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NEW METHOD FOR GENERATING TEST CODES TO DETECT MULTIPLE STUCK-AT-FAULTS IN COMBINATIONAL CIRCUITS[1]. PART 1

The article is devoted to a new method of generating test codes to detect multiple damages in digital combinational circuits, which is based on the artificial introduction of nonessential variables and the application of the procedure of q-partition of minterms of a given function. Due to the use of a numerical set-theoretic approach, the proposed method differs from the known ones in a relatively simpler practical implementation to detect stuck-at-faults (0/1) type both at one point and at several points simultaneously of the circuit under study.

Keywords: *ccombinational circuit, single and multiple stuck-at-faults (0/1) type damage, q-partition of minterms, nonessential variables, vector of test codes.*

Introduction

In [1, 2], a method for detecting stuck-at-faults (0/1) at any single point of a combinational circuit is proposed by determining (generating) vectors of test codes. This method is based on a set-theoretical approach, using simple procedures and operations on binary minterms of a given function [1] or system minterms of a given system of functions [2] describing the operation of the studied circuit.

[1] This article is a development of a topic published in the author's previous articles [1, 2].

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In the practice of microcircuit design, there are often situations where stuck-at-faults (0/1) type damages can occur not only at a single point in the circuit, but also at several different points in the circuit simultaneously, which may be interconnected. Such damage is more difficult to detect due to the random nature of their occurrence, and in addition, their binary values may differ. Determining them by pointwise search is not a convincing proof of the reliability of the obtained result, which consequently reduces the reliability of the design process.

Known methods for detecting multiple stuck-at-faults (0/1) at several points in a combinational circuit [3–12] are mainly based on modeling single errors to find the minimum number of test vector sets. However, this approach is quite cumbersome, as it requires the use of additional procedures, since the problem of detecting such faults is NP-complete with a complexity of at least $O(2^n)$ for n errors. To solve this complexity, in [6] it was proposed to use the SAT Solver algorithm to generate test sets in combinational circuits with several errors, that also complicates the problem.

Among the approaches to the diagnostics of digital circuit design for detecting multiple errors, in addition to modeling, symbolic methods are also known [3, 4]. However, they do not rely on test vectors, but usually use ordered binary solution diagrams (OBBD) to characterize the necessary and sufficient conditions of a potential error source as a Boolean function. Although symbolic approaches are more accurate than modeling approaches, the strong increase in the complexity of their implementation imposes practical limitations for detecting the above-mentioned errors during the design of digital circuits.

This article is devoted to a new method of generating test code vectors to detect stuck-at-faults (0/1) type damages both at one point and at several different points of a combinational circuit simultaneously. The method is based on the idea of artificially introducing into the Boolean space a completely specified function $f(x_1, x_2, ..., x_{\ddot{i}})$, which describes the operation of the studied circuit, a certain number (but not n) of a nonessential variables and applying the q-partition procedure of the minterms of the perfect STF Y^1 function f [12, 13]. Unlike the known methods of generating test codes, the proposed approach is relatively simpler in terms of practical implementation.

Theoretical Part

It is generally known that a Boolean function $f(x_1, x_2, ..., x_{\ddot{i}})$ significantly depends on the variable x_i if there are such values $\sigma_1, ..., \sigma_{i-1}, \sigma_{i+1}, ..., \sigma_n$ of the variables $x_1, ..., x_{i-1}, x_{i+1}, ..., x_n$ that

$$f(\sigma_1, ..., \sigma_{i-1}, 0, \sigma_{i+1}, ..., \sigma_n) \neq f(\sigma_1, ..., \sigma_{i-1}, 1, \sigma_{i+1}, ..., \sigma_n). \quad (1)$$

If condition (1) is not met, then the variable x_i is nonessential for the function *f*.

Instead, to transform an essential variable x_i of the function f into a nonessential one, which is assumed by the proposed method, it is necessary to replace the value σ_i with the opposite one in the domain of definition of the function f, that is, in the set 2^n of sets $\mathbf{E}_2^n = \{\langle \sigma_1, ..., \sigma_i, ..., \sigma_n \rangle : \sigma_i \in \{0,1\}\}$ of its variables. Then, the Boolean space of the function f will be divided into two equivalent subspaces, and the newly created function will differ from the original one in values on certain sets of variables. Accordingly, if r "nonessential" variables are simultaneously introduced, the Boolean space will be divided into 2^r equivalent subspaces, and those sets of variables, on which the values of the function f have changed, are precisely the desired vectors of test codes, which will allow to recognize the type and the location of stuck-at-faults (0/1) damage in the studied scheme.

Table 1 illustrates the changes in the values $f_0, f_1, ..., f_7$ (3^{rd} column) of the conditional function $f(x_1, x_2, x_3)$ in various intervention situations: when one "nonessential" variable is introduced, for example x_1 (4th and 5th columns) and x_2 (6th and 7th columns), and when the same variables are introduced simultaneously as "nonessentials" ones (8th, 9th, 10th and 11th columns).

In Table 2, this case is illustrated by an example of a function $f(x_1, x_2, x_3)$ that has a perfect STF $Y^1 = \{(001), (101), (110), (111)\}^1$ (*see* Example 1 in [1]). For the same "nonessential" variables x_1 and x_2, the elements of the pseudo-perfect STF differ from the elements of the perfect STF Y^1, which are highlighted here in bold. For example (*see* the 4th column), a function $f_{x_1/0}$ corrupted by an "nonessential" variable $x_1/0$ has a pseudoperfect STF $Y^1_{x_1/0} = \{(001), (101)\}^1$, where two elements on the sets of variables (110) and (111), which are vectors of test codes, are highlighted in bold.

In Table 2, the numbers in the 9th (log. 0) and 11th (log. 1) columns, highlighted in square frames, show that these test code vectors cannot be determined separately for the "nonessential" variables x_1 and x_2. Since such vectors are not present in the "damaged" functions $f_{x_1/\sim}$ and $f_{x_2/\sim}$,

Table 1

"10"	$x_1\,x_2\,x_3$	f	$f_{x_1/0}$	$f_{x_1/1}$	$f_{x_2/0}$	$f_{x_2/1}$	$f_{x_1x_2/00}$	$f_{x_1x_2/01}$	$f_{x_1x_2/10}$	$f_{x_1x_2/11}$
0	0 0 0	f_0	f_0	f_4	f_0	f_2	f_0	f_2	f_4	f_6
1	0 0 1	f_1	f_1	f_5	f_1	f_3	f_1	f_3	f_5	f_7
2	0 1 0	f_2	f_2	f_6	f_0	f_2	f_0	f_2	f_4	f_6
3	0 1 1	f_3	f_3	f_7	f_1	f_3	f_1	f_3	f_5	f_7
4	1 0 0	f_4	f_0	f_4	f_4	f_6	f_0	f_2	f_4	f_6
5	1 0 1	f_5	f_1	f_5	f_5	f_7	f_1	f_3	f_5	f_7
6	1 1 0	f_6	f_2	f_6	f_4	f_6	f_0	f_2	f_4	f_6
7	1 1 1	f_7	f_3	f_7	f_5	f_7	f_1	f_3	f_5	f_7

this confirms the need to search for possible multiple stuck-at-faults (0/1) in the studied circuit, which is important for diagnostics.

To determine (generate) vectors of test codes that will detect stuck-at-faults (0/1) type damage both at one point and at several different points of the combinational circuit simultaneously, it is convenient to apply the q-partition procedure of minterm of the perfect STF Y^1 of the Boolean function f, the essence of which is as follows.

As noted in [12, 13], the q-partition procedure (operator $\overset{P^q}{\Rightarrow}$) of an n-position binary minterm $m_i = (\sigma_1 \sigma_2 \cdots \sigma_n)$, $\sigma_i \in \{0,1\}$ of the function $f(x_1, x_2, ..., x_n)$ is a sampling of q positions ($q \in \{1, 2, ..., (n-1)\}$, $q < n$) from the minterm m_i. This procedure is performed by applying of a mask of literals $\{l_{\alpha_1} l_{\alpha_2} \cdots l_{\alpha_{n-q}} \mid l_{\beta_1} l_{\beta_2} \cdots l_{\beta_q}\}$ to the minterm m_i, formed by partitioning into two classes — the $(n-q)$-class and the q-class — the conjunction of literals $l_1 l_2 \cdots l_n$, $l_i \in \{\bar{x}_i, x_i\}$, where the dash | is the symbol of the q-partition.

