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ON ONE METHOD OF MULTIPLICATIVE MODELS ELABORATION DURING EXPERIMENTS

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Abstract. Description of functions in the vicinity of a point within the domain of a function is most often used for problems solution of mathematical physics as the Taylor series approximation. The reason for this approximation seems to be the application of function derivatives. The greater the order of derivatives, the more accurately the function in vicinity of the selected point could be presented. However, there is a necessity exists to define functions at the point of the mathematical models during experimental studies in a variety field of science. Mostly, two types of the models are used - additive and multiplicative ones. The multiplicative model is distinguished by the practical sense as well as widespread use.

The fact is that the nature of a particular function research for technology industrial problems consists in the sequential change of its parameters. The study of function change upon condition of a single parameter change involves the retention of other parameters of certain pre-selected values, i.e. at a certain point in the functional space of parameters. It is not always clear that the result of the experimental study is finding the values of the function exceptionally in the vicinity of this point, not within the function domain. Neglecting this circumstance along with attempts to find the values of the function far out beyond the vicinity of selected point leads to the values of the function with inappropriate error. The approximate representation of scalar functions in the multiplicative form in the vicinity of the point has a wide range of applications, especially for geomechanics.

It turned out that the approximate representation of scalar functions in a multiplicative form at a point within the domain could be extended to the whole domain. Moreover, the maximum error of a representation at the boundary of the domain for geotechnical problems, as a rule, does not exceed 5-7%, which is acceptable for engineering calculations.

To test an efficiency of the successive approximations method an applied geomechanical problem has been solved. The conclusion on the efficiency of method for geomechanical problem is made.

Keywords: mathematical model, successive approximations method, active experimental study, function.

1. Introduction

The results of experiments to identify certain properties of machines, mechanisms, devices, etc., often represent arrays of numerical data. As a rule, a mathematical model could be elaborated based on the results of experiments. It makes possible to generalize the process of its study. Either additive or multiplicative models could be selected to describe the quality function. Multiplicative models have been widely used in the processing of experimental or simulation data due to the clarity of determination for the influence of parameters.

The model can be represented by physical devices, systems of differential equations, a sequence of mathematical expressions as well as computational programming or application software that implement the model. The results of mathematical model research are usually matrices of numbers or data tables. Mathematical model is reproduced using its function values for the parameters grid (mesh). However, for complex tasks, obtaining the values of functions for the parameters mesh requires significant computation time, which often makes the problem of reproducing functions barely possible. Thus, the elaboration of a method that would allow to describe a mathematical model with a minimum number of quality function calculations taking into account the results of the experimental study (physical or numerical) is a relevant scientific problem.

Statement of the problem. In this study the (sequence) successive approximations method for obtaining mathematical model by a simplified procedure, as it usually involved in experimental research, has been used to confirm its efficiency for examples comparing with existing methods. According to this approach, obtaining of the mathematical models was not carried out for the parameters mesh, but for a certain point of the function domain. Thus, instead of calculation of the function values for the parameters mesh, they are calculated only for the coordinate lines that cross the selected point, which significantly reduces the number of calculations for the function. The sequence approximations method proposed by the authors allows to significantly reduce the computational expenditure for the reproduction of the function, but only in the vicinity of a certain point in the function domain.

2. Methods

Active experiment means the possibility of active influence on the process under consideration according to an experiment plan designed prior its conducting. Active experiments could be performed for both physical and mathematical models.

Passive experiment [1, 2] is based on the rerecording of a set of factors X and a set of effective features Y under the action of many random factors ς without interfering with the system itself. For the problems of passive multiple factor analysis, multidimensional methods of regression analysis are usually used. Thus, the Brandon algorithm known for its involvement for the mathematical models elaboration in economics, has been used to obtain multidimensional nonlinear models [1]. To study the parameters of multiple factor processes, multiplicative models are used:

$$Y = \alpha_0 \prod_{i-1}^{n} x_i^{a_i} , \tag{1}$$

where Y – effective factor; α_0, α_i – constants; x_i – process variables.

The multiple factor model (1) is known in econometrics as the Cobb – Douglas production function [3].

Active experiment provides the ability to affect actively on the process being studied according to an experiment plan designed prior its conducting. The number of active experimental studies is huge, the cost of their implementation varies from 10 up to 30% of the unit cost of serial production. Active experiment is one of the main ways to obtain information about internal relationships during the study of the processes of different nature.

