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ANALYTICAL METHOD FOR CALCULATION OF THE STRENGTH OF CYLINDRICAL ROCK SPECIMENS DURING THEIR LONGITUDINAL STRESS

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Abstract. The goal of this research is to develop a method for calculating the strength of cylindrical rock specimens under axial failure. This will allow for the management of the stress-strain state of rock masses, which is an important issue for many mining companies. To achieve this goal, analytical modeling of the process of failure of cylindrical rock specimens under axial failure was carried out. This was done using experimental values of four indicators of rock properties: shear strength, coefficients of internal and external friction, and elasticity modulus. The results of this research allow for the determination of the ultimate strength and residual strength of cylindrical rock specimens using the four property indicators. These indicators can be experimentally determined using simple methods in laboratory conditions of mining companies. The scientific novelty of this research lies in the fact that analytical modeling of the process of failure of cylindrical rock specimens under axial failure was conducted for the first time, taking into account internal and external friction. This allowed for new results to be obtained and provided a basis for the development of new methods for managing the state of rock masses. The practical significance of this research lies in the fact that the proposed method allows for the determination of the ultimate and residual strength of rock specimens using four property indicators. These indicators can be experimentally determined in mining company laboratories, making the calculation results applicable for the management of the state of rock masses and the efficient destruction of rocks during disintegration. Thus, this method has significant practical significance for the mining industry.

A method for calculating the strength of cylindrical specimens under longitudinal failure mode has been developed. The average convergence of calculated strength values with f_c = 0.5 to experimental data is 83.4%, which corresponds to a good level of reliability for rock materials. It has been shown that the self-organization of longitudinal failure mode in cylindrical rock specimens occurs in accordance with Coulomb's criterion of maximum effective shear stress, which has been improved to account for contact friction.

Keywords: rock; strength limit; destruction; crack; stress-strain diagram.

1. Introduction

One of the important information characteristics necessary for managing the stress-strain state of rock masses and their effective disintegration is the strength limit and residual strength of samples [1–8]. These characteristics have been measured since the 1960s on special presses that are available in certain research institutes, such as the Institute of Geotechnical Mechanics of the National Academy of Sciences of Ukraine and Kryvyi Rih National University. However, these works require highly qualified personnel, and the equipment is located far from the consumer, where operational information on the properties of rocks is needed. Therefore, there is a need to develop analytical methods for calculating the limits and residual strength of samples when knowing the indicators of rock properties that can be determined by simple methods available to mining enterprises. Attempts have been made to mathematically model the process of sample fracture [5–7]. However, these models have not been developed into completed analytical methods for calculating the strength of rock samples. Prismatic or cylindrical samples are used for experimental construction of stress-strain diagrams. Analytical methods for calculating the strength of prismatic samples are detailed in the book [8]. More widely than prismatic samples, cylindrical samples from core drilling of wells are used to determine stress-strain diagrams experimentally. Therefore, there is a need to develop analytical methods for calculating the strength of cylindrical samples.

2. Methods

It is known that under uniaxial compression of prismatic and cylindrical rock samples, a longitudinal form of their destruction is formed. The longitudinal form of fracture of samples refers to anomalous types of fracture. Therefore, it is called in the literature the paradox of P. Bridgman, named after the outstanding American scientist, Nobel Prize winner, who first discovered this type of destruction. The anomalous nature of this phenomenon (cracking along the compressive load) from the standpoint of the general theory of body deformation lies in the fact that "in the direction perpendicular to the plane of such cracks, there are no normal tensile (as well as none at all) stress" [9]. Perhaps our article is one of the first works that does not propose hypotheses, unlike other authors, [10] which described this phenomenon on the basis of the Coulomb criterion, improved by us taking into account contact friction [8] using the laws of solid body deformation mechanics.

In this article, the mechanism for the formation of this form of destruction is described and a physical explanation for this phenomenon is given. We do not dwell on the mechanism, but note that the crack is formed approximately in the trajectories of the maximum effective shear stresses (TMESS).

To describe the process of destruction of rocks, the Coulomb criterion is widely used - the maximum effective shear stress on the TMESS lines.