The q-partition procedure of minterms $m_1, m_2, ..., m_k$ of a perfect STF Y^1 of an arbitrary completely specified function f forms a set of partitioned minterms:

$$Y^1 = \{m_1, m_2, ..., m_k\}^1 \overset{P^q}{\Rightarrow} \{l_{\alpha_1} l_{\alpha_2} \cdots l_{\alpha_{n-q}} \mid l_{\beta_1} l_{\beta_2} \cdots l_{\beta_q}\} =$$

$$= \{m_{\alpha_1}^{n-q} \mid m_{\beta_1}^{q}, m_{\alpha_2}^{n-q} \mid m_{\beta_2}^{q}, ..., m_{\alpha_k}^{n-q} \mid m_{\beta_k}^{q}\}^1 \Rightarrow, \qquad (2)$$

where $m_i^{n-q} \mid m_i^q$ is the partitioned i-th minterm consisting of two subminterms: $(n-q)$-class — m_i^{n-q} and q-class — m_i^q; in binary format — $m_i^{n-q} \mid m_i^q = (\sigma_{\alpha_i} \cdots \sigma_{\alpha_{n-q}}) \mid (\sigma_{\beta_i} \cdots \sigma_{\beta_q})$. For example, the 2-partition of a binary minterm (10010) for mask of literal $\{l_2 l_3 l_4 \mid l_1 l_5\}$ looks like $(10010) \overset{P^2}{\Rightarrow} \{l_2 l_3 l_4 \mid l_1 l_5\} = (001 \mid 10)$.

As a result of the q-partition procedure of the binary minterms of the perfect STF Y^1 (2), a set of the partitioned of binary minterms is formed like that:

$$\Rightarrow \{(\sigma_{\alpha_1} \cdots \sigma_{\alpha_{n-q}}) \mid (\sigma_{\beta_1} \cdots \sigma_{\beta_q}), (\sigma_{\alpha_2} \cdots \sigma_{\alpha_{n-q}}) \mid (\sigma_{\beta_2} \cdots \sigma_{\beta_q}), ...,$$

$$(\sigma_{\alpha_k} \cdots \sigma_{\alpha_{n-q}}) \mid (\sigma_{\beta_k} \cdots \sigma_{\beta_q})\}^1 \overset{|\cup}{\Rightarrow}$$

Table 2

«10»	$x_1\, x_2\, x_3$	f	$f_{x_1/0}$	$f_{x_1/1}$	$f_{x_2/0}$	$f_{x_2/1}$	$f_{x_1 x_2/00}$	$f_{x_1 x_2/01}$	$f_{x_1 x_2/10}$	$f_{x_1 x_2/11}$
0	0 0 0	0	0	0	0	0	0	0	0	**1**
1	0 0 1	1	1	1	1	**0**	1	**0**	1	1
2	0 1 0	0	0	**1**	0	0	0	0	0	**1**
3	0 1 1	0	0	**1**	**1**	0	**1**	0	**1**	**1**
4	1 0 0	0	0	0	0	**1**	0	0	0	**1**
5	1 0 1	1	1	1	1	1	1	**0**	1	1
6	1 1 0	1	**0**	1	**0**	1	**0**	**0**	**0**	1
7	1 1 1	1	**0**	1	1	1	1	**0**	1	1

and, it can be further simplified first by the "rightwise union" procedure (operator $\overset{|\cup}{\Rightarrow}$), and if necessary, then by the "leftwise union" procedure (operator $\overset{\cup|}{\Rightarrow}$) [14].

Let the binary subminterms of the $(n-q)$-class reflect the values of "nonessential" variables and their number $r = 1,2,...,(n-1)$. Then the values of this subminterms in the $(n-q)$-class must form a complete set $\mathbf{E}_2^r$. Thus, the values of the "damaged" function *f* for each case of a one "nonessential" variable ($r=1$) or "nonessential" variables ($r = 2,3,...,(n-1)$) will form sets of pseudo-perfect STFs, accordingly.

Let us consider the proposed approach in more detail.

Let $q=(n-1)$. This means that one "nonessential" variable x_i is chosen, for the existence of which it is necessary to have in the $(n-q)$-class (here in the 1-class of the partition) two values of the subminterms $\mathbf{E}_2^1 = \{0,1\}$, namely:

$$Y^1 = \{m_1, m_2, ..., m_k\}^1 \overset{p^{n-1}}{\Rightarrow} \{l_{\alpha_i} \mid l_{\beta_1} l_{\beta_2} \cdots l_{\beta_{n-1}}\} \overset{|\cup}{\Rightarrow} \{(0 \mid N_0^{n-1}), (1 \mid N_1^{n-1})\}^1, \quad (3)$$

where N_0^{n-1}, N_1^{n-1} are sets of subminterms of the $(n-1)$-class formed by the $(n-1)$-partition procedure and the "rightwise union" operation.

Based on (3), two sets can be formed: a pseudoperfect STF $Y^1_{x_i/0}$ – when a "nonessential" variable x_i caused "damage" s-a-0 ($x_i/0$) and a pseudoperfect STF $Y^1_{x_i/1}$ – when a "nonessential" variable x_i caused "damage" s-a-1 ($x_i/1$). To determine the test codes, it is necessary to first perform the concatenation procedure on the partitioned minterms (3) $(0 \mid N_0^{n-1})$ and $(1 \mid N_1^{n-1})$, and then, to combine the resulting sets in a polynomial format with the given minterms, i.e. to form $\{Y^1_{x_i/0}, Y^1\}^{\oplus}$ and $\{Y^1_{x_i/1}, Y^1\}^{\oplus}$. As a result, the desired test code vectors are "filtered" into separate sets $C^1_{x_i/0}, C^0_{x_i/0}$ and $C^1_{x_i/1}, C^0_{x_i/1}$ [2]:

$$\begin{Bmatrix} Y^1_{x_i/0} \\ Y^1 \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} C^1_{x_i/0} \\ C^0_{x_i/0} \end{cases}, \quad (4)$$

$$\begin{Bmatrix} Y^1_{x_i/1} \\ Y^1 \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} C^1_{x_i/1} \\ C^0_{x_i/1} \end{cases}. \quad (5)$$

Let $q=(n-2)$. This means that two "nonessential" variables x_i and x_j are chosen, for the existence of which it is necessary to have four values of subminterms in the 2-partition class – $\mathbf{E}_2^2 = \{00,01,10,11\}$, namely:

$$Y^1 = \{m_1, m_2, ..., m_k\}^1 \overset{p^{n-2}}{\Rightarrow} \{l_{\alpha_i} l_{\alpha_j} \mid l_{\beta_1} l_{\beta_2} \cdots l_{\beta_{n-2}}\} \Rightarrow$$
$$\Rightarrow \{(00 \mid N_{00}^{n-2}), (01 \mid N_{01}^{n-2}), (10 \mid N_{10}^{n-2}), (11 \mid N_{11}^{n-2})\}^1, \quad (6)$$

where N_{00}^{n-2}, N_{01}^{n-2}, N_{10}^{n-2}, N_{11}^{n-2} are sets of subminterms of the $(n-2)$-class.

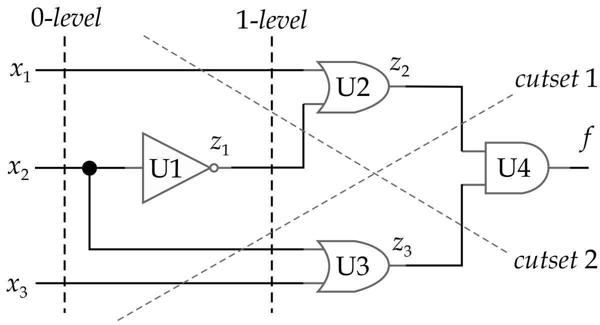

Fig. 1

From the set (6) similarly to (3) it is possible to form four pseudoperfect STFs $Y^1_{x_ix_j/00}$, $Y^1_{x_ix_j/01}$, $Y^1_{x_ix_j/10}$, $Y^1_{x_ix_j/11}$ and on their basis to obtain vectors of test codes for determining possible stuck-at-faults (0/1) type damages simultaneously at two points of the combinational circuit.

