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MODIFICATION OF THE ANALYTIC HIERARCHY PROCESS IN MINING INDUSTRY

Abstract. The Analytic Hierarchy Process (AHP), proposed by American mathematician Thomas Saaty, is widely used in various fields, including applied science, engineering, sociology, healthcare, ecology, and others. Thousands of papers have been published. The AHP is successfully used in mining industry, including for risk analysis at underground coal mining. Risk is defined as the product of the probability of a hazard and the severity of its consequences. Difficulties in decision-making in the mining industry are related to the reliability of geological data, long-term investments, strict safety standards, and the need to comply with environmental regulations. It is worth noting the complexity of equipment selection, which requires a harmonious combination of high performance, reliability in extreme conditions, energy efficiency, and adherence to safety standards. The diversity of indicators affecting operational efficiency necessitates finding trade-offs between various criteria, leading to multicriteria selection problems. The traditional approach in the AHP does not take into account the spread of components when comparing global priorities for different alternatives, and this, given minor differences that are acceptable to the decision maker, can lead to an irrational choice. The purpose of this work is to take into account the spread of components of global priorities. The complexity of multicriteria decision making lies in the fact that, as a rule, there is no alternative that is immediately superior across all criteria. As a compromise, it is proposed to give preference to an alternative with a higher additional priority value, which comprehensively considers the components of global priorities, in the case of small deviations in global priorities. In developing the AHP, global priorities are compared based on the magnitude of the relative global priority. An example of decision making using a modified AHP is given. In the traditional AHP, the optimal alternative is selected based on global priorities obtained by linearly convolving local priorities. It is proposed to additionally consider the priorities of the components of global priorities. The priority of the components is a multiplicative convolution of the relative components of global priorities. The practical significance of this work lies in the fact that the modified method enables decision making with minimal dispersion of global priorities, which makes it possible to consider their comprehensive impact on achieving the stated goal. The advantage of the proposed modification is its comprehensive consideration of the influence of global priorities on the choice of alternatives. The disadvantage is the permissibility of reducing global relative priorities by an acceptable amount.

Keywords: decision making, goal, criteria, alternatives, paired comparisons, consistency, priorities, global, convolution, linear, multiplicative.

1. Introduction

The Analytic Hierarchy Process (AHP) proposed by the American mathematician Thomas Saaty [1] is widely used in various fields of activity: applied science, engineering, sociology, healthcare, ecology, etc. [2, 3]. Thousands of papers have been published [4].

The AHP is used in mining [5–7], including for risk analysis in underground coal mining [8]. Risk is defined as the product of the probability of a hazard and the severity of its consequences if it occurs [8–10]. Difficulties in decision-making in the mining industry are associated with the reliability of geological data, long-term investments, strict safety standards, and the need to comply with environmental regulations. It is worth noting the complexity of equipment selection, which requires a harmonious combination of high performance, reliability in extreme conditions, energy efficiency, and adherence to safety standards. The variety of indicators affecting operational efficiency necessitates finding compromise solutions between various criteria, which leads to multicriteria selection problems.

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With the traditional approach, when comparing global priorities in AHP for different alternatives, the spread of their components is not taken into account, and this, given small differences (the definition of smallness is given below), leads to an irrational decision.

In this regard, the purpose of this study is to account for the variation in the components of global priorities.

Object of the study: the variation in the components of global priorities.

2. Methods

The following methods were used in this study: an analytical review of literature sources; probability theory; and mathematical statistics. Internet resources were used to analyze the current state of the art.

To achieve the set purpose, the following **tasks** should be solved:

– to define the multiplicative convolution of local priorities;

– to determine the priorities of the components of global priorities.

3. Theoretical and experimental parts

The Analytic Hierarchy Process is described in detail in the works of Thomas Saaty, a list of which is available on the website [11].

The AHP algorithm is as follows.

It is given: purpose, m_k criteria $K_1, K_2, ..., K_{m_k}$ and n_d alternatives $D_1, D_2, ..., D_{n_d}$.