First of all, let us describe the mathematical model of the process of cylindrical specimen fracture. According to Coulomb's criterion, when the maximum effective shear stresses reach the strength limit of the material, a crack is formed on the TMESS plane. Based on the plane strain model, the load-bearing part of the specimen can be determined at any moment by knowing the coordinates of the vertex of one or two cracks. This part is equal to the initial area of the specimen minus the part that is released from the load during crack propagation along the TMESS plane. To describe the formation of the longitudinal shape of the fracture of a cylindrical specimen, it is necessary to develop a law of distribution of contact stresses. For a prismatic specimen of unit width, L. Prandtl [9] represented this law by a formula.

$$\sigma_{yi} = \sigma_{y_0}\left(1+\frac{2f_c \cdot x}{h}\right), \quad (1)$$

where σ_{y_0} – vertical normal stress at the angle point of the sample, Pa; f_c – contact friction coefficient; x – abscissa of the considerate point, m; h – sample height, m.

Now it is necessary to bind the formula (1) to the area of the cylinder (Fig. 1). Let us describe the proposed approach to the law of distribution of normal contact stress on a cylindrical sample. The boundary of the destroyed part from the undestroyed one is the chord of the cross section of the cylindrical sample (Fig. 1), the length of which is equal to

$$a = 2\sqrt{u \cdot x - x^2}\ , \quad (2)$$

where x – abscissa of chord points, m; u– circle diameter, m.

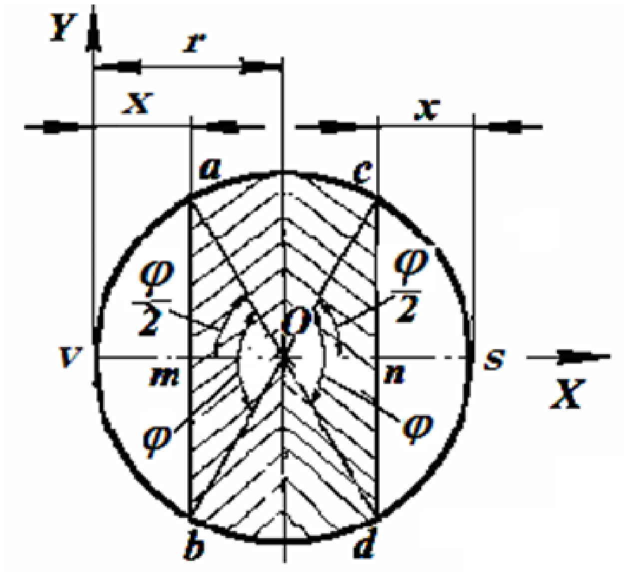

Figure 1 – Scheme of formation bearing area during development two symmetrical cracks in a cylindrical sample

Then, using expressions (1) and (2), we write the formula for the distribution of vertical stress on the contact surface of a cylindrical sample, while attaching the abscissa x to one of the points of the circle, for example, to point v (Fig. 1), Then the formula for the distribution of stresses on contact surface (Fig. 2) according to expression (1) has the form:

$$\sigma_{yi} = \sigma_y \left(\frac{2}{u} \sqrt{u \cdot x - x^2} + \frac{4 f_c}{u \cdot h} x \sqrt{u \cdot x - x^2} \right), \tag{3}$$

where σ_y – normal stress at the crack tip, Pa.

According to expression (3), we write the formula for the force of action on the part of the sample that comes out from under the load at the moment of crack development in the form:

$$P = \sigma_y \int_0^x \left(\frac{2}{u} \sqrt{u \cdot x - x^2} + \frac{4 f_c}{u \cdot h} x \sqrt{u \cdot x - x^2} \right) dx. \tag{4}$$

Then, based on expression (4) and the solution of its integrals, and taking into account that the area of the semicircle is equal to $\pi \cdot u^2/8$, we have the current value of the specific force on the bearing part of the semicircle of the sample during the formation of a crack:

$$p=\sigma_y\left(\frac{\pi\cdot u^2}{8}-2\int_0^x\left(\left(\left(2x-u\right)\frac{\sqrt{u\cdot x-x^2}}{4}-\frac{u^2}{8}\left(\frac{\pi}{2}-\arcsin 2\frac{\sqrt{u\cdot x-x^2}}{u}\right)\right)+\frac{2f_c}{h}\left(-\frac{\left(u\cdot x-x^2\right)\sqrt{u\cdot x-x^2}}{3}+\frac{u\left(2x-u\right)\sqrt{u\cdot x-x^2}}{8}-\frac{u^3}{16}\left(\frac{\pi}{2}-\arcsin 2\frac{\sqrt{u\cdot x-x^2}}{u}\right)\right)\right)\right)\Bigg/\left(\frac{\pi\cdot u^2}{8}-\frac{\left(2x-u\right)}{4}\sqrt{u\cdot x-x^2}-\frac{u^2}{8}\left(\frac{\pi}{2}-\arcsin 2\frac{\sqrt{u\cdot x-x^2}}{u}\right)\right). \quad (5)$$