It is obvious that by using the same algorithm, it is possible to generate sets of test code vectors for a larger number of introduced "nonessential" variables of the function *f*.

It should be noted, that in practice, when studying (diagnostics) combinational circuits for verification of possible stuck-at-faults (0/1) type damages, a situation may occur when $|Y^1|>|Y^0|$. In this case, due to the less cumbersome (simpler) implementation of the necessary procedures and operations, it is advisable to use a set Y^0 instead Y^1 (*see* example 2 below). Then, after the operation of combining pseudoperfect STFs $Y^0_{x_i/0}$ and $Y^0_{x_i/1}$ in inverse polynomial format (using the example of introducing an "nonessential" variable x_i) with a perfect STF Y^0, the desired sets of test code vectors $C^0_{x_i/0}$, $C^1_{x_i/0}$ and $C^0_{x_i/1}$, $C^1_{x_i/1}$ can be defined as follows:

$$\begin{Bmatrix} Y^0_{x_i/0} \\ Y^0 \end{Bmatrix}^{\bar{\oplus}} \Rightarrow \begin{cases} C^0_{x_i/0} \\ C^1_{x_i/0} \end{cases}, \tag{7}$$

$$\begin{Bmatrix} Y^0_{x_i/1} \\ Y^0 \end{Bmatrix}^{\bar{\oplus}} \Rightarrow \begin{cases} C^0_{x_i/1} \\ C^1_{x_i/1} \end{cases}. \tag{8}$$

Practical Part

Let us illustrate the proposed method with examples.

Example 1. Determine the vectors of test codes for detecting multiple stuck-at-faults (0/1) type damages at the 0- and 1-levels and at the intersection points of the cutset 1 and 2 of the logic circuit shown in Fig. 1. (*borrowing from* [1])

Solution. The scheme in Fig. 1 implements the function $f(x_1,x_2,x_3)$ given by Table 2.

Let us first define the vectors of test codes for the case of introducing one "nonessential" variable x_1.

At the 0-level of the scheme, as a result of the 2-partition of the minterms of the perfect STF Y^1 of the function f, we obtain sets of partitioned minterms, which after the "rightwise union" procedure will simplify to the following form:

$$Y^1 = \{(001),(101),(110),(111)\}^1 \overset{p^2}{\Rightarrow} \begin{Bmatrix} l_1 \,|\, l_2 l_3 \\ l_2 \,|\, l_1 l_3 \\ l_3 \,|\, l_1 l_2 \end{Bmatrix} = \begin{cases} \{(0\,|\,01),(1\,|\,01,10,11)\}^1 \\ \{(0\,|\,01,11),(1\,|\,10,11)\}^1 \\ \{(0\,|\,11),(1\,|\,00,10,11)\}^1 \end{cases}.$$

For each mask of literals from the two formed subsets, we obtain pseudoperfect STFs $Y^1_{x_i/0}$ and $Y^1_{x_i/1}$ after the concatenation operation (operator $\overset{con}{\Rightarrow}$). Each of these sets, similarly to ([1] see table 2), after combining in polynomial format their minterms with the given minterms of the perfect STF Y^1 of the function f and simplifying by eliminating identical pairs, will form the desired vectors of test codes (here and further we will highlight the value of the "nonessential" variable in bold):

$$Y^1_{x_1/0} = \{(\mathbf{0},1)\,|\,01\}^1 \overset{con}{\Rightarrow} \{(001),(101)\}^1 \Rightarrow \begin{Bmatrix} \cancel{001,101} \\ \cancel{001,101},110,111 \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{\varnothing\}^1 \\ \{110,111\}^0 \end{cases},$$

$$Y^1_{x_1/1} = \{(0,\mathbf{1})\,|\,01,10,11\}^1 \overset{con}{\Rightarrow} \{(001),(010),(011),(101),(110),(111)\}^1 \Rightarrow$$

$$\Rightarrow \begin{Bmatrix} \cancel{001},010,011,\cancel{101,110,111} \\ \cancel{001,101,110,111} \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{010,011\}^1 \\ \{\varnothing\}^0 \end{cases};$$

$$Y^1_{x_2/0} = \{(\mathbf{0},1)\,|\,(01,11)\}^1 \overset{con}{\Rightarrow} \{(001),(011),(101),(111)\}^1 \Rightarrow$$

$$\Rightarrow \begin{Bmatrix} \cancel{001},011,\cancel{101,111} \\ \cancel{001,101},110,\cancel{111} \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{011\}^1 \\ \{110\}^0 \end{cases},$$

$$Y^1_{x_2/1} = \{(0,\mathbf{1})\,|\,10,11\}^1 \overset{con}{\Rightarrow} \{(100),(101),(110),(111)\}^1 \Rightarrow$$

$$\Rightarrow \begin{Bmatrix} 100,\cancel{101,110,111} \\ 001,\cancel{101,110,111} \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{100\}^1 \\ \{001\}^0 \end{cases}.$$

In Table 2, columns 4–7 correspond to the pseudoperfect STFs $Y^1_{x_1/0}$, $Y^1_{x_1/1}$, $Y^1_{x_2/0}$ and $Y^1_{x_2/1}$.

$$Y^1_{x_3/0} = \{(\mathbf{0},1)\,|\,(11)\}^1 \overset{con}{\Rightarrow} \{(110),(111)\}^1 \Rightarrow$$

$$\Rightarrow \begin{Bmatrix} \cancel{110,111} \\ 001,101,\cancel{110,111} \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{\varnothing\}^1 \\ \{001,101\}^0 \end{cases},$$

$$Y^1_{x_3/1} = \{(0,\mathbf{1}) \mid 00,10,11\}^1 \overset{con}{\Rightarrow} \{(000),(001),(100),(101),(110),(111)\}^1 \Rightarrow$$

$$\Rightarrow \begin{Bmatrix} 000,\cancel{001},100,\cancel{101,110,111} \\ \cancel{001,101,110,111} \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{000,100\}^1 \\ \{\varnothing\}^0 \end{cases}.$$

Note that the resulting test code vectors are the same as in the article [1].

Let us apply the 1-partition of the given minterms to determine the test codes for the case of simultaneous stuck-at-faults (0/1) damage at two points on the 0-level of the circuit:

$$Y^1 = \{(001),(101),(110),(111)\}^1 \overset{p^1}{\Rightarrow} \begin{Bmatrix} l_1 l_2 \mid l_3 \\ l_1 l_3 \mid l_2 \\ l_2 l_3 \mid l_1 \end{Bmatrix} =$$

$$= \begin{cases} \{(00,10 \mid 1),(01 \mid \varnothing),(11 \mid 0,1)\}^1 \\ \{(00 \mid \varnothing),(01 \mid 0),(10 \mid 1),(11 \mid 0,1)\}^1 \\ \{(00 \mid \varnothing),(01 \mid 0,1),(10,11 \mid 1)\}^1 \end{cases},$$

$$Y^1_{x_1x_2/00,10} = \{(\mathbf{00},01,\mathbf{10},11) \mid 1\}^1 \overset{con}{\Rightarrow} \{(001),(011),(101),(111)\}^1 \Rightarrow$$

$$\Rightarrow \begin{Bmatrix} \cancel{001},011,\cancel{101,111} \\ \cancel{001,101},110,\cancel{111} \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{011\}^1 \\ \{110\}^0 \end{cases},$$

$$Y^1_{x_1x_2/01} = \{(00,\mathbf{01},10,11) \mid \varnothing\}^1 \overset{con}{\Rightarrow} \{\varnothing\}^1 \Rightarrow$$

$$\Rightarrow \begin{Bmatrix} \varnothing \\ 001,101,110,111 \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{\varnothing\}^1 \\ \{001,101,110,111\}^0 \end{cases},$$

$$Y^1_{x_1x_2/11} = \{(00,01,10,\mathbf{11}) \mid 0,1\}^1 \overset{con}{\Rightarrow}$$

$$\overset{con}{\Rightarrow} \{(000),(001),(010),(011),(100),(101),(110),(111)\}^1 \Rightarrow$$

$$\Rightarrow \begin{Bmatrix} 000,001,010,011,100,101,110,111 \\ \cancel{001,101,110,111} \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{000,010,011,100\}^1 \\ \{\varnothing\}^0 \end{cases}.$$

In Table 2, columns 8–11 correspond to pseudoperfect STFs $Y^1_{x_1x_2/00}$, $Y^1_{x_1x_2/01}$, $Y^1_{x_1x_2/10}$ and $Y^1_{x_1x_2/11}$, and the numbers, highlighted in bold, are vectors of test codes for cases of simultaneous stuck-at-faults (0/1) damage at the two specified points of the circuit.