In case when multiple factor processes follow the law of multiplicative or additive influence of factors, thus, the multidimensional surface of the studied factors can be described by expressions (1), the plan of active experiment is being elaborated as follows. The point within space of parameters [4, 5] is selected and the value of the effective feature Y0 is calculated. Furher, single factor tests are performed, but taking into account a condition that all single factor curves cross the node point. Availability of single factor models allows to solve the synthesis problem as well as elaborate a mathematical model or effective feature. But application of the method reveals problems with selection of the value for the ignorance factor. Thus, to increase accuracy it is recommended to select values at a point that equals the arithmetic mean, geometric mean, etc.

Thus, there is a wide range of applications in engineering practice for the representation of functions at a point. This range includes the presentation of mathematical models of experimental studies. The nature of a technical function study can be merely upon conditions of consistent parameters change. The study of function change upon condition of a single parameter change involves the retention of other parameters of certain pre-selected values, i.e. at a certain point in the function domain of parameters. It is not always clear that the result of the experimental study is finding the values of the function exceptionally in the vicinity of this point, not within the function domain. Neglecting this circumstance along with attempts to find the values of the function beyond the vicinity of selected point leads to the values of the function with inappropriate error. The approximate representation of scalar functions in the multiplicative form in the vicinity of the point has a wide range of applications, especially for the determination of mathematical models in experimental studies.

The problem of finding an unknown coefficient or multiplier k is a serious problem nowadays. This coefficient is sometimes called the ignorance factor. The importance of determining this coefficient for economic problems is that the value of the function is used to forecast or extrapolation the production capacity of enterprises. A small error in determining the production function at the boundary of the function domain, if used it in the forecast, can lead to completely unpredictable results. Hence, there are many ways to define it, but there is still no understanding of its nature.

Thus, ignorance of the nature of this factor is important not only in economics but also for experimental studies. Ignorance of the nature for this factor reduces the process of determining the mathematical model of the experimental study, despite the huge costs of its implementation.

The experience of successful use of sequence approximations method [6] has shown that the accuracy of the mathematical models obtained for geomechanics by means of abovementioned methods is satisfactory for engineering calculations within the function domain. This circumstance has inspired the author to formulate a theorem on the existence of such a representation in order to apply it to a wider range of problems.

3. Result and discussion

Applying of sequence approximations method in engineering practice will create calculation methods for certain range of technical problems. The following theorem gives an option of determining the ignorance factor or the approximation coefficient in general.

Theorem definition**:** suppose there is a scalar function $F(X,)=F(x_1,x_2,x_3,...x_n)$ which is limited, defined as well as continuous within a closed domain $\overline{D}$ of a scalar field P. Then for any point $\overline{M\subset D\forall M\in D;\forall\varepsilon\geq 0\exists U_{\varepsilon}(M)\subset \overline{D}}$ in the vicinity of the point $M_0(x_1^0,x_2^0,x_3^0,\dots x_n^0)$ (Fig. 1) function $F(X)$ can be represented as follows:

$$\left|F(X)-\varphi(X)\right|\leq\varepsilon\forall M_0\in U_{\varepsilon}(M_0)$$

where $\varphi(X)=\alpha\prod_{i=1}^{n} g_i(x_i)$ and $g_i(x_i)$ - approximation functions for $f_1, f_2, \dots f_n$ which are expressed as follows:

$$f_1(x_1)=F(x_1,\overset{0}{x_2},\overset{0}{x_3},\dots\overset{0}{x_n}),\ f_2(x_2)=F(\overset{0}{x_1},x_2,\overset{0}{x_3},\dots\overset{0}{x_n}),\ f_3(x_1)=F(\overset{0}{x_1},\overset{0}{x_2},x_3,\dots\overset{0}{x_n}),$$