For a complete solution of the problem, it is necessary to determine the normal stress σ_y at the top of the crack. The paper [8] describes a method for determining this stress on the basis of the theory of local destruction of rocks developed by the authors. Stress σ_y at the crack tip on TMESS ξ according to the method is determined by the system of equations:

$$\sigma_y=\frac{1}{\mu}\left[\frac{\left(k_n\cdot\left(1+\sin\rho\sqrt{1-b_\xi^{\ 2}}\right)\right)\cdot\exp\left(2\mu\left(\beta_\xi+\beta_b\right)\right)}{1-\sin\rho\cdot\sqrt{1-b_\xi^{\ 2}}}-k_b\right], \quad (6)$$

where

$$k_b=\frac{\left(k_n+\mu\cdot\sigma_y\right)\left(1-\sin\rho\sqrt{1-b_{\xi'}^{\ 2}}\right)}{\left(1+\sin\rho\sqrt{1-b_{b_\xi}^{\ 2}}\right)\cdot\exp\left(4\mu\beta_b\right)}; \quad (7)$$

$$b_\xi=\frac{f_c\left(1-\frac{2y}{h_1}\right)\cdot\sigma_{y\xi}\left(1+\frac{2f_c}{h}x_\xi\right)}{k_n+\mu\cdot\sigma_{y\xi}\left(1+\frac{2f_c}{h}x_\xi\right)}; \quad (8)$$

$$b_b=-\frac{f_c\cdot\sigma_{y\xi}\left(1+\frac{2f_c}{h}x_b\right)}{k_b+\mu\cdot\sigma_{y\xi}\left(1+\frac{2f_c}{h}x_b\right)}; \quad (9)$$

$$\beta_\xi=\frac{1}{2}\mathrm{arctg}\frac{b_\xi\cos\rho}{\sin\rho-\sqrt{1-b_\xi^2}}; \quad (10)$$

$$\beta_b = \frac{1}{2} arctg \frac{b_b \cos\rho}{\sin\rho - \sqrt{1-b_b^2}}, \tag{11}$$

where k_n – rock shear strength, Pa; k_b – effective shear stress at point b TMESS ξ exit on the contact surface, Pa; ρ =arctg μ angle, rad and μ – coefficient of internal friction; y – point ordinate on TMESS ξ (at the top of the crack), m; x_ξ – crack tip abscissa ξ, m; x_b – abscissa point b, m; β_ξ and β_b – rotation angles TMESS ξ at the crack tip and at the point b, rad.

We assume that deformations develop in pairs along TMESS: on the left side of TMESS ξ and on the right side of TMESS ξ'. We denote the angle between the direction of the tangent to TMESS ξ at point a, measured clockwise, as α. The truncated wedge shape [8] is the basic form for longitudinal sample failure. With this form, TMESS η, and TMESS η and η' develop from the corners of the sample.

Now we need to determine the parameters β_ξ, β_b, b_ξ, b_b, k_b. The values of these parameters will be specified by the corresponding conditions along TMESS ξ and the line ob1 during top-down failure, and along η and the line o_1d_1 during bottom-up failure (Fig. 2). Let us consider the development of the crack along TMESS η. The angle of inclination of TMESS η is determined by the formula [9]:

$$\alpha_\eta = \frac{3\pi}{4} - \frac{\rho}{2} + \beta_\eta, \tag{12}$$

where β_η – angle of rotation TMESS η from contact friction, equal to

$$\beta_\eta = -\frac{1}{2} \operatorname{arctg} \frac{b_\eta \cos\rho}{\sin\rho - \sqrt{1-b_\eta^2}}, \tag{13}$$

$$b_\eta = \frac{f_c\left(1 - \frac{2\cdot y}{h}\right)\cdot \sigma_{y_\eta}\left(1 + \frac{2\cdot f_c \cdot x}{h}\right)}{k_s + \mu\cdot\sigma_{y_\eta}\left(1 + \frac{2\cdot f_c\cdot x}{h}\right)}. \tag{14}$$