For other pairs of "nonessential" variables at the 0-level of the circuit, we obtain the following result:

$$Y^1_{x_1x_3/00} = \{(\mathbf{00},01,10,11)\,|\,\varnothing\}^1 \overset{con}{\Rightarrow} \{\varnothing\}^1 \Rightarrow$$

$$\Rightarrow \left\{\begin{matrix}\varnothing \\ 001,101,110,111\end{matrix}\right\}^{\oplus} \Rightarrow \left\{\begin{matrix}\{\varnothing\}^1 \\ \{001,101,110,111\}^0\end{matrix}\right.,$$

$$Y^1_{x_1x_3/01} = \{(00,\mathbf{01},10,11)\,|\,0\}^1 \overset{con}{\Rightarrow} \{(000),(001),(100),(101)\}^1 \Rightarrow$$

$$\Rightarrow \left\{\begin{matrix}000,\cancel{001},100,\cancel{101} \\ \cancel{001},\cancel{101},110,111\end{matrix}\right\}^{\oplus} \Rightarrow \left\{\begin{matrix}\{000,100\}^1 \\ \{110,111\}^0\end{matrix}\right.,$$

$$Y^1_{x_1x_3/10} = \{(00,01,\mathbf{10},11)\,|\,1\}^1 \overset{con}{\Rightarrow} \{(010),(011),(110),(111)\}^1 \Rightarrow$$

$$\Rightarrow \left\{\begin{matrix}010,011,\cancel{110},\cancel{111} \\ 001,101,\cancel{110},\cancel{111}\end{matrix}\right\}^{\oplus} \Rightarrow \left\{\begin{matrix}\{010,011\}^1 \\ \{001,101\}^0\end{matrix}\right.,$$

$$Y^1_{x_1x_3/11} = \{(00,01,10,\mathbf{11})\,|\,0,1\}^1 \overset{con}{\Rightarrow}$$

$$\overset{con}{\Rightarrow} \{(000),(001),(010),(011),(100),(101),(110),(111)\}^1 \Rightarrow$$

$$\Rightarrow \left\{\begin{matrix}000,001,010,011,100,101,110,111 \\ \cancel{001},\cancel{101},\cancel{110},\cancel{111}\end{matrix}\right\}^{\oplus} \Rightarrow \left\{\begin{matrix}\{000,010,011,100\}^1 \\ \{\varnothing\}^0\end{matrix}\right.;$$

$$Y^1_{x_2x_3/00} = \{(\mathbf{00},01,10,11)\,|\,\varnothing\}^1 \overset{con}{\Rightarrow} \{\varnothing\}^1 \Rightarrow$$

$$\Rightarrow \left\{\begin{matrix}\varnothing \\ 001,101,110,111\end{matrix}\right\}^{\oplus} \Rightarrow \left\{\begin{matrix}\{\varnothing\}^1 \\ \{001,101,110,111\}^0\end{matrix}\right.,$$

$$Y^1_{x_2x_3/01} = \{(00,\mathbf{01},10,11)\,|\,0,1\}^1 \overset{con}{\Rightarrow}$$

$$\overset{con}{\Rightarrow} \{(000),(001),(010),(011),(100),(101),(110),(111)\}^1 \Rightarrow$$

$$\Rightarrow \left\{\begin{matrix}000,001,010,011,100,101,110,111 \\ \cancel{001},\cancel{101},\cancel{110},\cancel{111}\end{matrix}\right\}^{\oplus} \Rightarrow \left\{\begin{matrix}\{000,010,011,100\}^1 \\ \{\varnothing\}^0\end{matrix}\right.,$$

$$Y^1_{x_2x_3/10,11} = \{(00,01,\mathbf{10},\mathbf{11})\,|\,1\}^1 \overset{con}{\Rightarrow} \{(100),(101),(110),(111)\}^1 \Rightarrow$$

$$\Rightarrow \left\{\begin{matrix}100,\cancel{101},\cancel{110},\cancel{111} \\ 001,\cancel{101},\cancel{110},\cancel{111}\end{matrix}\right\}^{\oplus} \Rightarrow \left\{\begin{matrix}\{100\}^1 \\ \{001\}^0\end{matrix}\right..$$

At the 1-level of the scheme we have a function $f(x_1,z_1,x_3)$, which has a perfect STF $Y^1 = \{3,4,5,7\}^1$. After performing the 2-partition procedure over its minterms, we obtain the same test code vectors as in ([1], *see* table 4):

$$Y^1=\{(011),(100),(101),(111)\}^1 \overset{p^2}{\Rightarrow} \begin{bmatrix} l_1 \mid l_2 l_3 \\ l_2 \mid l_1 l_3 \\ l_3 \mid l_1 l_2 \end{bmatrix} = \begin{cases} \{(0\mid 11),(1\mid 00,01,11)\}^1 \\ \{(0\mid 10,11),(1\mid 01,11)\}^1 \\ \{(0\mid 10),(1\mid 01,10,11)\}^1 \end{cases},$$

$$Y^1_{x_1/0}=\{(\mathbf{0},1)\mid 11\}^1 \overset{con}{\Rightarrow} \{(011),(111)\}^1 \Rightarrow \begin{Bmatrix} \sout{011,111} \\ \cancel{011},100,101,\cancel{111} \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{\varnothing\}^1 \\ \{100,101\}^0 \end{cases},$$

$$Y^1_{x_1/1}=\{(0,\mathbf{1})\mid 00,01,11\}^1 \overset{con}{\Rightarrow} \{(000),(001),(011),(100),(101),(111)\}^1 \Rightarrow$$
$$\Rightarrow \begin{Bmatrix} 000,001,011,100,101,111 \\ \sout{011,100,101,111} \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{000,001\}^1 \\ \{\varnothing\}^0 \end{cases};$$

$$Y^1_{z_1/0} \Rightarrow \begin{cases} \{110\}^1 \\ \{011\}^0 \end{cases},\; Y^1_{z_1/1} \Rightarrow \begin{cases} \{001\}^1 \\ \{100\}^0 \end{cases},\; Y^1_{x_3/0} \Rightarrow \begin{cases} \{\varnothing\}^1 \\ \{011,111\}^0 \end{cases},\; Y^1_{x_3/1} \Rightarrow \begin{cases} \{010,110\}^1 \\ \{\varnothing\}^0 \end{cases}.$$

In case of simultaneous stuck-at-faults (0/1) damage at two points on the 1-level of the circuit, after performing the 1-partition procedure of the minterms of the function $f(x_1,z_1,x_3)$, we obtain the following result:

$$Y^1=\{(011),(100),(101),(111)\}^1 \overset{p^1}{\Rightarrow} \begin{bmatrix} l_1 l_2 \mid l_3 \\ l_1 l_3 \mid l_2 \\ l_2 l_3 \mid l_1 \end{bmatrix} = \begin{cases} \{(00\mid\varnothing),(01,11\mid 1),(10\mid 0,1)\}^1 \\ \{(00\mid\varnothing),(01\mid 1),(10\mid 0),(11\mid 0,1)\}^1; \\ \{(00,01\mid 1),(10\mid\varnothing),(11\mid 0,1)\}^1 \end{cases}$$

$$Y^1_{x_1 z_1/00}=\{\varnothing\}^1 \Rightarrow \begin{Bmatrix} \varnothing \\ 011,100,101,111 \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{\varnothing\}^1 \\ \{011,100,101,111\}^0 \end{cases},$$

$$Y^1_{x_1 z_1/01,11}=(00,\mathbf{01},10,\mathbf{11})\mid 1\}^1 \overset{con}{\Rightarrow} \{(001),(011),(101),(111)\}^1 \Rightarrow$$
$$\Rightarrow \begin{Bmatrix} 001,\sout{011,101,111} \\ \cancel{011},100,\sout{101,111} \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{001\}^1 \\ \{100\}^0 \end{cases},$$