α – the approximation coefficient is determined as follows:

$$\alpha=\frac{F(M_0)}{g_1(x_1^0)g_2(x_2^0),\dots g_n(x_n^0)}$$

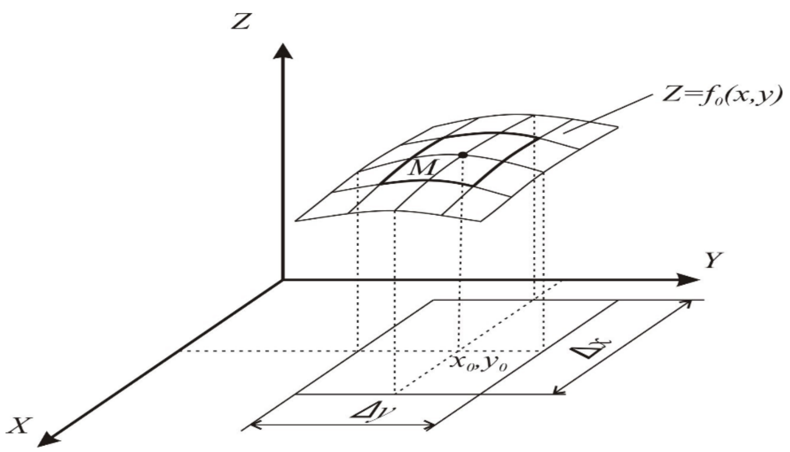

Figure 1– Vicinity selection for the point (x_0,y_0)

Experience of applying of this approach to the representation of the function $F(X,)=F(x_1,x_2,x_3,\dots x_n)$ in the vicinity of the point $M_0(x_1^0,x_2^0,x_3^0,\dots x_n^0)$ has sufficient accuracy for engineering calculations within the domain $\overline{D}$.

The algorithm for the application of successive approximations method can be represented by a sequence of the following steps:

Step 1. Selection of a point from the function domain $M=M(\overset{0}{x_1},\overset{0}{x_2},\overset{0}{x_3},\dots\overset{0}{x_n}), M\in\overline{D}$;

Step 2. Function make-up $f_1(x_1)=F(x_1,\overset{0}{x_2},\overset{0}{x_3},\dots\overset{0}{x_n})$;

Step 3. Finding a type of the function $g_1(x_1)$, which is the best approximation for the function $f_1(x_1)$;

Step 4. Finding $\varphi_1(x_1)$ according to step 1: $\varphi_1(x_1)=\alpha_1 g_1(x_1)$, where α_1 – approximation coefficient;

Step 5. Determining the function in the vicinity of the point M from the equality $F(x_1)\approx\varphi_1(x_1)$.

Reiteration of steps 2–5 successively for the variables and getting required representation:

$$F(x_1, x_2, x_3, \dots x_n) \approx \varphi(x_1, x_2, x_3, \dots x_n) = \alpha_n g_1(x_1) g_2(x_2) \dots g_n(x_n),$$

Location of the point $M = M(\overset{0}{x_1}, \overset{0}{x_2}, \overset{0}{x_3}, \dots \overset{0}{x_n}), M \in \overline{D};$ within the function domain significantly depends on its topology and therefore affects the type of its representation. Selection of a point within the function domain is determined by prior definition of its features and is determined by the qualification of the researcher. In case of complex functions and lack of prior definition of the behavior of the effective function, it is proposed to select it in the center of the domain, i.e. to determine coordinates as follows

$$x_j = \frac{b_j - a_j}{2},$$

where a_j, b_j represent the start point as well as end point of the interval of the parameter x_j change respectively.

Thus, having a transparent algorithm, there is only one thing i.e. to apply it in practice.

The widespread application of successive approximations method in practice has shown that the use of the power functions is particularly effective for assessing the impact of parameters on the quality function. The method of impact assessment is a reproduction of the solution for the problem as the product of power functions and comparison of their indicators. The greater the exponent of power for the function, the stronger the influence of the parameter on the function. To test the efficiency of the method for problems of geomechanics the following problem was solved.

Bins that averaging up cargo stream play an important role for the underground transportation of coal mines. They are usually mounted within working section as well as within places of reloading of stope cargo streams to combined conveyors (Fig. 2).

