures in rocks on a macroscopic scale and the complexity of experimental observations due to the low electrical conductivity of coal necessitates an additional theoretical study.

In this paper, an approach is proposed that uses a hydrodynamic analogy between an electric current flowing around fractures and the motion of an ideal incompressible fluid in a potential flow around solids proposed earlier in our previous studies. The problem is reduced to solving a boundary value problem by the Cauchy-type integral method using the Sokhotsky-Plemel formulas. Integral dependences of the relative additional electrical resistance on the fracturing coefficient of the coal sample for an arbitrary ratio between the dimensions of coal sample and dimensions of fractures are found. The dependences of the additional electrical resistance on the fracturing coefficient of the coal sample were numerical calculated. Based on these calculations, the influence of the shape of cracks and the dimensions of a coal sample on the additional electrical resistance caused by fractures was analyzed. The nonlinear behavior of the dependence of the relative change in the additional electrical resistance of coal on the coefficient of fracturing and the thickness of the sample was established. It is shown that with an increase in the fracture coefficient, the growth of the electrical resistance of the sample occurs according to the root law, and further - according to the power law. An inverse relationship was established between the gaping of fractures and the electrical resistance of a coal sample.

The results obtained can be used as a noninvasive method for determining the fracturing coefficient not only for coal, but also for other rocks that conduct electric current.

Keywords: electrical resistance, coal sample, gas-filled cracks, hydrodynamic analogy, fracturing coefficient.

1. Introduction

A characteristic property of most rocks and, in particular, coal, is their heterogeneous structure due to the presence of long defects in them, such as fractures, pores, and filtration channels. The current experimental methods used to determine the defectiveness of minerals and rocks are very ineffective.

There are certain theoretical studies on the influence of fracturing on the conductivity of coal [1–3], but they are also incomplete. The sizes of cracks in rocks can be of microscopic level. And the model of the effective medium may be unsuitable to describe their influence on the electrophysical properties of the medium. This is of especial importance for rocks such as coal seams, because coal is an electrically conductive material. The weak electrical conductivity of coal makes it difficult to be measured by experimental methods. Therefore, an effective quantitative assessment of the influence of non-conductive (gas-filled) cracks on the electrical conductivity of coal is needed. And this determines the relevance and demand of the theoretical study of the effect of fractures on the electrical resistance of coal at direct current.

The presence of various voids in coal has been studied before. In particular, the effect of pores on the electrical conductivity of coal was theoretically studied in [3] (2010). However, the degree of porosity of the coal substance, as a rule, turns out to be less than the degree of its fracturing. Thus, there is necessary to find additional electrical resistance of coal caused by cracks. As for the effect of fractures on the electrical conductivity of solid states, these issues were considered in [4, 5], where transfer processes in solid states were studied. However, the influence of the ratio of coal sample sizes and fracture sizes on the electrical conductivity of the material was not analyzed. In addition, these papers considered the approximation of a small concentration of fractures, when the average distance between them is quite large.

The purpose of this investigation is a theoretical study of the effect of fractures on the static electrical conductivity of coal. It should be noted that some results related to the effect of fractures on the electrical resistance of a conductive material were obtained by the authors in [6, 7], however, they require further detailing and clarification.

2. Methods

2.1 Statement of the problem.

In the theoretical study the effect of gas-filled fractures on the electrical conductivity of coal can be considered as a heterogeneous system with inclusions, when the volume fraction of inclusions (fractures) is relatively small. At the same time, we will not limit ourselves to the case when the characteristic sizes of the fractures are small. A feature of gas-filled fractures is that they are non-conductive. Therefore, they can cause a significant impact on the effective characteristics of the environment due to a significant difference in the properties of the environment inside and outside them, despite the fact that they occupy a relatively small volume. Therefore, any approaches that operate only with the volume fraction of inclusions are unsuitable for fractures. Let us assume that the distance between the fractures is large enough so that their mutual influence can be neglected. Thus, it is advisable to first consider the problem of the effect of a single isolated gas-filled fracture on the resistance of a coal sample. And then it can be generalized to a sample containing a whole set of similar fractures that are randomly distributed in its volume.