$$Y^1_{x_1 z_1/10}=\{(00,01,\mathbf{10},11)\mid 0,1\}^1 \overset{con}{\Rightarrow}$$
$$\overset{con}{\Rightarrow} \{(000),(001),(010),(011),(100),(101),(110),(111)\}^1 \Rightarrow$$
$$\Rightarrow \begin{Bmatrix} 000,001,010,011,100,101,110,111 \\ \sout{011,100,101,111} \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{000,001,010,110\}^1 \\ \{\varnothing\}^0 \end{cases};$$

$$Y^1_{x_1 x_3/00}=\{\varnothing\}^1 \Rightarrow \begin{Bmatrix} \varnothing \\ 011,100,101,111 \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{\varnothing\}^1 \\ \{011,100,101,111\}^0 \end{cases},$$

$$Y^1_{x_1x_3/01} = (00,\mathbf{01},10,11)\,|\,1\}^1 \overset{con}{\Rightarrow} \{(010),(011),(110),(111)\}^1 \Rightarrow$$

$$\Rightarrow \begin{Bmatrix} 010,\cancel{011},110,\cancel{111} \\ \cancel{011},100,101,\cancel{111} \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{010,110\}^1 \\ \{100,101\}^0 \end{cases}$$

$$Y^1_{x_1x_3/10} = (00,01,\mathbf{10},11)\,|\,0\}^1 \overset{con}{\Rightarrow} \{(000),(001),(100),(101)\}^1 \Rightarrow$$

$$\Rightarrow \begin{Bmatrix} 000,001,\cancel{100,101} \\ 011,\cancel{100,101},111 \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{000,001\}^1 \\ \{011,111\}^0 \end{cases},$$

$$Y^1_{x_1x_3/11} = \{(00,01,10,\mathbf{11})\,|\,0,1\}^1 \overset{con}{\Rightarrow}$$

$$\overset{con}{\Rightarrow} \{(000),(001),(010),(011),(100),(101),(110),(111)\}^1 \Rightarrow$$

$$\Rightarrow \begin{Bmatrix} 000,001,010,\cancel{011,100,101},110,\cancel{111} \\ \cancel{011,100,101,111} \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{000,001,010,110\}^1 \\ \{\varnothing\}^0 \end{cases};$$

$$Y^1_{z_1x_3/00,01} = (\mathbf{00},\mathbf{01},10,11)\,|\,1\}^1 \overset{con}{\Rightarrow} \{(100),(101),(110),(111)\}^1 \Rightarrow$$

$$\Rightarrow \begin{Bmatrix} \cancel{100,101},110,\cancel{111} \\ 011,\cancel{100,101,111} \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{110\}^1 \\ \{011\}^0 \end{cases},$$

$$Y^1_{z_1x_3/10} = (00,01,\mathbf{10},11)\,|\,\varnothing\}^1 \overset{con}{\Rightarrow} \{\varnothing\}^1 \Rightarrow$$

$$\Rightarrow \begin{Bmatrix} \varnothing \\ 011,100,101,111 \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{\varnothing\}^1 \\ \{011,100,101,111\}^0 \end{cases},$$

$$Y^1_{z_1x_3/11} = \{(00,01,10,\mathbf{11})\,|\,0,1\}^1 \overset{con}{\Rightarrow}$$

$$\overset{con}{\Rightarrow} \{(000),(001),(010),(011),(100),(101),(110),(111)\}^1 \Rightarrow$$

$$\Rightarrow \begin{Bmatrix} 000,001,010,\cancel{011,100,101},110,\cancel{111} \\ \cancel{011,100,101,111} \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{000,001,010,110\}^1 \\ \{\varnothing\}^0 \end{cases}.$$

Table 3

2-partition	0-level			1-level		
Stuck-at-fault	$x_1/\sim$	$x_2/\sim$	$x_3/\sim$	$x_1/\sim$	$z_1\sim$	$x_3\sim$
s-a-0	$\begin{pmatrix}110\\111\end{pmatrix}^0$	$(011)^1$ $(110)^0$	$\begin{pmatrix}001\\101\end{pmatrix}^0$	$\begin{pmatrix}100\\101\end{pmatrix}^0$	$(110)^1$ $(011)^0$	$\begin{pmatrix}011\\111\end{pmatrix}^0$
s-a-1	$\begin{pmatrix}010\\011\end{pmatrix}^1$	$(100)^1$ $(001)^0$	$\begin{pmatrix}000\\100\end{pmatrix}^1$	$\begin{pmatrix}000\\001\end{pmatrix}^1$	$(001)^1$ $(100)^0$	$\begin{pmatrix}010\\110\end{pmatrix}^1$

Table 3 contains vectors of test codes at 0- and 1-levels of the circuit for the case of stuck-at-faults (0/1) type damage at one point, and Table 4 for the case of multiple damage at two points of the circuit.

For the cutset 2 we have the function $f(x_1, z_1, z_3) = x_1 z_3 \vee z_1 z_3 \Rightarrow$ $\Rightarrow \{(1-1),(-11)\}^1$ that has a perfect STF $Y^1 = \{3,5,7\}^1$ $(Y^0 = \{0,1,2,4,6\}^0)$. After performing a 2-partition of its minterms, we define the vectors of test codes:

$$Y^1 = \{(011),(101),(111)\}^1 \overset{p^2}{\Rightarrow} \begin{Bmatrix} l_1 \mid l_2 l_3 \\ l_2 \mid l_1 l_3 \\ l_3 \mid l_1 l_2 \end{Bmatrix} = \begin{cases} \{(0 \mid 11),(1 \mid 01,11)\}^1 \\ \{(0 \mid 11),(1 \mid 01,11)\}^1 \\ \{(0 \mid \varnothing),(1 \mid 01,10,11)\}^1 \end{cases};$$

$$Y^1_{x_1/0} = \{(\mathbf{0},1) \mid 11\}^1 \overset{con}{\Rightarrow} \{(011),(111)\}^1 \Rightarrow \begin{Bmatrix} \cancel{011,111} \\ \cancel{011},101,\cancel{111} \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{\varnothing\}^1 \\ \{101\}^0 \end{cases}$$

$$Y^1_{x_1/1} = \{(0,\mathbf{1}) \mid 01,11\}^1 \overset{con}{\Rightarrow} \{(001),(011),(101),(111)\}^1 \Rightarrow$$
$$\Rightarrow \begin{Bmatrix} 001,\cancel{011,101,111} \\ \cancel{011,101,111} \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{001\}^1 \\ \{\varnothing\}^0 \end{cases},$$

Table 4

1-partition	0-level			1-level		
Stuck-at-fault	$x_1x_2/\sim\sim$	$x_1x_3/\sim\sim$	$x_2x_3/\sim\sim$	$x_1z_1/\sim\sim$	$x_1x_3/\sim\sim$	$z_1x_3/\sim\sim$
s-a-00	$(011)^1$ $(110)^0$	$\begin{pmatrix}001\\101\\110\\111\end{pmatrix}^0$	$\begin{pmatrix}001\\101\\110\\111\end{pmatrix}^0$	$\begin{pmatrix}011\\100\\101\\111\end{pmatrix}^0$	$\begin{pmatrix}011\\100\\101\\111\end{pmatrix}^0$	$(110)^1$ $(011)^0$
s-a-01	$\begin{pmatrix}001\\101\\110\\111\end{pmatrix}^0$	$\begin{pmatrix}000\\100\end{pmatrix}^1$ $\begin{pmatrix}110\\111\end{pmatrix}^0$	$\begin{pmatrix}000\\010\\011\\100\end{pmatrix}^1$	$(001)^1$ $(100)^0$	$\begin{pmatrix}010\\110\end{pmatrix}^1$ $\begin{pmatrix}100\\101\end{pmatrix}^0$	$(110)^1$ $(011)^0$
s-a-10	$(011)^1$ $(110)^0$	$\begin{pmatrix}010\\011\end{pmatrix}^1$ $\begin{pmatrix}001\\101\end{pmatrix}^0$	$(100)^1$ $(001)^0$	$\begin{pmatrix}000\\001\\010\\110\end{pmatrix}^1$	$\begin{pmatrix}000\\001\end{pmatrix}^1$ $\begin{pmatrix}011\\111\end{pmatrix}^0$	$\begin{pmatrix}011\\100\\101\\111\end{pmatrix}^0$
s-a-11	$\begin{pmatrix}000\\010\\011\\100\end{pmatrix}^1$	$\begin{pmatrix}000\\010\\011\\100\end{pmatrix}^1$	$(100)^1$ $(001)^0$	$(001)^1$ $(100)^0$	$\begin{pmatrix}000\\001\\010\\110\end{pmatrix}^1$	$\begin{pmatrix}000\\001\\010\\110\end{pmatrix}^1$

