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THE USE OF SEQUENTIAL APPROXIMATION METHOD FOR RISK DETERMINATION IN PROBLEMS OF GEOTECHNICAL MECHANICS

Krukovskyi O.P., Larionov H.I., Zemlyana Yu.V., Hvorostyan V.O., Golovko S.A.
M.S. Poliakov Institute of Geotechnical Mechanics of the National Academy of Sciences of Ukraine

Abstract. Most processes in technical improvements are deterministic, therefore, the concept of risk as a product of the probability of an accident occurrence on the financial costs of its elimination, which is proposed in most articles and regulatory documents, is not acceptable, since none of the project parameters is a random variable. In this regard, it is proposed to define risk as a technical system exceeding the values permissible by technical specifications, that is going beyond the operational capability. Before determining the degree of influence of parameters on the risk amount, it is necessary to determine sensitivity to their changing. Sensitivity analysis allows identifying parameters with the greatest influence on the risk of criterion going beyond the operational capability. However, in practice, it is not always possible to determine criterion sensitivity to the change of one or another parameter. In practice, typical situation is a problem to determine risk under conditions of simultaneous change of all parameters. Thus, a relevant method for risk calculation would be a method which allows determining risk sensitivity to the change of parameters and, at the same time, calculating the risk with simultaneous changes in all parameters. The sequential approximation method (SAM) makes it possible to calculate the risk with simultaneous changes of other parameters within a certain range using the information obtained during determining the risk sensitivity to the change of parameters. In the SAM, risk is represented in a multiplicative form, where the components of the product are the functions of one parameter. If the risk approximation is carried out in the form of a product of power functions, each of which depends on only one parameter, then the risk sensitivity to the change of the parameters can be approximately determined by the power indicators. The higher is the power, the greater is the influence of parameter on the risk. In this way, it is possible not only to make an approximate assessment of the influence of parameters on the criterion itself, but also to make conclusions about the importance of the influence of the system exceeding the permissible limits on the risk.

In this work, the efficiency of the SAM method for determining the risks of parameters exceeding the permissible limits is demonstrated by the results of solving a classic problem of determining the stress-strain state in the neighborhood of the roadway with a circular cross-section by the finite element method. An algorithm for calculating risks based on specific examples is presented. In order to demonstrate the satisfactory accuracy of the criterion calculations, surfaces of the tangent stress intensity function obtained by the SAM method is compared with the interpolation surfaces obtained by numerical results. Conclusion is made about the ability of the method to determine the risks of the criterion exceeding the permissible limits and to provide satisfactory accuracy of the obtained results.

Keywords: risk of loss of operational capacity, multiplicative form of representation, sensitivity of the function, change of parameters, neighborhood of a point, tangential stresses intensity.

1. Introduction

Modeling is one of the most widespread means of studying processes and phenomena of any nature. Today, numerous modeling methods and tools are known and widely used in scientific researchs and engineering practice. A distinction is made between physical, simulation and mathematical modeling. In physical modeling, a model repeats the process under study and preserves its physical nature [1]. Mathematical modeling is understood as a method for researching processes of different nature by studying phenomena of different physical nature, but which are described by the same mathematical relationships. In the simplest cases, the well-known analogies between mechanical, electrical, thermal and other phenomena are used.

An essential issue is that when studying any process with the help of mathematical modeling, it is necessary to have, first of all, its mathematical description or its mathematical model. As a rule, there are three forms of representation of such models: additive, multiplicative and combined. In the case of choosing an additive model, the coefficients of the terms of the sum are determined by conducting the necessary number of experiments. The multiplicative model is advantageously different from the additive model. Its main advantage is that in order to determine the risk for a great number of system parameters, there is no need to repeat the process of defining functions of the parameters that have already been determined. To create a new model, it is enough to add one more factor to the existing product. Besides, the multiplicative form of the model representation demonstrates more clearly the relationship between the parameters [2].

The procedure for constructing a mathematical model of any device in the multiplicative form is well known and consists in defining a function by a set of parameter values, by which the model adequately describes operation of the device. Determination of the function values by changing the parameters starts with changing of one parameter by a certain step and leaving the others in an unchanged initial state. When changing the next parameter, the previous one takes its initial value, and all other parameters also remain in their initial state. That is, before defining the function, the curves formed by the intersection of its surface with the planes of the corresponding parameters are approximated. Then the function is represented in an analytical form, for example, in a multiplicative form, in the form of a product of the obtained functions of the curve approximations [2,3]. Acting in this way, the functional dependence of the chosen criterion and, then, of the risk are determined. Class of approximation functions is found in the class of elementary functions, because class of elementary functions is wide and allows choosing product functions of any form. However, this form of representation has a rather big drawback. The fact is that before the product there is an approximation coefficient, which is sometimes called a multiplier. The nature of this coefficient is complex, and attempts to choose it depending on functions did not give results. For today, it is considered to be uncertain, and therefore it is sometimes called the "ignorance" coefficient. To determine it, even an additional series of experiments should be conducted. Though, it is known that the costs of experimental research of technical wares make up to 30% of their production cost. Therefore, this procedure is expensive and is valid only for concrete type of the ware without any generalizations. The answer to this can only be the application of mathematical methods and representations.