We consider, as before in [6, 7], a coal sample in the form of a plane-parallel plate of thickness d, containing a system of randomly distributed non-conductive inclusions (gas-filled fractures) with an average length of $2l$. For the simplicity, we assume that the cracks are located parallel to each other and parallel to the surface of the plate (Fig. 1 in [7]).

We will take advantage of the fact that the carbon substance is a conductive environment, albeit with a rather weak electrical conductivity. If we apply some potential difference $V_2 - V_1$ to the opposite sides of the plate, then a constant current I will flow in the external circuit. According to Ohm's law for the electrical resistance of the section of the circuit containing the carbon sample, we can write the ratio

$$R = \frac{V_2 - V_1}{I}. \tag{1}$$

The presence of even one fracture in the sample will lead to a decrease in the current density through the sample. Let us denote the perturbed current density as $j' = j - \Delta j$, where Δj is the change in current density caused by a fracture. Then, by using expression (1), it is easy to show that the relative change in resistance of the sample caused by an isolated fracture is equal to

$$\left(\frac{R_f - R}{R}\right)_0 = \left(\frac{\Delta R}{R}\right)_0 = \frac{j - j'}{j'} = \frac{j - (j - \Delta j)}{(j - \Delta j)} = \frac{\Delta j}{j - \Delta j}. \qquad (2)$$

Here, R is an electrical resistance of an ideal sample without inclusions, R_f is an electrical resistance with a fracture, $\Delta R = R_f - R$ is an additional electrical resistance of the sample due to the interaction of the current with the fracture; Δj is a change in the initial current strength due to the fracture. The index 0 in expression (2) means that we are talking about one isolated crack. An important distinguishing feature of expression (2) for the additional resistivity is that it is exact. This is provided by the rejection of the assumption we used in [6] that the current density disturbance caused by the fracture is small compared to the value of the current density in an ideal sample, i.e. $\Delta j/j \ll 1$. Thus, the expression (2) can be used in the case when the disturbance of the initial current density caused by fracture-like inclusions is significant.

2.2 The effect of an isolated fracture on the electric current in a coal sample.

Let us first consider the situation when there is only one fracture in the coal sample, which is parallel to the surface of the plate. In the case of inclined cracks, the obtained result should be multiplied by $\cos^2\alpha$, where α is the angle between the normal to the outer surface of the plate and the normal to the plane of the fracture.

In order to determine the effect of a collection of randomly distributed fractures on the electrical resistance of a coal sample, the change in current density under the influence of one isolated fracture should be considered. And then the obtained result can be averaged on the whole sample volume.

A hydrodynamic analogy between the movement of an ideal incompressible fluid with potential flow around solid bodies [8] and an electric current through a conductor containing long fractures was used to solve the problem of the flow of an electric current in a coal sample with gas-filled fractures, [6, 7]. The scheme of electric current movement around one fracture (two-dimensional case) was given before (Fig. 2 in [6]).

As it is shown in [6, 7], for a plane flow of liquid with a speed **u** in the direction of the Oy axis, it is possible to introduce the quantity $\Phi(z)$ which has the meaning of the complex speed of this flow. A fracture of length $2l$ represents a section of the complex plane along the real axis Ox in the interval $[-l, l]$. It is necessary to find the perturbation of the flow rate of an ideal incompressible fluid created by this fracture.

It follows from the theory of potential that the solution of this boundary value problem is determined by an integral of the Cauchy type [9]. Here, the density in the Cauchy integral is determined by the expression:

$$\varphi(\xi) = \frac{2u\xi}{\sqrt{l^2 - \xi^2}}. \qquad (3)$$

Specification of the source as a section along the real axis in the interval $[-l, l]$

$$w(\xi) = -2u\sqrt{l^2 - \xi^2}\,, \tag{4}$$

shows that, that the derivative of this function by ξ gives the function φ(ξ) determined by expression (3). Namely

$$\frac{dw}{d\xi} = \frac{d}{d\xi}\left(-2u\sqrt{l^2 - \xi^2}\right) = \frac{2u\xi}{\sqrt{l^2 - \xi^2}} \equiv \varphi(\xi)\,. \tag{5}$$

Thus, taking into account (5), the expression for the complex fluid velocity, as in [7], takes the form

$$\Phi(z) = \frac{u}{\pi i}\int_{-l}^{l}\frac{\xi d\xi}{(\xi - z)\sqrt{l^2 - \xi^2}}\,. \tag{6}$$

As it is shown in [7], using (6) and the Sohotsky-Plemel formulas [8], it is possible to find the real and imaginary parts of the complex velocity of the fluid on the edges of the fracture.