$$Y^1_{z_1/0} = \{(\mathbf{0},1)\,|\,11\}^1 \overset{con}{\Rightarrow} \{(101),(111)\}^1 \Rightarrow \left\{\begin{matrix} \cancel{101,111} \\ 011,\cancel{101,111} \end{matrix}\right\}^{\oplus} \Rightarrow \left\{\begin{matrix} \{\varnothing\}^1 \\ \{011\}^0 \end{matrix}\right.,$$

$$Y^1_{z_1/1} = \{(0,\mathbf{1})\,|\,01,11\}^1 \overset{con}{\Rightarrow} \{(001),(011),(101),(111)\}^1 \Rightarrow$$
$$\Rightarrow \left\{\begin{matrix} 001,\cancel{011,101,111} \\ \cancel{011,101,111} \end{matrix}\right\}^{\oplus} \Rightarrow \left\{\begin{matrix} \{001\}^1 \\ \{\varnothing\}^0 \end{matrix}\right.,$$

$$Y^1_{z_3/0} = \{(\mathbf{0},1)\,|\,\varnothing\}^1 \overset{con}{\Rightarrow} \{\varnothing\}^1 \Rightarrow \left\{\begin{matrix} \varnothing \\ 011,101,111 \end{matrix}\right\}^{\oplus} \Rightarrow \left\{\begin{matrix} \{\varnothing\}^1 \\ \{011,101,111\}^0 \end{matrix}\right.;$$

$$Y^1_{z_3/1} = \{(0,\mathbf{1})\,|\,01,10,11\}^1 \overset{con}{\Rightarrow} \{(010),(011),(100),(101),(110),(111)\}^1 \Rightarrow$$
$$\Rightarrow \left\{\begin{matrix} 010,\cancel{011},100,\cancel{101},110,\cancel{111} \\ \cancel{011,101,111} \end{matrix}\right\}^{\oplus} \Rightarrow \left\{\begin{matrix} \{010,100,110\}^1 \\ \{\varnothing\}^0 \end{matrix}\right..$$

The vectors of test codes with simultaneous stuck-at-faults (0/1) damage at two points of the circuit can be obtained after performing the 1-partition of the minterms of the function $f(x_1, z_1, z_3)$:

$$Y^1 = \{(011),(101),(111)\}^1 \overset{P^1}{\Rightarrow} \left\{\begin{matrix} l_1l_2\,|\,l_3 \\ l_1l_3\,|\,l_2 \\ l_2l_3\,|\,l_1 \end{matrix}\right\} = \left\{\begin{matrix} \{(00\,|\,\varnothing),(01,10,11\,|\,1)\}^1 \\ \{(00,10\,|\,\varnothing),(01\,|\,1),(11\,|\,0,1)\}^1 \\ \{(00,10\,|\,\varnothing),(01\,|\,1),(11\,|\,0,1)\}^1 \end{matrix}\right.;$$

$$Y^1_{x_1z_1/00} = \{(\mathbf{00},01,10,11)\,|\,\varnothing\}^1 \Rightarrow \left\{\begin{matrix} \varnothing \\ 011,101,111 \end{matrix}\right\}^{\oplus} \Rightarrow \left\{\begin{matrix} \{\varnothing\}^1 \\ \{011,101,111\}^0 \end{matrix}\right.,$$

$$Y^1_{x_1z_1/01,10,11} = \{(00,\mathbf{01},\mathbf{10},\mathbf{11})\,|\,1\}^1 \overset{con}{\Rightarrow} \{(001),(011),(101),(111)\}^1 \Rightarrow$$
$$\Rightarrow \left\{\begin{matrix} 001,\cancel{011,101,111} \\ \cancel{011,101,111} \end{matrix}\right\}^{\oplus} \Rightarrow \left\{\begin{matrix} \{001\}^1 \\ \{\varnothing\}^0 \end{matrix}\right.;$$

$$Y^1_{x_1z_3/00,10} = \{(\mathbf{00},01,\mathbf{10},11)\,|\,\varnothing\}^1 \Rightarrow \left\{\begin{matrix} \varnothing \\ 011,101,111 \end{matrix}\right\}^{\oplus} \Rightarrow \left\{\begin{matrix} \{\varnothing\}^1 \\ \{011,101,111\}^0 \end{matrix}\right.;$$

$$Y^1_{x_1z_3/01} = \{(00,\mathbf{01},10,11)\,|\,1\}^1 \overset{con}{\Rightarrow} \{(010),(011),(110),(111)\}^1 \Rightarrow$$
$$\Rightarrow \left\{\begin{matrix} 010,\cancel{011},110,\cancel{111} \\ \cancel{011},101,\cancel{111} \end{matrix}\right\}^{\oplus} \Rightarrow \left\{\begin{matrix} \{010,110\}^1 \\ \{101\}^0 \end{matrix}\right.,$$

$$Y^1_{x_1z_3/11} = \{(00,01,10,\mathbf{11})\,|\,0,1\}^1 \overset{con}{\Rightarrow}$$
$$\overset{con}{\Rightarrow}\{(000),(001),(010),(011),(100),(101),(110),(111)\}^1 \Rightarrow$$
$$\Rightarrow\left\{\begin{matrix}000,001,010,\cancel{011},100,\cancel{101},110,\cancel{111}\\ \cancel{011,101,111}\end{matrix}\right\}^{\oplus} \Rightarrow \left\{\begin{matrix}\{000,001,010,100,110\}^1\\ \{\varnothing\}^0\end{matrix}\right.,$$

$$Y^1_{z_2z_3/00,10} = \{(\mathbf{00},01,\mathbf{10},11)\,|\,\varnothing\}^1 \Rightarrow \left\{\begin{matrix}\varnothing\\ 011,101,111\end{matrix}\right\}^{\oplus} \Rightarrow \left\{\begin{matrix}\{\varnothing\}^1\\ \{011,101,111\}^0\end{matrix}\right.;$$

$$Y^1_{z_2z_3/01} = \{(00,\mathbf{01},10,11)\,|\,1\}^1 \overset{con}{\Rightarrow}\{(100),(101),(110),(111)\}^1 \Rightarrow$$
$$\Rightarrow\left\{\begin{matrix}100,\cancel{101},110,\cancel{111}\\ 011,\cancel{101,111}\end{matrix}\right\}^{\oplus} \Rightarrow \left\{\begin{matrix}\{100,110\}^1\\ \{011\}^0\end{matrix}\right.,$$

$$Y^1_{z_2z_3/11} = \{(00,01,10,\mathbf{11})\,|\,0,1\}^1 \overset{con}{\Rightarrow}$$
$$\overset{con}{\Rightarrow}\{(000),(001),(010),(011),(100),(101),(110),(111)\}^1 \Rightarrow$$
$$\Rightarrow\left\{\begin{matrix}000,001,010,\cancel{011},100,\cancel{101},110,\cancel{111}\\ \cancel{011,101,111}\end{matrix}\right\}^{\oplus} \Rightarrow \left\{\begin{matrix}\{000,001,010,100,110\}^1\\ \{\varnothing\}^0\end{matrix}\right..$$

The test code vectors obtained for the cutset 2 for damage at one point of the circuit are placed in Table 5, and for simultaneous damage at two points of the circuit — in Table 6.