If you carefully follow the algorithm for creating a mathematical model, it becomes clear that the model we create will demonstrate the behavior of our process, but only in neighborhood of the point we have chosen. It is not worth looking for the value of the criterion far from this point, because the error in determining these values can be huge.

The problem of representing the criteria, in our case it is the tangential stresses intensity (TSI), in the multiplicative form, where the product functions are one-dimensional representations of the intersections of the function space by the corresponding planes, is solved by realization of the following hypothesis [4].

2. Theoretical and/or experimental parts

Let there exist a scalar function $F(X)=F(x_1,x_2,x_3,...,x_n)$ which is bounded, defined and continuous in a closed domain $\overline{D}$ of the scalar field P. Then, for any point $\forall M \in D; \forall \varepsilon \geq 0 \exists U_{\varepsilon}(M) \subset \overline{D}$ in the neighborhood of the point $U_{\varepsilon}(M_0)$ where $M_0(x_1^0, x_2^0, x_3^0, ..., x_n^0)$, the function $F(X)$ can be represented in the form:

$$\left|F(X) - \varphi(X)\right| \leq \varepsilon, \forall M_0 \in U_{\varepsilon}(M_0),$$

where $U_{\varepsilon}(M_0)$ is the neighborhood of the function; $\varphi(X) = \alpha \prod_{i=1}^{n} g_i(x_i)$; $g_i(x_i)$ are approximation functions for $f_1, f_2, ..., f_n$, which are given as follows:

$$f_1(x_1) = F(x_1, x_2^0, x_3^0, ..., x_n^0),\ f_2(x_2) = F(x_1^0, x_2, x_3^0, ..., x_n^0),\ f_3(x_3) = F(x_1^0, x_2^0, x_3, ..., x_n^0);$$

α is the approximation coefficient, which is determined by the formula

$$\alpha = \frac{F(M_0)}{g_1(x_1^0) g_2(x_2^0), ..., g_n(x_n^0)}.$$

The experience of successful use of the sequential approximation method (SAM) in applied problems of mechanics [2, 3, 4, 5] allows not only to obtain an analytical form of the criterion at a point, but also allows the solution of practical problems to be extended to the entire domain of the function definition. Errors of such an extension, as a rule, do not exceed the value of 5–7%, which is sufficient for most applied problems of geomechanics. The accuracy of the criterion definition can be improved to the required value by way of narrowing the area of its definition, i.e., shortening the intervals for the parameters change.

As it follows from the performed studies [5], risk definition was replaced for deterministic systems. It is formulated in such a way that risk for a deterministic process is a deviation of the chosen criterion beyond its permissible value. The quantitative risk amount for each parameter can be calculated by the formula [5]:

$$R_i = \frac{C_i^0 - C_i}{C_i^0} 100\%, \tag{1}$$

where C_i^0, C_i is the permissible and current value of the chosen criterion for parameter i; R_i is the risk of deviation of the criterion beyond the permissible value for parameter i. That is, in order to calculate the risk according to this formula, no changes in the methodology for calculating technical criteria are required for users. However, if it is impossible to provide conditions for determining the criterion for each of the parameters, the SAM method can be used, which represents the risk in an analytical form and make it possible to determine the risk in case of simultaneous changes of the parameters within a certain range.

Therefore, the live scientific task is to study and analyze the results of the SAM method use for calculating risks in the neighborhood of a point from the definition area for any number of parameters simultaneously changed with using previously obtained results of their sensitivity to change, which does not need significant costs for risk calculation and does not require from the user deep knowledge of mathematical analysis, but instead is based on deep knowledge of the process being studied.

3. Methods

To use the SAM method to assess the risks of criterion exceeding the permissible limits due to changes of parameters or to determine the risk of loss of roadway stability due to changes of the parameters of the depth of the roadway location *H* and the depth of sinking into the depth from the roadway surface *h*.

As a demonstration of the possibilities of using the SAM method, a classical problem is chosen (fig. 1). The calculation scheme is presented in fig. 1. The problem is solved in a flat elastic-plastic formulation [6, 7]. A flow model based on the Coulomb-Mohr strength theory is used to characterize the strength properties of rocks [8–10].