Point o_1 is formed according to the formula (12) with $\alpha_\eta = \frac{3\pi}{4} - \frac{\rho}{2} - \beta_\eta = \frac{\pi}{2}$, if TMESS develops upward from point o_1 to point d_1 and exits into the contact area of the upper plane, we change the indexing from TMESS η to TMESS ξ_1, as the crack propagates downward. Accordingly, the indexing of TMESS in Fig. 2 has been changed. Then, the condition at which longitudinal failure occurs is represented by

$$\beta_{\xi_1} = -\left(\frac{\pi}{4} - \frac{\rho}{2}\right). \tag{15}$$

Here it is necessary to define b_{ξ_1} with $\beta_{\xi_1} = -\left(\frac{\pi}{4} - \frac{\rho}{2}\right)$ according to the formula (8). From which it was obtained that a longitudinal fracture is formed on the contact surface under the condition:

$$b_{\xi_1} = \frac{\cos^2\rho}{1+\sin^2\rho} = \frac{f_c \cdot \sigma_{y\xi}}{k_s + \mu \cdot \sigma_{y\xi}}. \tag{16}$$

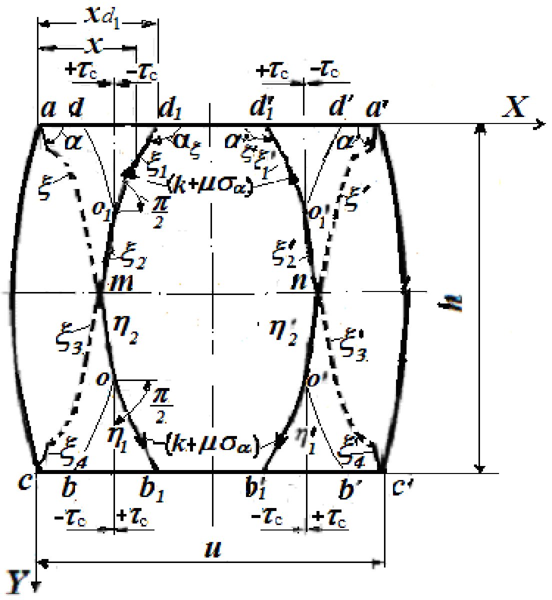

Figure 2 – The initial scheme of self-organization TMESS in the sample with its longitudinal form of destruction, [11]

As in points o_1 and o_1' (Fig. 2) tangential friction stress changes sign, then at these points the parameters in the formulas (6) and (7) b_b=0, β_b=0, and TMESS η in the upper part acquire properties TMESS ξ_1 and TMESS ξ_1', because the tension σ_y necessary for the development of cracks at points d and d' becomes the smallest in comparison with other points. It should be noted that at the joining points of different TMESS values b_b did not exceed k_n.

Thus, all the expressions necessary to determine the normal stresses on the left slip lines ξ(1-4) by equation (6) are presented. Due to symmetry, the same stress values will be present on the right side of TMESS (1-4). Moreover, the same pattern will occur when solving the problem from bottom to top. Then, a complex surface of effective shear stresses is obtained in the form of (Fig. 2) when they are formed from top to bottom and bottom to top. The cracks will develop from top to bottom and from bottom to top towards each other. When four cracks, d_1m, and b_1m, develop pairwise towards each other, the sample will split into three parts.

To calculate the current strength value on the load-bearing part of the upper left half of the sample, the obtained specific force values by formula (5) should be multiplied by the area that has not been released from the load, which is equal to:

$$S = \frac{\pi \cdot u^2}{8} - \frac{2(0.5\cdot u - x_{d_1} + x) - u}{4}\sqrt{u\cdot(0.5\cdot u - x_{d_1} + x) - (0.5\cdot u - x_{d_1} + x)^2} - \\ - \frac{u^2}{8}\left(\frac{\pi}{2} - \arcsin 2\frac{\sqrt{u\cdot(0.5\cdot u - x_{d_1} + x) - (0.5\cdot u - x_{d_1} + x)^2}}{u}\right) \tag{17}$$

Using expressions (9–17), it is possible to determine the parameters of conditional analytical diagrams "stress-strain" by iterations on TMESS, which researchers obtain on presses with a longitudinal form of destruction of cylindrical samples. Now we compare the calculated values of ultimate strength σ in the case of longitudinal fracture with the experimental data (Table 1) [14]. As can be seen, the average convergence of the calculated values of the ultimate strength at f_c = 0.5 with experimental data is 83.4%, which corresponds to a good level of reliability for rocks according to the classification of L.I. Baron [15], especially if we take into account the complexity of the formation of the longitudinal form of destruction of rock samples.