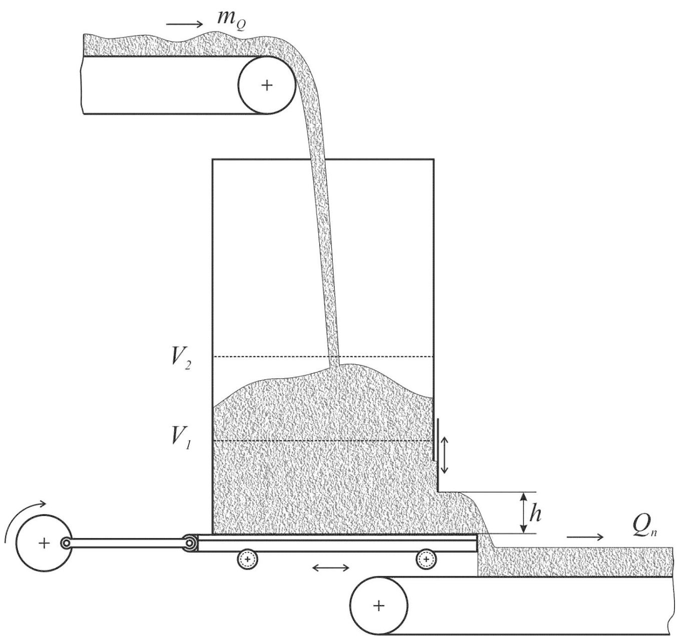

Figure 2 – Scheme of operation for the reloader bin

In order to prevent the destruction of mining equipment due to falling of large pieces of rock into the bin, it is necessary to maintain a protective cargo layer [7]. To provide it, depending on the cargo inflow parameters, bin capacity as well as the volume of the protective cargo layer, it is necessary to determine the productivity of unloading. In case of the bin operation in the mode of maintenance of a protective cargo layer within the bin, cargo stream unloaded from the bunker is stopped when the cargo quantity in the bin reaches out its maximum value V_m (m^3). Cargo stream is resumed if the cargo quantity within the bin becomes less than acceptable minimum value V_p (m^3).

The aim of the research [7] was to develop an engineering relationship for calculation of the bin average volume and assessing the impact on the average volume of its parameters as well as the selection of variables for the controller management for stabilization of the average volume. The relationships for the calculation of the bin average volume are as follows:

$$\Delta V = V_m - V_p;$$

$$A_{11} = \frac{1 + \left(\alpha \left(1 - \frac{k_1}{2} \right) \frac{\gamma \Delta V}{m_{q1}^2} + \frac{2k_1}{m_{q1}} \right) \sigma_\theta}{1 + (\alpha \frac{\gamma \Delta V}{m_{q1}^2} - \frac{2k_1}{m_{q1}}) \sigma_\theta};$$

$$A_{12} = \frac{1 + \left(\alpha \left(1 - \frac{k_1}{2} \right) \frac{\gamma \Delta V}{m_{q2}^2} + \frac{2k_1}{m_{q2}} \right) \sigma_\theta}{1 + (\alpha \frac{\gamma \Delta V}{m_{q2}^2} - \frac{2k_1}{m_{q2}}) \sigma_\theta};$$

$$A_1 = \frac{A_{11} + A_{12}}{2};$$

$$A_{21} = \frac{1 - \left(\alpha \left(1 - \frac{k1}{2} \right) \frac{\gamma \Delta V}{(m_{q1} - Q_p)^2} - \frac{2k_1}{(m_{q1} - Q_p)} \right) \sigma_\theta}{1 - \left(\alpha \frac{\gamma \Delta V}{(m_{q1} - Q_p)^2} - \frac{2k_1}{(m_{q1} - Q_p)} \right) \sigma_\theta};$$

$$A_{22} = \frac{1 - \left(\alpha \left(1 - \frac{k_1}{2} \right) \frac{\gamma \Delta V}{(m_{21} - Q_p)^2} - \frac{2k_1}{(m_{q2} - Q_p)} \right) \sigma_\theta}{1 - \left(\alpha \frac{\gamma \Delta V}{(m_{q21} - Q_p)^2} - \frac{2k_1}{(m_{q2} - Q_p)} \right) \sigma_\theta};$$

$$A_2 = \frac{A_{21} + A_{22}}{2};$$

$$t_z = \frac{\gamma \Delta V}{m_q} A_1;$$

$$t_v = \frac{\gamma \Delta V_p}{\left(m_q - Q_p\right)} A_2;$$

$$V_m = V_p + \Delta V;$$

$$V_c = \frac{m_q t_z^2 - \left(Q_P - m_q\right) t_v^2}{2\left(t_z - t_v\right)} + \frac{V_p t_z + V_m t_v}{\left(t_z + t_v\right)},$$

where m_q – average productivity of the main cargo stream, t / min; α – correlation coefficient of the outflow cargo stream; σ_θ – standard deviation of the outflow cargo stream, t/min; V_m – maximum capacity of the reloading bin, m^3; V_p – minimum capacity of the reloading bin, m^3; V_c – average capacity of the reloading bin, m^3; γ – cargo density, t/m^3; Q_p – reloader bin productivity, t/min, ΔV – volume of the reloading bin except its minimum volume, m^3.