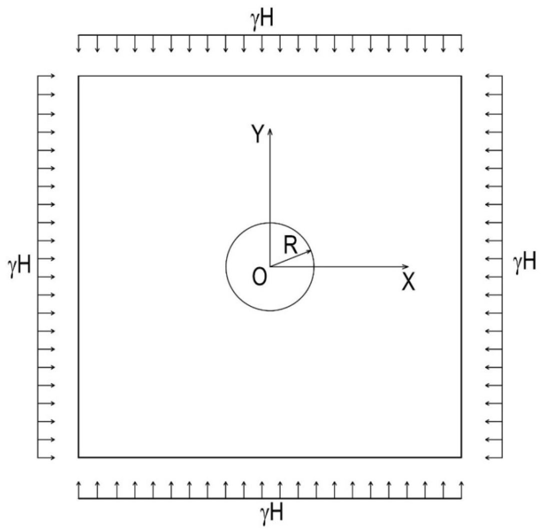


Figure 1 – Calculation scheme

The calculations are performed in the Cartesian OXY coordinate system.

Axes OX and OY are directed horizontally and vertically down, respectively. The chosen size of the rectangular research area in the rock massif is: 60×60×60 m, the boundary conditions of the load are set for the edge planes of the area. Calculations were carried out with different variants of the depths of the roadway location. For the main variant of the calculation, vertical component of the initial stress field in the rock mass is equal to γH, and the horizontal component is equal to $\lambda\gamma H$, under the condition that the lateral pressure coefficient is $\lambda = 1$. In the center of the area for the calculation, there is a roadway of rounded cross-sectional shape with a radius of $R = 2.5$ m.

Layering is an important factor affecting the properties of the rock massif [11–13]. The physical and mechanical properties of both coal seams and rock seams containing coal differ. But at the stage of this research, we accept that the massif is a continuous isotropic homogeneous medium.

Physical and mechanical properties of the rocks surrounding the roadway are given in Table 1.

Table 1 – Physical and mechanical properties of the rocks surrounding the roadway

Rock	Elastic modulus E ×10^{-4}, MPa	Poisson's ratio, μ	Adhesion C, MPa	Angle of internal friction φ, degrees	Tensile strength σ_p, MPa
Siltstone	1,5	0,25	3,5	30	-2

Criterion for assessing the state of the rocks in the neighborhood of the roadway is the tangential stresses intensity (TSI), which is calculated according to the formula

$$\tau_i = \frac{1}{\sqrt{6}}\sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)}.$$

The analysis of the stress-strain state and the analysis of the process of rock surface destruction in the neighborhood of the roadways gave grounds for choosing the TSI (tangential stress intensity) as a criterion for the stability of the roadways and the strength of the rocks around it. The criterion, in our case it is the risk to be studied and presented as a product of one-dimensional approximations, meets the requirements of the hypothesis, that is, it is a scalar quantity. Therefore, it is necessary to determine the risk of parameters exceeding the permissible limits or the loss of roadway stability due to the changes of the parameters of the depth of the roadway location H and the depth of sinking into the depth from the roadway surface h.

In this example, the number of parameters makes it possible to make a clear demonstration of the data chosen for approximation and the methodology of defining risk due to the change of one or another parameter.

The interpretation of the problem formulation can be expressed in the following way: to determine the risk of the TSI going beyond the operational capability when setting a roof bolt in the rock at a depth from the roadway surface h and the depth of the roadway location H. We present the research plan for determining the risk as follows:

1. Taking into account knowing of the technology of the process of roadway supporting by roof-bolt setting on the surface of the roadway and deformation of contour rocks depending on the depth of the roadway location, we choose the point M_1(2.4 m; 100 m), in the neighborhood of which we will perform the representation of the TSI function. The intervals for h are (0.5 m–4.7 m) and for H are (50 m–200 m); when the defined parameters change in these intervals, the rock deformation of elastic nature occurs.

2. In order to determine the risks of the TSI going beyond the operational capability due to changes of parameters, it is necessary to have the TSI distribution within

the defined intervals for their change. A set of the TSI values for the parameter changes will allow their approximation. There are no restrictions on the procedures for their determination and numerical methods to be used.

3. Then we perform the procedure for determining sensitivity of the TSI function due to the changes of the chosen parameters. After this we define the TSI function in the specified intervals and define their approximation functions $\tau(h, H_0)$ and $\tau(h_0, H)$, respectively.

4. The TSI function due to the parameter changes in chosen intervals are represented as a product of one-dimensional representations, as stated in the formulated hypothesis.

5. Next, according to the given hypothesis, we determine the risks of roof-bolt setting for two situations:

a) We determine the risk of the TSI criterion deviation from the permissible one in the neighborhood of the roof bolt when the roof bolt is not set to full depth, namely 2.1 m .

b) We determine the risk of the TSI criterion deviation from the permissible one in the neighborhood of the roadway in the case of using the chart of roof-bolt setting to the depth of 2.4 m, but with depth 200 m for the roadway location.