A matrix of pairwise comparisons for all criteria is formed

$$\mathbf{K} = \begin{pmatrix} k_{11} & k_{12} & ... & k_{1m_k} \\ k_{21} & k_{22} & ... & k_{2m_k} \\ ... & ... & ... & ... \\ k_{m_k 1} & k_{m_k 2} & ... & k_{m_k m_k} \end{pmatrix}, \quad (1)$$

where k_{ij} – a positive number indicating how many times the criterion K_i is preferable (in the Saaty scale) to criterion K_j. The pairwise comparison matrix is diagonal and inversely symmetric.

The normalized eigenvector of the matrix $\mathbf{K}$ (vector of local priorities) is calculated

$$W_i = \sqrt[m_k]{\prod_{j=1}^{m_k} k_{ij}} \Bigg/ \sum_{i=1}^{m_k} \sqrt[m_k]{\prod_{j=1}^{m_k} k_{ij}} . \quad (2)$$

Then the maximum eigenvalue is determined

$$\lambda_{\max} = \sum_{j=1}^{m_k} \sum_{i=1}^{m_k} k_{ij} W_j \,. \tag{3}$$

To check the consistency of the matrix, the consistency index is calculated

$$I_c = \frac{\lambda_{\max} - 1}{m_k - 1} \tag{4}$$

and the consistency relation

$$O_c = \frac{I_c}{I_y}\,, \tag{5}$$

where I_y – the average value of the consistency index of a random matrix of pairwise comparisons, which is taken from the table [12].

If $O_c \le 0{,}1$ the matrix is consistent, then $0.1 < O_c \le 0.2$ The consistency is considered acceptable [12]. If the consistency conditions are not met, the pairwise comparison matrix should be adjusted.

For each criterion, a pairwise comparison of alternatives is performed. As a result, we obtain *k* matrices

$$\mathbf{D}^k = \begin{pmatrix} d_{11}^k & d_{12}^k & \dots & d_{1n}^k \\ d_{21}^k & d_{22}^k & \dots & d_{2n}^k \\ \dots & \dots & \dots & \dots \\ d_{n_d 1}^k & dn_{dn2}^k & \dots & d_{n_d n_d}^k \end{pmatrix}. \tag{6}$$

Here k – a superscript indicating the number of the criterion to which the value with this index relates.

After this the normalized eigenvectors of matrices $\mathbf{D}^k$ (vectors of local priorities) are calculated

$$W_i^k = \sqrt[n_d]{\prod_{j=1}^{n_d} d_{ij}} \Bigg/ \sum_{i=1}^{n_d} \sqrt[n_d]{\prod_{j=1}^{n_d} d_{ij}} \,. \tag{7}$$

Then the maximum eigenvalue is determined

$$\lambda_{\max}^k = \sum_{j=1}^{n_d} \sum_{i=1}^{n_d} d_{ij} W_j^k \,. \tag{8}$$

The index and consistency ratio is calculated

$$I_c^k = \frac{\lambda_{\max}^k - 1}{n_d - 1}, \quad O_c^k = \frac{I_c^k}{I_y}. \tag{9}$$

Global priorities are determined

$$G_i = \sum_{j=1,k=j}^{m_k} W_{ij}^k W_j , \quad i = 1, 2, ..., n \ . \tag{10}$$

The best alternative is the one with the highest global priority, which we will denote as $G_{\max}$.

The complexity of multi-criteria decision-making lies in the fact that, as a rule, there is no alternative that is immediately superior across all criteria.

As a compromise, it is proposed that, for small deviations in global priorities, preference be given to an alternative with a higher additional priority value that comprehensively takes into account the components of the global priorities. Let's consider the implementation of this approach.

In developing the AHP, we will compare global priorities based on the magnitude of the relative global priority.

$$G_{i,rel} = \frac{G_i}{G_{\max}} . \tag{11}$$

The maximum value $G_{i,rel}$ is one, so the deviation $G_{i,rel}$ from the maximum possible value is

$$\varepsilon_{i,rel} = 1 - G_{i,rel} . \tag{12}$$

Let the decision maker (DM) accept a deviation of the value δ_G, that is

$$\varepsilon_{i,rel} \le \delta_G . \tag{13}$$

It is in this sense that a small deviation from the maximum value of global priority is understood.