According to the hydrodynamic analogy, the initial current density j is proportional to the fluid flow rate u in the direction of the Oy axis. Then formula (7) allows us to find the correction to the normal component of the current density $\Delta j_y \propto \upsilon_y(x,y)$ due to the fracture (that is, the expression for the vertical component of the density current in integral form):

$$\upsilon_y(x,y) = -\frac{u}{\pi}\int_{-l}^{l}\frac{\xi - x}{(\xi - x)^2 + y^2}\frac{\xi d\xi}{\sqrt{l^2 - \xi^2}}\,. \tag{7}$$

Here, the integral is taken in the sense of principal value.

2.3 Effect of the fractures system on electric current.

In order to determine the effect of the entire system of randomly located fractures on the electric current, it is necessary to introduce the concept of the density of the number of fractures per unit volume of the coal sample, which in the flat (two-dimensional) case is equal to $1/\rho_c^2$, where ρ_c is the average distance between the centers of fractures. Then, after the integration, we will get along the Oy axis from 0 to d the average change in current density due to the totality of all fractures in the sample $\langle \Delta j_y \rangle$. Here, the integration along x from $-\infty$ to ∞ can be replaced by the integration within the range from $-\rho_c$ to ρ_c.

Based on the assumption of statistical homogeneity of the distribution of fractures in the coal sample, the average value of the flow velocity normal to the fracture $\langle \upsilon_y \rangle$ does not depend on the x and y coordinates. Modification of the expression (2) and calculation of the corresponding integrals in x and y gives us the ratio for the relative change in the electrical resistance of the coal sample, which is due to the entire set of fractures that it contains

$$\frac{\Delta R}{R}=\frac{\left\langle \upsilon_y \right\rangle / u}{1-\left\langle \upsilon_y \right\rangle / u}=\frac{k}{1-k}. \tag{8}$$

Here, the dimensionless expression for k is determined by the integral

$$k=\frac{ly}{\pi h}\int_0^1 \Phi(\xi,\alpha,\delta,\gamma)\frac{\xi d\xi}{\sqrt{1-\xi^2}}. \tag{9}$$

Here, the following function is introduced under the integral for brevity:

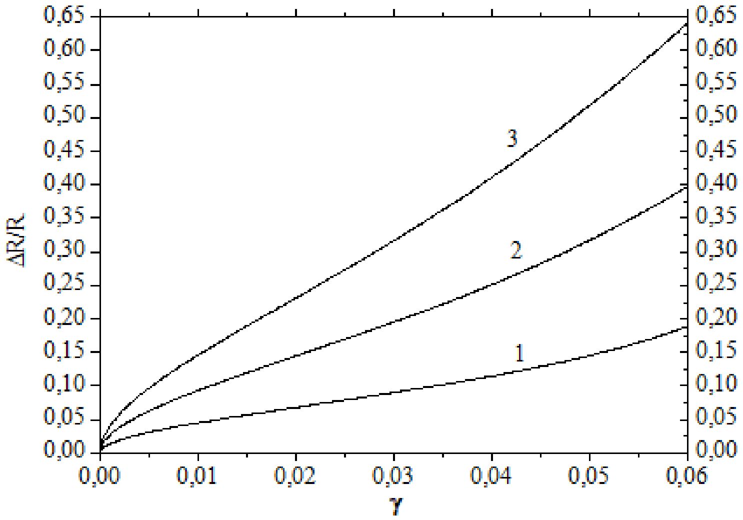
$$\Phi(\xi,\alpha,\delta,\gamma)=\frac{\delta}{2}ln\frac{\left(\xi+\sqrt{2\alpha/\gamma}\right)^2+\delta^2}{\left(\xi-\sqrt{2\alpha/\gamma}\right)^2+\delta^2}+\left(\xi+\sqrt{\frac{2\alpha}{\gamma}}\right)arctg\frac{\delta}{\xi+\sqrt{2\alpha/\gamma}}-$$
$$-\left(\xi-\sqrt{\frac{2\alpha}{\gamma}}\right)arctg\frac{\delta}{\xi-\sqrt{2\alpha/\gamma}}. \tag{10}$$