6. Then we determine the risk of deviation of the criterion beyond the permissible limits for the case when parameters *h* and *H* change simultaneously, so that h_1=3.3m, H_1=200m.

7. After this, we analyze the obtained results and draw the conclusions.

4. Results and discussion

According to the hypothesis and the algorithm for it, we find the approximation functions $\tau_i(h, H_0)$ and $\tau_i(H, h_0)$, where *h* is the distance to the depth from the roadway surface, *H* is the depth of the roadway location, and parameters H_0 and h_0 are fixed at a point M_1(2.4 m; 100 m).

Let's study the change of the TSI criterion in the intervals *h*(0.5m; 4.7m) and *H*(50m, 200m) and find the approximation functions (fig. 2, 3).

The chosen approximation functions have the form of power functions:

$$\tau(h, H_0) = \frac{0.15227}{h^{0.43}} \quad (3)$$

and

$$\tau(h_0, H) = \frac{0.382}{H^{1.110^{-11}}}. \quad (4)$$

Having the expressions for TSI in the form of power functions, it is easy to determine the sensitivity of the function to changes of the *h* and *H* parameters by way of comparing the power indicators.

In case of simultaneous change of the chosen parameters, the TSI function can be defined by formulas (3) and (4), which were used to determine the sensitivity of the TSI to parameter changes, namely:

$$\tau(h,H) = A\,\tau(h,H_0)\tau(h_0,H) \text{ or } \tau(h,H) = A\frac{0.15227}{h^{0.43}}\frac{0.382}{H^{1.110^{-11}}}. \qquad (5)$$