By denoting that $z_{ij}^k = W_{ij}^k W_j$, we will represent formula (10) in the following form

$$G_i = \sum_{j=1,k=j}^{m_k} z_{ij}^k . \tag{14}$$

The relationships between the components of the *i*-th alternative will be characterized by relative components

$$z_{ij,rel}^k = \frac{z_{ij}^k}{\max(z_{ij}^k)} . \tag{15}$$

To take into account the complex influence $z_{ij,rel}^{k}$ on global priority, we perform a multiplicative convolution [13]

$$Z_i = \prod_{j=1,k=j}^{m_k} z_{ij,rel}^{k}. \tag{16}$$

We'll call this convolution the priority of the components.

To solve a multi-criteria problem, we need to select an alternative that best takes each criterion into account, and this is achieved by maximizing the priority of the components.

Three priority-based situations are possible.

First, one of the alternatives has the highest global and component priorities. This alternative is the best.

Second, one of the alternatives has the highest global priority, and there is also a second alternative that is not much different from the first $\varepsilon_{i,rel} \le \delta_G$. If the second alternative has a higher component priority than the first, then it is preferred.

Third, there are several alternatives that satisfy the conditions of the second situation. In this case, preference should be given to the alternative with the highest component priority.

Example.

Let's consider a model example of using the advanced Analytic Hierarchy Process. A mining company needs to purchase an excavator using the following criteria: K_1– productivity; K_2– reliability; K_3– price. Alternatives: D_1, D_2, D_3– accordingly the models of excavators conventionally designated as 1, 2, 3. Permissible deviation is $\delta_G = 0.1$. The hierarchical structure is shown in Fig.1.

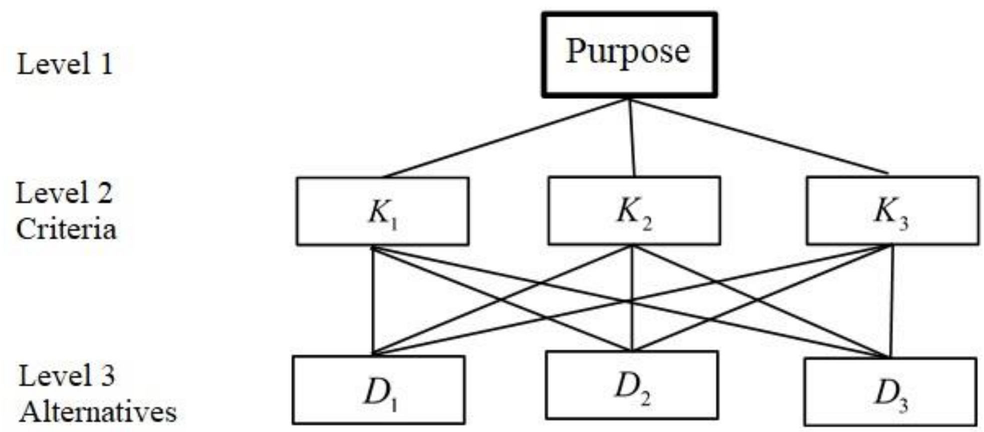

Figure 1 – Hierarchical representation of the excavator acquisition task

Let us assume that as a result of expert assessments the following matrices of pairwise comparison of criteria and alternatives were obtained:

$$\mathbf{K} = \begin{pmatrix} 1 & 3 & 7 \\ 1/3 & 1 & 3 \\ 1/7 & 1/3 & 1 \end{pmatrix}, \quad \mathbf{D}^1 = \begin{pmatrix} 1 & 1/2 & 7 \\ 2 & 1 & 7 \\ 1/7 & 1/7 & 1 \end{pmatrix},$$

$$\mathbf{D}^2 = \begin{pmatrix} 1 & 2 & 7 \\ 1/2 & 1 & 6 \\ 1/7 & 1/6 & 1 \end{pmatrix}, \quad \mathbf{D}^3 = \begin{pmatrix} 1 & 4 & 5 \\ 1/4 & 1 & 3 \\ 1/5 & 1/3 & 1 \end{pmatrix}.$$

The results of calculations using formulas (1–9) are presented in Table 1.