A dimensionless fracturing coefficient $\gamma=2hl/r_c^2$, where h is gaping of fracture, and dimensionless parameters $\alpha = h/l$ and $\delta = d/l$ are introduced into expressions (9-10).

3. Results and discussion

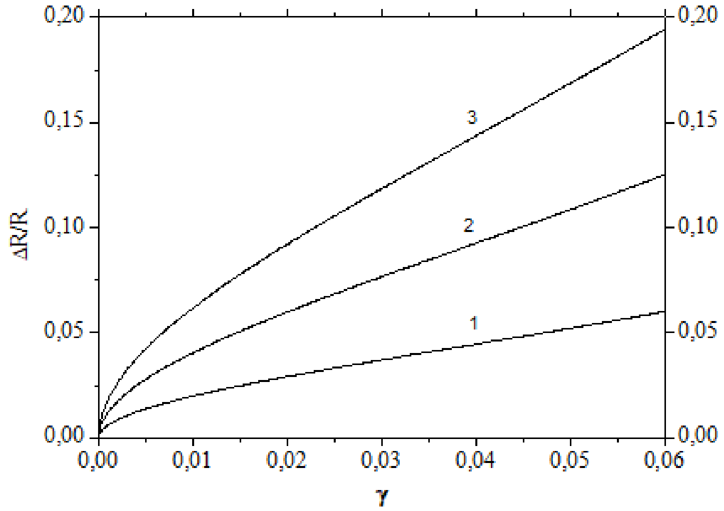
With using expressions (8), (9) and (10), we will analyse the effect of fracturing on the electrical resistance of coal for thin and thick samples and the effect of the ratio of the size of the coal sample and the size of the fractures on the electrical conductivity of the material. In other words, we will find the relative value of the additional resistance of the coal sample due to the presence of gas-filled fractures in it. To do this, it is necessary to calculate the integral (10), which depends on the coal fracturing coefficient γ, as a parameter, and substitute the obtained values in expression (8). The integral (9) containing the function (10) cannot be found analytically. The approximate behaviour of the function (8) was found in [7] only for certain limiting cases. Therefore, the actual behaviour of the additional electrical resistance due to fractures required an accurate calculation of the integral (9) with taking into account (10). In this paper, it became possible to numerically integrate the integral (9) with subsequent graphical visualization of the obtained results. Situations where fractures had various shapes were considered. In our case, the shape of the fractures is characterized by the parameter $\alpha = h/l$, that is, the ratio of the gaping of fracture to its half-length. The cases with the thin and thick coal sample were analysed separately, i.e.

when the parameter $\delta = d/l$ had different values. The dependences of the relative additional electrical resistance on the fracturing coefficient for the thin coal sample at a fixed value of the dimensionless parameter $\alpha = h/l = 0.03$ and $\alpha = h/l = 0.05$, which characterizes the ratio of gaping of fracture to its length were shown in Fig. 1 and Fig. 2, respectively.



Curves 1 – 3 correspond to thickness of the sample $\delta = d/l$: {0.1, 0.2, 0.3} respectively

Figure 1 – Dependences of the relative additional electrical resistance on the fracturing coefficient for the thin coal sample with a fixed fracture shape when $\alpha = h/l = 0.03$