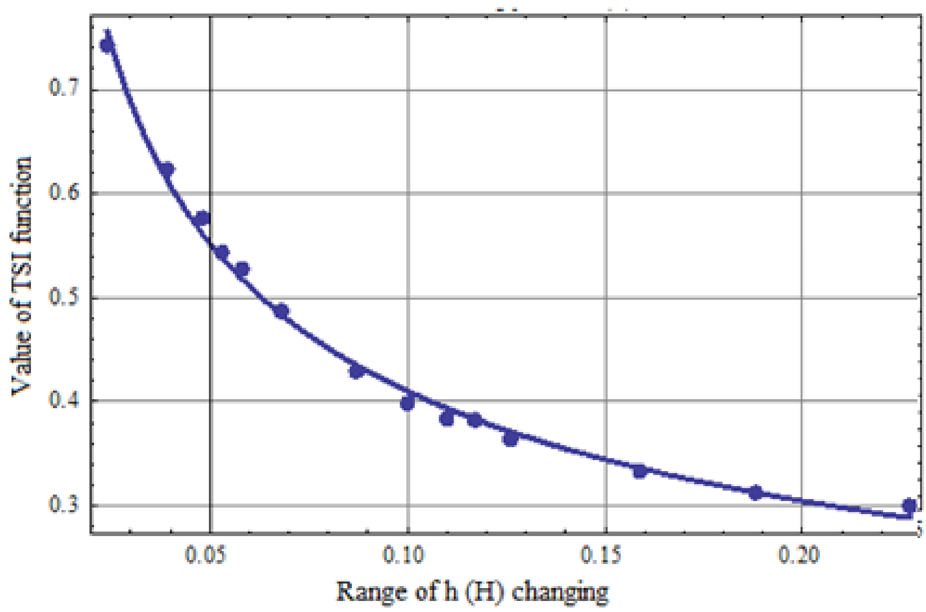


Figure 2 – Dependence of the TSI on sinking depth *h*

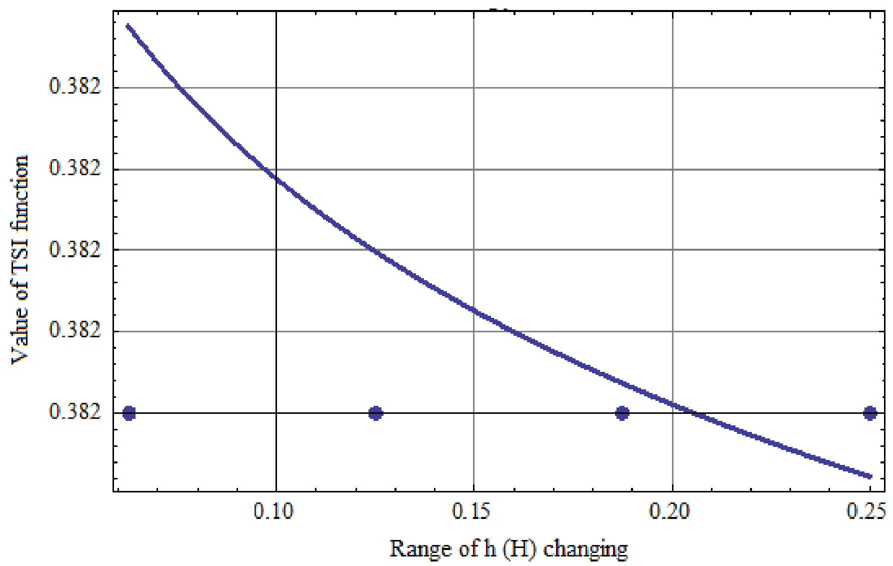


Figure 3 – Dependence of the TSI on the depth of roadway location *H*

Since parameters of the problem are geometric, the problem of the location of the roof-bolt lock can be formulated as follows:

Suppose that at a depth of 100 m, the lock of the metal-polymer roof bolt was sunk in the massif only to the depth of h_1= 2.1 m, while the TSI, as it is demonstrated by the graphic dependence, is presented in fig. 1 as equal to $\tau(h_1, H_0)$ = 0.398.

The lock of the metal-polymer roof bolt can be located only at the depth where strong rock or a layer of rocks are found. Thus, for a sinking depth of 2.4 m, the permissible level of tangential stress intensity is equal to $\tau(h_0, H_0)$ = 0.382

Let's calculate the risk of the criterion change, which will arise for the parameter h under the condition that the other parameter H is fixed at the point M_1(2.4 m; 100 m), i.e. $H=H_0$.

We define that:

$$R_1 = \frac{\tau(h_0,H_0) - \tau(h_1,H_0)}{\tau(h_0,H_0)} 100\% \text{ or } R_h = \left(\frac{0.382 - 0.398}{0.382}\right) 100\% = 4.2\%. \quad (6)$$

For the location of the lock at a depth of 2.4 m, and for the depth of the roadway location H_1=200 m, we have $\tau(h_0, H_1)$=0.383. Let's find the risk of deviation of the TSI criterion due to this change.

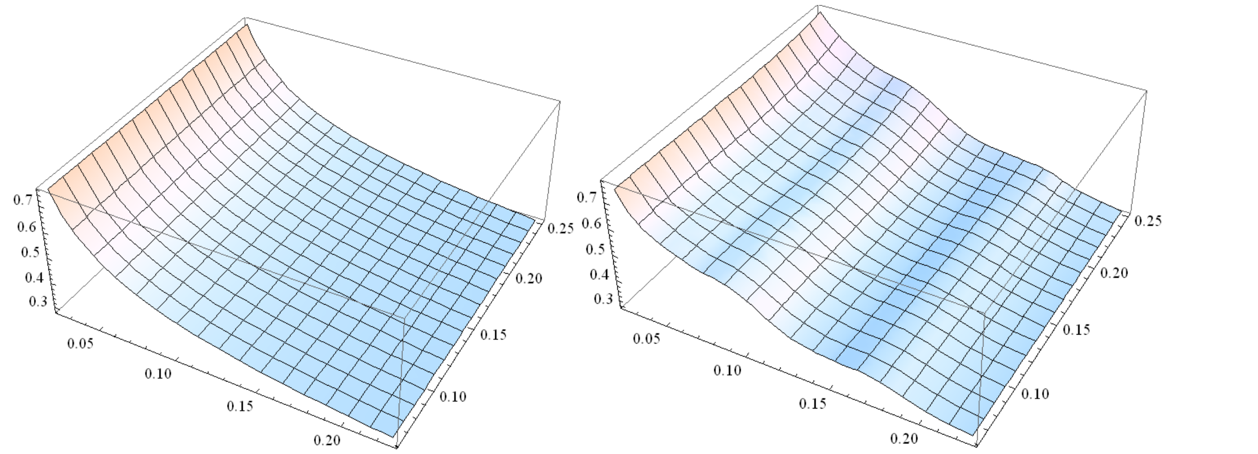
Noteworthy is graphic inconsistency of the presented approximation. The explanation of this fact should be sought in the engineering approach to obtaining a solution to the problem. It should be mentioned that range of the criterion change is very small, and this makes it possible to choose as an approximation function a function that is convenient for the engineer, in our case we chose a power function.

Let's calculate the risk of criterion change, which will arise for the parameter H under the condition of fixing the other parameter h at the point M_1(2.4 m; 100 m), i.e. when $h=h_0$.