The use of the function approximation in vicinity the point *M* as product of the power functions allows not only derive an engineering relationship but also estimate the impact of every single parameter on the original function:

$$V_c = \frac{0.999841 \cdot m_q^{0.000164105} \cdot V_p^{0.694993} \cdot \Delta V^{0.303491}}{q_p^{0,00263142} \cdot \alpha^{0,00107564} \cdot \gamma^{0,00108262} \cdot \sigma_\theta^{0,00153557}}.$$

From the point of view for the risk assessment of the influence of the parameters listed on the loss of average volume for the reloader bin it is possible to note the following: the analysis of indices for power functions shows its insignificant influence on change of average volume for parameter m_q, i.e. average productivity of the main cargo stream, t/min.

As a result, given the small values of the parameters, the above relationship could be simplified as follows:

$$V_c = a \frac{V_p^{0.694993} \cdot \Delta V^{0.303491}}{q_p^{0.00263142}},$$

where *a* – approximation coefficient.

The point *M*, in vicinity of which the approximation was performed as well as simplified relationship for the average volume of the reloader bin was obtained, has been selected as the center of the problem definition domain (Table 1).

Table 1 – Ranges of parameter changes

Parameters	Initial value	Value at point *M*	Finite value
γ [t/m^3]	0.8	1.4	2.0
ΔV [m^3]	2.0	4.0	6.0
V_p [m^3]	4.0	4.5	5.0
α [s^{-1}]	0.1	0.14	0.18
m_q [t/min]	3.0	3.5	4.0
Q_p [t/min]	4.8	8.4	12.0
σ_θ [t/min]	1.0	1.23	1.46

4. Conclusions

Analysis of the study results shows that for engineering calculations of quality functions in the vicinity of the domain of a function could be represented by means of the the successive approximations method. The advantage of the representation is the omitting of the function derivatives application that significantly expands the scope of its application.

Since active experimental investigation includes elaboration of the mathematical model by means of numerical methods, its application extends to the means of mathematical modeling of a wide range of engineering problems, in particular, geomechanics. This feature is provided by the fact that despite the representation of the function is considered at a single point, it can be extended to the domain of a function. Thus, it is possible to obtain solutions of the problems by means of a simplified procedure with accuracy sufficient for engineering calculations. An example of the geomechanical problem solution is described as well as advantages of the developed method are demonstrated.

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ПРО ОДИН МЕТОД СТВОРЕННЯ МУЛЬТИПЛІКАТИВНИХ МОДЕЛЕЙ ПРИ ПРОВЕДЕНІ ЕКСПЕРИМЕНТІВ

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Анотація. Представлення функцій у околі точки із області її визначення найчастіше використовують у задачах математичної фізики у вигляді ряду Тейлора. Причиною тому є використання похідних функції, причому чим більший порядок таких похідних, тим точніше може бути представлена шукана функція у околі точки. Проте існує широка сфера застосування в інженерній практиці представлення функцій в точці. До такої сфери відноситься представлення математичних моделей проведених експериментів. Справа в тім, що природа досліджень тієї чи іншої технічної функції може бути лише за умов послідовної зміни параметрів. Дослідження зміни функції за зміни одного параметру передбачає утримання інших параметрів певних заздалегідь обраних значень, тобто у певній точці функціонального простору параметрів. Не всі розуміють, що результатом експерименту є знаходження значень функції лише у околі цієї точки, а не на всій області її визначення. Нерозуміння цієї обставини, та спроба пошуку значень функції далеко за межами околу обраної точки призводить до отримання значень функції з великою похибкою. Наближене представлення скалярних функцій у мультиплікативному вигляді у околі точки має широку сферу застосування у прикладних задачах геомеханіки.

Виявилось, що наближене представлення скалярних функцій в точці, що знаходиться в центрі області визначення функції, може бути розширено на всю область визначення. Причому максимальна похибка такого представлення на межі області визначення , як правило, для задач геомеханіки не перевищує 5-7%, що є достатнім для інженерних розрахунків. У роботі наведено результати використання запропонованого методу до розв'язання однієї задачі геотехнічної механіки Зроблено висновок про ефективність застосування запропонованого методу до розв'язання інженерних задач гірничої механіки.

Ключові слова: математична модель, метод послідовної апроксимації, активний експеримент, функція.

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