Table 1 – Convergence of the calculated tensile strengths for the longitudinal form of fracture

Rock type	Experimental			Calculated		Cadastre [17, p.]
	k_n, MPa	ρ, degree	σ_e, MPa	σ_c, MPa	Relative error, %	
Monzonite	6.3	55	47	57.1	21.4	105
Monzonite	6.2	55	50	56.4	12.8	105
Monzonite	5.1	56	38	47.8	25.8	105
Monzonite	4.5	56	76	42.5	44.1	103
Feldspar	6.0	57	59	57.8	2.0	105
Monzonite	19.5	62	167	221	32.3	105
Monzonite	3.4	64	37.5	41.8	11.5	105
Average relative error					21.4	

3. Conclusions

1. A method has been developed for calculating the strength of cylindrical specimens with a longitudinal fracture. The average convergence of the calculated values of the tensile strength at f_c = 0.5 with experimental data is 83.4%, which corresponds to a good level of reliability according to the classification of L.I. Baron for rocks.

2. It is shown that the self-organization of the longitudinal form of destruction of cylindrical rock samples occurs in accordance with the criterion of maximum effective shear stress of Coulomb, improved by taking into account contact friction.

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АНАЛІТИЧНИЙ МЕТОД ДЛЯ РОЗРАХУНКУ МІЦНОСТІ ЦИЛІНДРИЧНИХ ЗРАЗКІВ ГІРСЬКОЇ ПОРОДИ ПРИ ЇХ ПРОДОЛЬНОМУ НАПРУЖЕННІ

Васильєв Л.М., Васильєв Д.Л., Маліч М.Г., Катан В.О., Різо З.М.

Анотація Мета даного дослідження полягає у розробці методу розрахунку міцності циліндричних зразків гірських порід за їх поздовжньої форми руйнування. Це, у свою чергу, дозволить керувати напружено-деформованим станом гірничого масиву, що є важливим питанням для багатьох гірничих підприємств. Для досягнення цієї мети було проведено аналітичне моделювання процесу руйнування циліндричних зразків гірських порід за їх поздовжньої форми руйнування. Це було зроблено з використанням експериментальних значень чотирьох показників властивостей гірських порід: межі опору зсуву, коефіцієнтів внутрішнього та зовнішнього тертя та модуля пружності. Результати цього дослідження дозволяють визначити межу міцності та залишкову міцність циліндричних зразків гірських порід, використовуючи чотири показники властивостей. Ці показники можуть бути встановлені експериментально за допомогою простих методів у лабораторних умовах гірничих підприємств. Наукова новизна цього дослідження полягає в тому, що вперше було проведено аналітичне моделювання процесу руйнування циліндричних зразків гірських порід за їх поздовжньої форми руйнування з урахуванням внутрішнього та зовнішнього тертя. Це дозволило отримати нові результати та дало підставу для розробки нових методів керування станом гірничого масиву. Практична значимість цього дослідження полягає в тому, що запропонований метод дозволяє визначити межу і залишкову міцність зразків гірських порід, використовуючи чотири показники властивостей. Ці показники можуть бути експериментально встановлено в лабораторіях гірничих підприємств, що робить результати розрахунку оперативно застосовними для управління станом гірничого масиву та ефективного руйнування порід при дезінтеграції. Таким чином, цей метод має велику практичну значимість для гірничодобувної промисловості. Розроблений метод розрахунку міцності циліндричних зразків при довготривалій формі руйнування. Середня збіжність розрахункових значень межі міцності при f c = 0,5 з експериментальними даними становить 83,4%, що відповідає високому рівню достовірності для гірських порід. Показано, що самоорганізація довготривалої форми руйнування циліндричних зразків гірських порід відбувається відповідно до критерію максимальних ефективних тангенціальних напружень Кулона, удосконаленого з урахуванням контактного тертя.

Ключові слова: гірська порода, межа міцності, руйнування, тріщина, діаграма напруження-деформація.