For the cutset 1 of the scheme in Fig. 1, we have the function $f(z_2, x_2, x_3) = = z_2x_2 \vee z_2x_3 \Rightarrow \{(11-),(1-1)\}^1$ that has

Table 5

Stuck-at-fault	2-partition		
	$x_1/\sim$	$z_1/\sim$	$z_3/\sim$
s-a-0	$(101)^0$	$(101)^0$	$\begin{pmatrix}011\\101\\111\end{pmatrix}^0$
s-a-1	$(001)^1$	$(001)^1$	$\begin{pmatrix}010\\100\\110\end{pmatrix}^1$

Table 6

Stuck-at-fault	1-partition		
	$x_1z_1/\sim\sim$	$x_1z_3/\sim\sim$	$z_1z_3/\sim\sim$
s-a-00	$\begin{pmatrix}011\\101\\111\end{pmatrix}^0$	$\begin{pmatrix}011\\101\\111\end{pmatrix}^0$	$\begin{pmatrix}011\\101\\111\end{pmatrix}^0$
s-a-01	$(001)^1$	$\begin{pmatrix}010\\110\end{pmatrix}^1$ $(101)^0$	$\begin{pmatrix}100\\110\end{pmatrix}^1$ $(011)^0$
s-a-10	$(001)^1$	$\begin{pmatrix}011\\101\\111\end{pmatrix}^0$	$\begin{pmatrix}011\\101\\111\end{pmatrix}^0$
s-a-11	$(001)^1$	$\begin{pmatrix}000\\001\\010\\100\\110\end{pmatrix}^1$	$\begin{pmatrix}000\\001\\010\\100\\110\end{pmatrix}^1$

a perfect STF $Y^1=\{5,6,7\}^1$ ($Y^0=\{0,1,2,3,4\}^0$). After performing the 2-partition procedure of minterms, we determine the vectors of test codes for the case of damage at the intersection points of section 1:

$$Y^1=\{(101),(110),(111)\}^1 \overset{P^2}{\Rightarrow} \begin{Bmatrix} l_1\,|\,l_2l_3 \\ l_2\,|\,l_1l_3 \\ l_3\,|\,l_1l_2 \end{Bmatrix} = \begin{cases} \{(0\,|\,\varnothing),(1\,|\,01,10,11)\}^1 \\ \{(0\,|\,11),(1\,|\,10,11)\}^1 \\ \{(0\,|\,11),(1\,|\,10,11)\}^1 \end{cases};$$

$$Y^1_{z_2/0}=\{(\mathbf{0},1)\,|\,\varnothing\}^1 \overset{con}{\Rightarrow} \{\varnothing\}^1 \Rightarrow \begin{Bmatrix} \varnothing \\ 101,110,111 \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{\varnothing\}^1 \\ \{101,110,111\}^0 \end{cases},$$

$$Y^1_{z_2/1}=\{(0,\mathbf{1})\,|\,(01,10,11)\}^1 \overset{con}{\Rightarrow} \{(001),(010),(011),(101),(110),(111)\}^1 \Rightarrow$$
$$\Rightarrow \begin{Bmatrix} 001,010,011,\cancel{101,110,111} \\ \cancel{101,110,111} \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{001,010,011\}^1 \\ \{\varnothing\}^0 \end{cases};$$

$$Y^1_{x_2/0}=\{(\mathbf{0},1)\,|\,11\}^1 \overset{con}{\Rightarrow} \{(101),(111)\}^1 \Rightarrow \begin{Bmatrix} \cancel{101,111} \\ \cancel{101},110,\cancel{111} \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{\varnothing\}^1 \\ \{110\}^0 \end{cases},$$

$$Y^1_{x_2/1}=\{(0,\mathbf{1})\,|\,(10,11)\}^1 \overset{con}{\Rightarrow} \{(100),(101),(110),(111)\}^1 \Rightarrow$$
$$\Rightarrow \begin{Bmatrix} 100,\cancel{101,110,111} \\ \cancel{101,110,111} \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{100\}^1 \\ \{\varnothing\}^0 \end{cases};$$

$$Y^1_{x_3/0}=\{(\mathbf{0},1)\,|\,11\}^1 \overset{con}{\Rightarrow} \{(110),(111)\}^1 \Rightarrow \begin{Bmatrix} \cancel{110,111} \\ 101,\cancel{110,111} \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{\varnothing\}^1 \\ \{101\}^0 \end{cases},$$

$$Y^1_{x_3/1}=\{(0,\mathbf{1})\,|\,(10,11)\}^1 \overset{con}{\Rightarrow} \{(100),(101),(110),(111)\}^1 \Rightarrow$$
$$\Rightarrow \begin{Bmatrix} 100,\cancel{101,110,111} \\ \cancel{101,110,111} \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{100\}^1 \\ \{\varnothing\}^0 \end{cases}.$$

After performing 1-partition of the minterms of the function $f(z_2,x_2,x_3)$, we obtain vectors of test codes for detecting damage at two points of the circuit:

$$Y^1=\{(101),(110),(111)\}^1 \overset{P^1}{\Rightarrow} \begin{Bmatrix} l_1l_2\,|\,l_3 \\ l_1l_3\,|\,l_2 \\ l_2l_3\,|\,l_1 \end{Bmatrix} = \begin{cases} \{(00,01\,|\,\varnothing),(10\,|\,1),(11\,|\,0,1)\}^1 \\ \{(00,01\,|\,\varnothing),(10\,|\,1),(11\,|\,0,1)\}^1 \\ \{(00\,|\,\varnothing),(01,10,11\,|\,1)\}^1 \end{cases};$$

$$Y^1_{z_2x_2/00,01} = \{(\mathbf{00},\mathbf{01},10,11)\,|\,\varnothing\}^1 \overset{con}{\Rightarrow} \{\varnothing\}^1 \Rightarrow \left\{\begin{matrix}\varnothing \\ 101,110,111\end{matrix}\right\}^{\oplus} \Rightarrow \left\{\begin{matrix}\{\varnothing\}^1 \\ \{101,110,111\}^0\end{matrix}\right.,$$

$$Y^1_{z_2x_2/10} = \{(00,01,\mathbf{10},11)\,|\,1\}^1 \overset{con}{\Rightarrow} \{(001),(011),(101),(111)\}^1 \Rightarrow$$
$$\Rightarrow \left\{\begin{matrix}001,011,\cancel{101,111} \\ \cancel{101},110,\cancel{111}\end{matrix}\right\}^{\oplus} \Rightarrow \left\{\begin{matrix}\{001,011\}^1 \\ \{110\}^0\end{matrix}\right.,$$

$$Y^1_{z_2x_2/11} = \{(00,01,10,\mathbf{11})\,|\,0,1\}^1 \overset{con}{\Rightarrow}$$
$$\overset{con}{\Rightarrow} \{(000),(001),(010),(011),(100),(101),(110),(111)\}^1 \Rightarrow$$
$$\Rightarrow \left\{\begin{matrix}000,001,010,011,100,\cancel{101,110,111} \\ \cancel{101,110,111}\end{matrix}\right\}^{\oplus} \Rightarrow \left\{\begin{matrix}\{000,001,010,011,100\}^1 \\ \{\varnothing\}^0\end{matrix}\right.;$$

$$Y^1_{z_2x_3/00,01} = \{(\mathbf{00},\mathbf{01},10,11)\,|\,\varnothing\}^1 \overset{con}{\Rightarrow} \{\varnothing\}^1 \Rightarrow \left\{\begin{matrix}\varnothing \\ 101,110,111\end{matrix}\right\}^{\oplus} \Rightarrow \left\{\begin{matrix}\{\varnothing\}^1 \\ \{101,110,111\}^0\end{matrix}\right.,$$

$$Y^1_{z_2x_3/10} = \{(00,01,\mathbf{10},11)\,|\,1\}^1 \overset{con}{\Rightarrow} \{(010),(011),(110),(111)\}^1 \Rightarrow$$
$$\Rightarrow \left\{\begin{matrix}010,011,\cancel{110,111} \\ 101,\cancel{110},\cancel{111}\end{matrix}\right\}^{\oplus} \Rightarrow \left\{\begin{matrix}\{010,011\}^1 \\ \{101\}^0\end{matrix}\right.,$$

$$Y^1_{z_2x_3/11} = \{(00,01,10,\mathbf{11})\,|\,0,1\}^1 \overset{con}{\Rightarrow}$$
$$\overset{con}{\Rightarrow} \{(000),(001),(010),(011),(100),(101),(110),(111)\}^1 \Rightarrow$$
$$\Rightarrow \left\{\begin{matrix}000,001,010,011,100,\cancel{101,110,111} \\ \cancel{101,110,111}\end{matrix}\right\}^{\oplus} \Rightarrow \left\{\begin{matrix}\{000,001,010,011,100\}^1 \\ \{\varnothing\}^0\end{matrix}\right.;$$

$$Y^1_{x_2x_3/00} = \{(\mathbf{00},01,10,11)\,|\,\varnothing\}^1 \overset{con}{\Rightarrow} \{\varnothing\}^1 \Rightarrow \left\{\begin{matrix}\varnothing \\ 101,110,111\end{matrix}\right\}^{\oplus} \Rightarrow \left\{\begin{matrix}\{\varnothing\}^1 \\ \{101,110,111\}^0\end{matrix}\right.,$$

$$Y^1_{x_2x_3/11} = \{(00,\mathbf{01},\mathbf{10},\mathbf{11})\,|\,1\}^1 \overset{con}{\Rightarrow} \{(100),(101),(110),(111)\}^1 \Rightarrow$$
$$\Rightarrow \left\{\begin{matrix}100,\cancel{101,110,111} \\ \cancel{101,110,111}\end{matrix}\right\}^{\oplus} \Rightarrow \left\{\begin{matrix}\{100\}^1 \\ \{\varnothing\}^0\end{matrix}\right..$$

The obtained vectors of test codes for the cutset 1 for damage at one point of the circuit are placed in Table 7, and for simultaneous damage at two points of the circuit — in Table 8.