Table 1 – Local priority vectors

	Matrices			
	K	D^1	D^2	D^3
Vector of local priorities	0.669 0.243 0.088	0.361 0.574 0.065	0.582 0.348 0.069	0.674 0.226 0.101
The consistency relation	0.006 < 0.1	0046 < 0.1	0.028 < 0.1	0.074 < 0.1

Global priorities are calculated using the formula (10)

$$G_1 = 0.361 \cdot 0.669 + 0.582 \cdot 0.243 + 0.674 \cdot 0.088 =$$
$$= 0.24151 + 0.14143 + 0.05931 = 0.44225;$$
$$G_2 = 0.574 \cdot 0.669 + 0.348 \cdot 0.243 + 0.226 \cdot 0.088 =$$
$$= 0.38401 + 0.08456 + 0.01989 = \mathbf{0.48846};$$
$$G_3 = 0.065 \cdot 0.669 + 0.069 \cdot 0.243 + 0.101 \cdot 0.088 =$$
$$= 0.04349 + 0.01677 + 0.00889 = 0.06914.$$

Maximum global priority is G_2 (it is highlighted in bold), therefore, in accordance with the traditional AHP, the optimal alternative is D_2. Let's calculate relative global priorities (11)

$$G_{1,rel} = 0.905, \quad G_{2,rel} = 1, \quad G_{3,rel} = 0.142$$

and deviations (13)

$$\varepsilon_{1,rel} = 0.095, \quad \varepsilon_{2,rel} = 0, \quad \varepsilon_{3,rel} = 0.858.$$

Using formulas (15, 16), we determine the relative components of alternatives and their priorities.

$$z^1_{11,rel} = 1\ , \quad z^2_{12,rel} = 0.586, \quad z^3_{13,rel} = 0.246,$$
$$z^1_{21,rel} = 1, \quad z^2_{22,rel} = 0.220, \quad z^3_{23,rel} = 0.052,$$
$$z^1_{31,rel} = 1, \quad z^2_{32,rel} = 0.386, \quad z^3_{33,rel} = 0.204,$$
$$Z_1 = 0.1442, \quad Z_2 = 0.0114, \quad Z_3 = 0.0787.$$

Since $\varepsilon_{1,rel} < 0.1$, then the comparison of the priorities of the components for the alternatives D_1 and D_2 gave the following result

$$(Z_1 = 0.1442) > (Z_2 = 0.0114).$$

Thus, taking into account the additional priority of the components, preference should be given to the first alternative D_1.

The practical significance of the work lies in the fact that the modification of the method allows for decision-making with a minimal spread of global priorities, which makes it possible to take into account their complex impact on achieving the set purpose.

4. Conclusions

The Analytic Hierarchy Process, proposed by American mathematician Thomas Saaty is widely used in various fields: applied science, engineering, sociology, healthcare, ecology, and others.

The AHP is successfully used in mining, including for risk analysis at underground coal mining. Risk is defined as the product of the probability of a hazard and the severity of its consequences. Difficulties in decision-making in the mining industry include the reliability of geological data, long-term investment, strict safety standards, and the need to comply with environmental regulations. Equipment selection is challenging, requiring a harmonious balance of high performance, reliability under extreme conditions, energy efficiency, and adherence to safety standards.

The diversity of indicators affecting performance necessitates finding trade-offs between various criteria, leading to multi-criteria selection problems. The traditional approach, when comparing global priorities in the AHP for different alternatives, fails to take into account the variance of their components. However, the minor differences, even being acceptable for the decision maker, can lead to irrational choices.