From formula (4), we find that $\tau(h_0, H_1)$=0.383, then using formula (1) we get:

$$R_H = \frac{\tau(h_0,H_0) - \tau(h_0,H_1)}{\tau(h_0,H_0)} 100\% \text{ or } R_H = \left(\frac{0.383 - 0{,}382}{0.382}\right) 100\% = 0.26\% \quad (7)$$

Thus, knowing the dependence of the chosen criterion on the parameters, we can easily determine the risks caused by their change (deviation). If the number of parameters is more than two, the table construction procedure becomes more complicated. In addition, sometimes the construction of such a table requires considerable time. As for the errors in the determination of the criterion by the obtained formulas in comparison with those obtained by numerical procedures, a graphical representation of the comparison is given below. For clarity, figures 4 and 5 represent three-dimensional images of the TSI function: fig. 4,a – the image of the obtained approximate representation in the form of a product of two functions; 3, b – representation of the interpolation surface of the TSI and fig. 5 – superimposed representation of two surfaces.



a) b)

Figure 4 – Representation of surfaces a) the surface is represented by the product of two functions; b) the TSI interpolation surface;

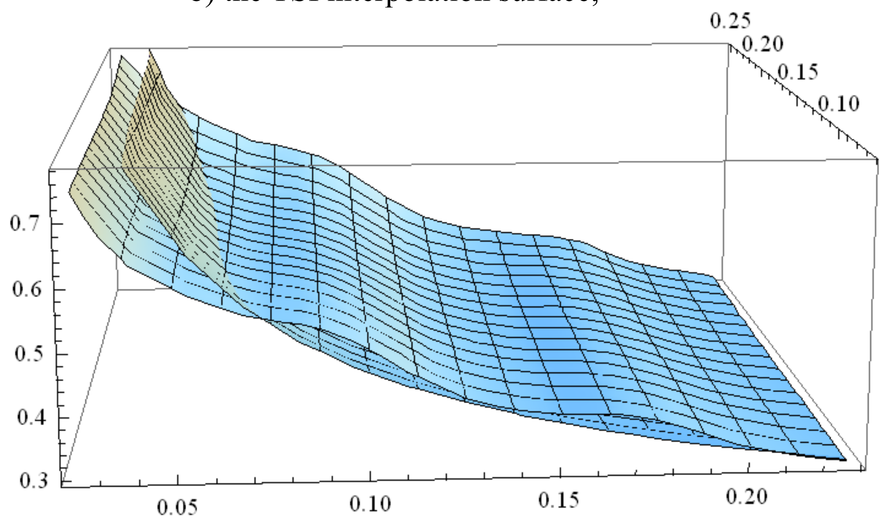


Figure 5 – The superimposed representation of the interpolation and obtained surfaces

The above examples of determining the risks of deviations of the criterion from the permissible values are based on the fact that when determining the risk of the criterion deviation caused by a change in one parameter, the other parameter remains fixed. In practice, such cases are not frequent.

Let's consider the variant of determining the risk of criterion deviations from the permissible value in the case when two parameters feature deviation simultaneously.

For this case, the formula for determining risks is formulated above, in the hypothesis. In practice, the formula for it is obtained as follows.

Then, in order to obtain the formula for representing the criterion (in our case, it is the TSI) in multiplicative form with simultaneous changes of two parameters, it is enough to make their product (5):

$$\tau(h,H)=A\,\tau(h,H_0)\tau(h_0,H) \text{ or } \tau(h,H)=A\frac{0.15227}{h^{0.43}}\frac{0.382}{H^{1.110^{-11}}},$$

where A is the approximation coefficient or multiplier and is determined according to the hypothesis, namely:

$$A = \frac{\tau(h_0, H_0)}{\tau(h, H_0)\tau(h_0, H)}. \quad (8)$$

For our case, that is, for neighborhood of M_1(2.4m; 100m), we have: $A_1=2.61$.

Since the approximation functions of the dependence of the TSI criterion on the parameters are power functions, it is possible to use the procedure of rapid determination of degree of the influence of criterion change due to the parameter. To do this, it is enough to estimate the indicator of its degree and determine the descending sequence of the values of their influence on the chosen criterion. This sequence will be decisive for determining the risks of criterion due to the parameters and for focusing on observing the conditions of manufacture and installation technology in the support setting project.

Let's formulate the problem of defining risks under the condition of simultaneous changes of the parameters of depth of the roadway location H and the depth of sinking into the depth from the roadway surface h.