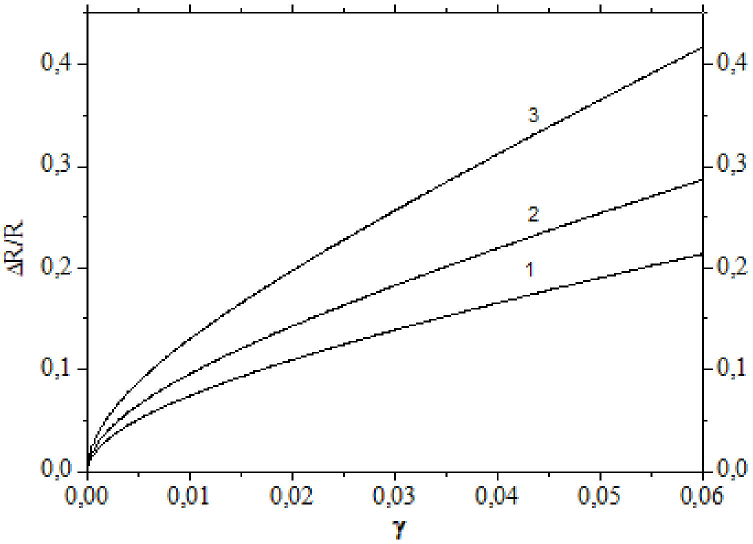
Curves 1 – 3 correspond to the thickness of the sample $\delta = d/l$: {0.1, 0.2, 0.3} respectively

Figure 2 – Dependences of the relative additional electrical resistance on the fracturing coefficient for the thin coal sample with a fixed fracture shape when $\alpha = h/l = 0.05$

As we can see, these dependences have a complex nonlinear character. At small values of the fracturing coefficient, $\gamma < 0.03$, the additional electrical resistance caused by fracture grows rather slowly and corresponds to the root law. With the growth of the fracturing coefficient, when $\gamma > 0.03$ and small gaping of fractures are

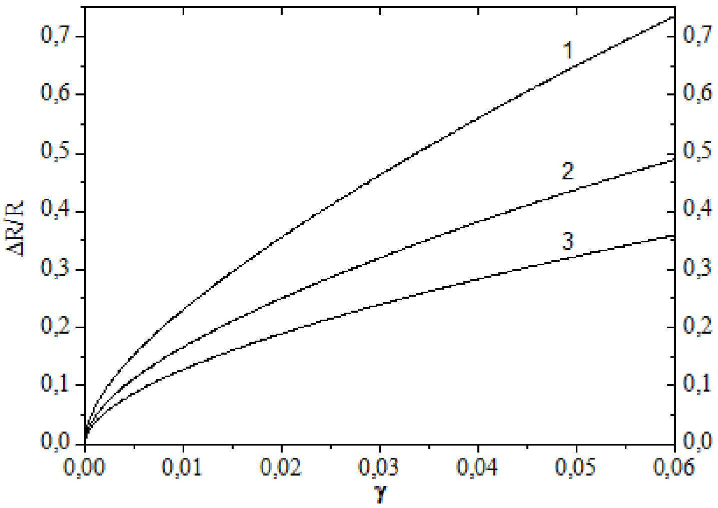
$\alpha = 0.03$, the dependence of the relative additional electrical resistance results in a rapid power-law growth (Fig. 1). But with a larger value of gaping of fractures $\alpha = 0.05$, the dependence of the relative additional electrical resistance on the cracking coefficient at $\gamma > 0.03$ has a linear character (Fig. 2).

Dependences of the relative additional electrical resistance on the fracturing coefficient for the thick coal sample at a fixed value of its thickness $\delta = d/l = 0.6$ and $\delta = d/l = 1$, are presented in Fig. 3 and Fig, 4 respectively.



Curves 1 – 3 correspond to different values of the dimensionless parameter $\alpha = h/l$: {0.05, 0.06, 0.07} respectively

Figure 3 – Dependences of the relative additional electrical resistance on the fracturing coefficient for the thick coal sample at $\delta = d/l = 0.6$



Curves 1 – 3 correspond to different values of dimensionless parameter $\alpha = h/l$: {0.05, 0.06, 0.07} respectively

Figure 4 – Dependences of the relative additional electrical resistance on the fracturing coefficient for the thick coal sample at $\delta = d/l = 1$.

They show that at small values of the fracturing coefficient, $\gamma < 0.03$, the relative additional resistance grows slowly, according to the root law. If the degree of fracturing of the coal sample exceeds $\gamma > 0.03$, then the electrical resistance caused by the cracks becomes linear. Comparison of the curves in Fig. 3 and Fig. 4 shows that an increase in the thickness of the sample affects only an increase in the value of the relative additional electrical resistance, without changing the nature of the dependence.