The complexity of multi-criteria decision-making lies in the fact that, as a rule, there is no alternative that is immediately superior across all criteria. As a compromise, it is proposed that, for small deviations in global priorities, preference be given to the alternative with a higher additional priority value that comprehensively considers the components of the global priorities.

In developing the AHP, global priorities are compared based on the magnitude of the relative global priority.

An example of decision making using a modified AHP is provided.

In the traditional AHP, the optimal alternative is selected based on global priorities obtained through a linear convolution of local priorities.

It is proposed to additionally consider the priorities of the components of global priorities.

The priority of the components is the multiplicative convolution of the relative components of global priorities.

The practical significance of this work lies in the fact that the modified method enables decision-making with minimal variation in global priorities, making it possible to consider their complex impact on achieving the stated goal.

The advantage of the proposed modification is its comprehensive consideration of the influence of global priorities on the choice of alternatives. Its disadvantage is the permissibility of reducing global relative priorities by an acceptable amount.

The obtained results will be used for decision-making in mining industry.

Conflict of interest

Authors state no conflict of interest.

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МОДИФІКАЦІЯ МЕТОДУ АНАЛІЗУ ІЄРАРХІЙ У ГІРНИЧІЙ СПРАВІ

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Анотація. Метод аналізу ієрархій (МАІ), запропонований американським математиком Томасом Сааті, широко застосовується в різних сферах діяльності: прикладна наука, інженерія, соціологія, охорона здоров'я, екологія тощо. Опубліковано тисячі робіт. МАІ успішно використовується в гірничій справі, зокрема для аналізу ризиків при шахтному видобутку вугілля. Під ризиком розуміється добуток ймовірності небезпеки на серйозність наслідків у разі її реалізації. Проблеми прийняття рішень у гірничодобувній галузі пов'язані з достовірністю геологічних даних, довгостроковістю інвестицій, жорсткими стандартами техніки безпеки та необхідністю дотримання екологічних норм. Слід зазначити складність вибору обладнання, яке потребує гармонійного поєднання високої продуктивності, надійності в екстремальних умовах, енергоефективності та відповідності стандартам безпеки. Різноманітність показників, які впливають ефективність діяльності, зумовлює необхідність пошуку компромісних рішень між різними критеріями, що призводить до виникнення багатокритеріальних завдань вибору. При традиційному підході до МАІ при порівнянні глобальних пріоритетів для різних альтернатив не враховується розкид їх складових, а це при незначних відмінностях, які є допустимими для особи, яка приймає рішення, може призводити до ірраціонального вибору. Мета – облік розкиду складових глобальних пріоритетів. Складність багатокритеріального прийняття рішення у тому, що, зазвичай, немає альтернативи відразу кращої за всіма критеріями. Як компроміс пропонується при малих відхиленнях глобальних пріоритетів віддавати перевагу альтернативі з великим значенням додаткового пріоритету, комплексно враховує складові глобальних пріоритетів. У розвиток МАІ порівняння світових пріоритетів виконано за величиною відносного глобального пріоритету. Наведено приклад ухвалення рішення за допомогою модифікованого МАІ. У традиційному МАІ вибір оптимальної альтернативи проводиться на основі глобальних пріоритетів, отриманих у результаті лінійного згортання локальних пріоритетів. Запропоновано додатково враховувати пріоритети складових глобальних пріоритетів. Пріоритет складових – мультиплікативний пакунок відносних складових глобальних пріоритетів. Прикладне значення роботи полягає в тому, що модифікація методу дозволяє прийняти рішення з мінімальним розкидом глобальних пріоритетів, що дає можливість враховувати їх комплексний вплив на досягнення поставленої мети. Перевага запропонованої модифікації – комплексний облік впливу глобальних пріоритетів на вибір альтернатив. Недолік – допустимість зменшення глобальних відносних пріоритетів прийнятну величину.

Ключові слова: ухвалення рішення, мета, критерії, альтернативи, парні порівняння, узгодженість, пріоритети, глобальні, згортка, лінійна, мультиплікативна.