We have already determined that $\tau(h_0, H_0)$=0.382 for the location of the lock at a depth of h_0=2.4 m and the depth of the roadway location H_0=100 m. Let's define the risk of the TSI criterion deviation due to the simultaneous change of parameters, for example, when h_0=3.3m, H_0=200m. Thus, for the sinking depth of 2.4 m, the permissible level of tangential stress intensity is equal to $\tau(h_0, H_0)$=0.382. Let's define the risk of criterion changing due to the simultaneous change of parameters (5):

$$\tau(h, H) = A\frac{0.15227}{h^{0.43}}\frac{0.382}{H^{1.110^{-11}}}.$$

For our case, that is, for M_1 (2.4m; 100m), we have: A_1=2.61

With the new parameter values h_2=3.3m, H_2=200m, we have $\tau(h_2, H_2) = 0.342$. Then, according to formula (1), we obtain:

$$R_{H,h} = \frac{\tau(h_0, H_0) - \tau(h_2, H_2)}{\tau(h_0, H_0)}100\% \text{ or } R_{H,h} = \left(\frac{0.382 - 0{,}342}{0.382}\right)100\% = 10.47\% \quad (9)$$

The dependence of the relative errors of the function values $\tau_i(h, H)$ on the curves formed by the intersection of the surface of the product function, in its multiplicative representation, with the corresponding planes did not exceed 1%, which is sufficient for engineering practice to solve the problems of geotechnical mechanics.

The results presented in fig. 4, a, b, demonstrate the superimposition of the interpolation surface and the surface represented by the product of two functions, each of which is dependent on only one parameter. The TSI function in the neighborhood of the point M_1(2.4 m; 100 m) or in the dimensionless form of the point M_1(0.117; 0.125) is represented by formula (5):

$$\tau(h,H)=A\frac{0.15227}{h^{0.43}}\frac{0.382}{H^{1.110^{-11}}}.$$

It can be asserted that this function for TSI is valid for the ranges of parameter changes within the intervals of (1.4–4.7 m) for *h* and (50–200 m) for *H,* and the error of the approximation curves did not exceed 1%.

The superimposition of surfaces presented in fig. 5 demonstrates the qualitative nature of the location of the surface obtained by the given formula, in comparison with the interpolation surface. Deviations of the surface obtained by the obtained formula in comparison with the interpolation surface are generally insignificant. However, at the boundaries of the intervals, changes of the surface deviation parameters are significantly greater (fig. 5). This circumstance requires shortening of these intervals in order to avoid great errors. Thus, the interval of changes for the depth of sinking into the rock mass from the roadway surface *h* (0.5–4.7 m) should be shortened to (1.4–4.7 m) to ensure the required accuracy of 1%.

Regarding the risks of the system exceeding the permissible limit, the following should be mentioned:

The analysis of the TSI sensitivity according to formulas (3) and (4) showed that the main and most important parameter, which significantly affects the risk of the TSI going beyond the operational capability and can lead to an emergency situation, is parameter *h*. The depth of the roadway location *H* in the chosen interval (50–200 m) does not significantly affect the risk of the TSI going beyond the operational capability. Therefore, during the technological processes of drilling holes for roof bolts, all attention should be focused on their careful observance.

These results make it possible to obtain an analytical representation of the risks of the criteria exceeding the permissible limits as a function of the chosen parameters in the neighborhood of the chosen point. In the problems of geotechnical mechanics, where the initial data are obtained with an error of up to 20%, a relative error of 5–7% does not seem like something significant, and it is sufficient for engineering calculations. In the event of an unsatisfactory error, which reaches its maximum at the boundary of the definition area, it is necessary to improve it by shortening the intervals of the parameter changes. Unfortunately, the relative errors of approximation of the curves obtained by the intersection of the TSI surface do not fully characterize the maximum error of such representation. That is, approximations $\tau_i(h,H_0);\tau_i(h_0,H)$ are not decisive for reducing the neighborhoods of the chosen points. The areas between them, i.e. $\tau_i(h,H)$, are not the subject of this study.

Despite the fact that a significant number of problems in various fields of technical sciences were solved by using the above mentioned data, they are based only on the hypothesis of the existence of representation of function at a point in a multiplicative form. It would probably be expedient to prove the theorem about the existence of the representation of functions in neighborhood of the point in multiplicative form. Researches in this direction are in process.

5. Conclusions

1. The obtained results clearly demonstrate the effectiveness of the SAM use to determine the risks of deviation of the chosen criterion due to changes of its parameters in the neighborhood of the roadway of a circular cross-section located at a certain depth.

2. It was established that relative errors increase with approaching the boundary of the definition area. Thus, it is obvious from the fig. 5 that at the boundary of the definition area, the error shows unsatisfactory values and it is necessary to shorten the interval of change of the parameter h, that is, for the distance from the roadway surface into the depth, to shorten it from the value of 0.24 in dimensionless coordinates, that is, for the value of 0.5m. Therefore, the formula for TSI will be valid only for the interval from 1.4 m to 4.7 m instead of the interval from 0.5 m to 4.7 m.

3. Choosing the point provides results with a minimum relative error in the neighborhood of the chosen point.

4. In case of an unsatisfactory error of the obtained results, it is necessary to reduce the area of the function definition by shortening the interval of parameter change.