4. Conclusions

The effect of fracturing on the electrical resistance of coal was analyzed, in particular, the dependence of this effect on the ratio of the sizes of the coal sample and the sizes of the fractures on the electrical conductivity of the material. From the conducted numerical studies, it can be concluded that the relative change in electrical resistance of coal, due to fractures, mainly nonlinearly depends on the coefficient of cracking of coal.

Results of numerical analysis show that in the case of small fractures and an extremely thin coal plate, at low values of the coal fracturing coefficient, i.e., when $\gamma < 0.03$, the relative additional electrical resistance grows relatively slowly, close to the root law (Figs. 1, 2). In the case of a further increase in the degree of fracturing of the coal sample, that is, for $\gamma < 0.03$, the growth of the relative additional electrical resistance becomes power-law dependent.

And, finally, in the situation when the length of the fractures is close to the thickness of the plate, root growth is also observed for the relative additional electrical resistance of the sample caused by the fractures (Figs. 3, 4). But the additional electrical resistance increases additively with the increase in the thickness of the coal sample. In addition, the obtained dependences indicate that fractures with smaller gaping significantly affect the resistance of the coal sample.

Graphical dependences represented by curves in Fig. 1 – Fig. 4 were obtained by numerical integration of the general expression (9) with taking into account (8) and (10). They clearly demonstrate the behavior of the relative additional electrical resistance caused by cracks, depending on the fracturing coefficient.

We note that the obtained dependences of the additional specific electrical resistance on the fracture coefficient of the sample are not only of fundamental importance. They are also of practical interest, because they can be used as a non-invasive method of determining the fracturing coefficient not only of coal, but also of other rocks that conduct electric current.

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ВПЛИВ ГАЗОНАПОВНЕНИХ ТРІЩИН НА ЕЛЕКТРОПРОВІДНІСТЬ ВУГІЛЛЯ НА ПОСТІЙНОМУ СТРУМІ

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Анотація. Стаття присвячена теоретичному вивченню впливу тріщин на статичну електропровідність вугілля для розробки та обгрунтування неруйнівного способу визначення тріщинуватості вугілля та інших гірських порід, що проводять електричний струм. Нездатність моделі ефективного середовища описати поведінку тріщин у гірських породах у макроскопічному масштабі та складність експериментальних спостережень через низьку електропровідність вугілля обумовлює необхідність проведення додаткового теоретичного дослідження.

У цій роботі було використано підхід, який базується на використанні гідродинамічної аналогії між електричним струмом, що обтікає тріщини, і рухом ідеальної нестискаємої рідини при потенційному обтіканні твердих тіл, запропонований раніше у наших роботах. Проблема зводиться до вирішення крайової задачі методом інтеграла типу Коші з використанням формул Сохоцького-Племеля. Знайдено інтегральні залежності відносного додаткового електроопору від коефіцієнта тріщинуватості вугільного зразка для довільного співвідношення між розмірами вугільного зразка та розмірами тріщин. Проведено чисельні розрахунки залежностей додаткового електроопору від коефіцієнта тріщинуватості вугільного зразка. На основі цих розрахунків проведено аналіз впливу форми тріщин та розмірів вугільного зразка на додатковий електроопір, зумовлений тріщинами. Було встановлено нелінійний характер залежності відносної зміни додаткового електроопору вугілля від коефіцієнта тріщинуватості та товщини зразка. Показано, що зі зростанням коефіцієнта тріщинуватості збільшення електроопору зразка відбувається за кореневим законом, а надалі – за ступеневим. Як з'ясувалося між зяянням тріщин та їх впливом на електроопір вугільного зразка існує зворотна залежність.

Отримані результати можуть бути застосовані у якості неінвазивного методу визначення коефіцієнта тріщинуватості не тільки вугілля, але й інших гірських порід, що проводять електричний струм.

Ключові слова: електроопір, вугільний зразок, газонаповнені тріщини, гідродинамічна аналогія, коефіцієнт тріщинуватості.

The manuscript was submitted 24.07.2022