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About the authors

Krukovskyi Oleksandr Petrovych, Corresponding Member of the National Academy of Sciences of Ukraine, Doctor of Technical Sciences, Deputy Director, M.S. Poliakov Institute of Geotechnical Mechanics of the National Academy of Sciences of Ukraine (IGTM of the NAS of Ukraine), Dnipro, Ukraine, igtm@ukr.net

Larionov Hryhorii Ivanovich, Doctor of Technical Sciences (D. Sc), Senior Researcher in Rock Mechanics Department, Senior Researcher, M.S. Poliakov Institute of Geotechnical Mechanics of the National Academy of Sciences of Ukraine (IGTM of the NAS of Ukraine), Dnipro, Ukraine, larionovgrigoriydtn@gmail.com.

Zemliana Yuliia Valeriivna, Master of Science, Chief Technologist in Rock Mechanics Department, M.S. Poliakov Institute of Geotechnical Mechanics of the National Academy of Sciences of Ukraine (IGTM of the NAS of Ukraine), Dnipro, Ukraine, zemlanaaulia19@gmail.com.

Khvorostian Viktor Oleksiiovych, Junior Researcher in Rock Mechanics Department, M.S. Poliakov Institute of Geotechnical Mechanics of the National Academy of Sciences of Ukraine (IGTM of the NAS of Ukraine), Dnipro, Ukraine

Holovko Sofia Askhativna, Junior Researcher in Rock Mechanics Department, M.S. Poliakov Institute of Geotechnical Mechanics of the National Academy of Sciences of Ukraine (IGTM of the NAS of Ukraine), Dnipro, Ukraine, sofyag16@gmail.com.

ЗАСТОСУВАННЯ МЕТОДУ ПОСЛІДОВНОЇ АПРОКСИМАЦІЇ ДО ВИЗНАЧЕННЯ РИЗИКІВ В ЗАДАЧАХ ГЕОТЕХНІЧНОЇ МЕХАНІКИ

Круковський О.П., Ларіонов Г.І., Земляна Ю.В., Хворостян В.О., Головко С.А.

Анотація. Більшість процесів в технічних доробках є детермінованими, тому поняття ризику як добутку ймовірності виникнення аварії на фінансові затрати на її ліквідацію, яке пропонується у більшості статтях та нормативних документах, не є прийнятними, оскільки жоден з параметрів проекту не є випадковою величиною. В зв'язку з цим пропонується визначати ризик як вихід технічної системи за допустимі технічними умовами межі, тобто межі працездатності. Попередньо для визначення степені впливу параметрів на величину ризику необхідно виконати визначення чутливості до їх варіації. Аналіз чутливості дозволяє виділити найбільш впливові параметри на ризик виходу за межі працездатності. Проте визначення чутливості критерію до варіації того чи іншого параметру не завжди вдається забезпечити в практичній ситуації. Типовою ситуацією на практиці є проблема визначення ризику за умов одночасної зміни всіх. Таким чином, актуальним методом обчислення ризику був би метод який би одночасно дозволяв виявляти і чутливість ризику до варіації параметрів і обчислення його за одночасної зміни всіх параметрів. Метод послідовної апроксимації (МПА) дозволяє обчислювати ризик за одночасної зміни інших параметрів у певному діапазоні використовуючи інформацію отриману при визначенні чутливості ризику до варіації параметрів. Ризик у МПА подається у мультиплікативній формі, де складовими добутку є функції одного параметру. Якщо апроксимацію ризику здійснювати у вигляді добутку степеневих функцій, кожна з яких залежить лише від одного параметру, то за показниками степеню можна наближено встановити чутливість до варіації параметрів. Чим більший показник степеню, тим більший вплив на ризик має той параметр. Таким чином можна не тільки здійснити наближену оцінку впливу параметрів на сам критерій, але й зробити висновки стосовно важливості впливу на ризик виходу системи за допустимі межі.

У даній роботі працездатність МПА методу до визначення ризиків виходу параметрів за допустимі межі продемонстровано на результатах розв'язку класичної задачі про визначення напружено-деформованого стану у околі виробки кругового поперечного перерізу методом скінчених елементів. Представлено алгоритм обчислення ризиків на конкретних прикладах. Для наочності задовільної точності обчислень критерію виконано порівняння поверхонь функції інтенсивності дотичних напружень, отриманих за МПА методом, та поверхнями інтерполяції чисельних результатів Зроблено висновки про спроможність методу визначати ризики виходу критерію за допустимі межі та задовільну точність отриманих результатів.

Ключові слова: ризик втрати працездатності, мультиплікативна форма представлення, чутливість функції, варіація параметрів, окіл точки, інтенсивність дотичних напружень.