Example 2. The proposed method is to determine the vectors of test codes to detect single and simultaneous stuck-at-fault (0/1) damage at points *B* and *C* of the circuit in Fig. 2 (*borrowed from* [4], p. 594, Fig. 3).

Solution. The given circuit is described by the function $f(a,b,c,d)=$, $=\overline{\overline{\overline{ab}c}\,\overline{b\overline{cd}}}$ which has a perfect STF $Y^1=\{2,3,4,5,6,7,10,11,12,13,14\}^1$ or $Y^0=\{0,1,8,9,15\}^0$. Since $|Y^1|>|Y^0|$, then we will solve the example taking into account the perfect STF Y^0.

We will determine the vectors of test codes for single damage at points *B* and *C* of the circuit, applying the 3-partition of the minterms of the perfect STF Y^0 with respect to the variables *b* and *c*:

Table 7

Stuck-at-fault	2-partition		
	$z_2/\sim$	$x_2/\sim$	$x_3/\sim$
s-a-0	$\begin{pmatrix}101\\110\\111\end{pmatrix}^0$	$(110)^0$	$(101)^0$
s-a-1	$\begin{pmatrix}001\\010\\011\end{pmatrix}^1$	$(100)^1$	$(100)^1$

Table 8

Stuck-at-fault	1-partition		
	$z_2x_2/\sim\sim101$	$z_2x_3/\sim\sim$	$x_2x_3/\sim\sim$
s-a-00	$\begin{pmatrix}101\\110\\111\end{pmatrix}^0$	$\begin{pmatrix}101\\110\\111\end{pmatrix}^0$	$\begin{pmatrix}101\\110\\111\end{pmatrix}^0$
s-a-01	$\begin{pmatrix}101\\110\\111\end{pmatrix}^0$	$\begin{pmatrix}101\\110\\111\end{pmatrix}^0$	$(100)^1$
s-a-10	$\begin{pmatrix}001\\011\end{pmatrix}^1$, $(110)^0$	$\begin{pmatrix}010\\011\end{pmatrix}^1$, $(101)^0$	$(100)^1$
s-a-11	$\begin{pmatrix}000\\001\\010\\011\\100\end{pmatrix}^1$	$\begin{pmatrix}000\\001\\010\\011\\100\end{pmatrix}^1$	$(100)^1$

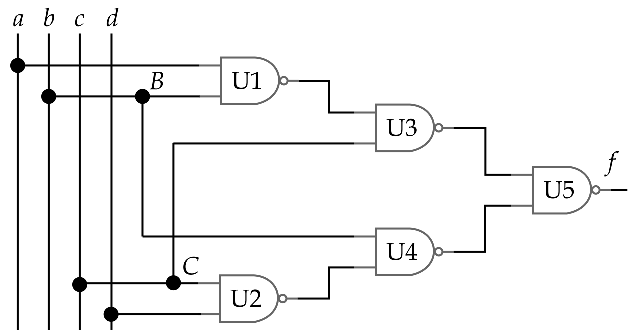

Fig. 2

$$Y^0=\{(0000),(0001),(1000),(1001),(1111)\}^0\overset{P^3}{\Rightarrow}\left\{\begin{matrix} l_2\,|\,l_1l_3l_4 \\ l_3\,|\,l_1l_2l_4\end{matrix}\right\}=$$

$$=\begin{cases}\{(0|000),(0|001),(0|100),(0|101),(1|111)\}^0 \\ \{(0|000),(0|001),(0|100),(0|101),(1|111)\}^0\end{cases}\overset{|\cup}{\Rightarrow}$$

$$\overset{|\cup}{\Rightarrow}\begin{cases}\{0|(000,001,100,101),1|111\}^0 \\ \{0|(000,001,100,101),1|111\}^0\end{cases}.$$

Considering these sets and procedures (7) and (8), we obtain the desired test codes:

$$Y^0_{b/0}=\{(\mathbf{0},1)|(000,001,100,101)\}^0\overset{con}{\Rightarrow}$$

$$\overset{con}{\Rightarrow}\{(0000),(0001),(0100),(0101),(1000),(1001),(1100),(1101)\}^0\Rightarrow$$

$$\Rightarrow\left\{\begin{matrix}\cancel{0000,0001},0100,0101,\cancel{1000,1001},1100,1101 \\ \cancel{0000,0001,1000,1001},1111\end{matrix}\right\}^{\bar{\oplus}}\Rightarrow\begin{cases}(0100,0101,1100,1101)^0 \\ (1111)^1\end{cases},$$

$$Y^0_{b/1}=\{(0,\mathbf{1})|111\}^0\Rightarrow\{(1011),(1111)\}^0\Rightarrow$$

$$\Rightarrow\left\{\begin{matrix}1011,\cancel{1111} \\ 0000,0001,1000,1001,\cancel{1111}\end{matrix}\right\}^{\bar{\oplus}}\Rightarrow\begin{cases}(1011)^0 \\ (0000,0001,1000,1001)^1\end{cases};$$

$$Y^0_{c/0}=\{(\mathbf{0},1)|(000,001,100,101)\}^0\overset{con}{\Rightarrow}$$

$$\overset{con}{\Rightarrow}\{(0000),(0001),(0010),(0011),(1000),(1001),(1010),(1011)\}^0\Rightarrow$$

$$\Rightarrow\left\{\begin{matrix}\cancel{0000,0001},0010,0011,\cancel{1000,1001},1010,1011 \\ \cancel{0000,0001,1000,1001},1111\end{matrix}\right\}^{\bar{\oplus}}\Rightarrow\begin{cases}(0010,0011,1010,1011)^0 \\ (1111)^1\end{cases},$$

$$Y^0_{c/1}=\{(0,\mathbf{1})|111\}^0\Rightarrow\{(1101),(1111)\}^0\Rightarrow$$

$$\Rightarrow\left\{\begin{matrix}1101,\cancel{1111} \\ 0000,0001,1000,1001,\cancel{1111}\end{matrix}\right\}^{\bar{\oplus}}\Rightarrow\begin{cases}(1101)^0 \\ (0000,0001,1000,1001)^1\end{cases}.$$

The test codes in the case of simultaneous stuck-at-fault (0/1) damage at points B and C of the circuit are obtained after applying the 2-partition of minterms and procedures (7) and (8):

$$Y^0=\{(0000),(0001),(1000),(1001),(1111)\}^0 \overset{P^2}{\Rightarrow}\{l_2l_3\,|\,l_1l_4\}=$$

$$=\{(00|00),(00|01),(00|10),(00|11),(11|11)\}^0\overset{|\cup}{\Rightarrow}$$

$$\overset{|\cup}{\Rightarrow}\{(00|(00,01,10,11)),(01|\varnothing),(10|\varnothing),(11|11)\}^0\overset{\cup|}{\Rightarrow}$$

$$\overset{\cup|}{\Rightarrow}\{(00|(00,01,10,11),((01,10)|\varnothing),(11|11)\}^0$$

$$Y^0_{bc/00}=\{(\mathbf{00},01,10,11)|(00,01,10,11)\}^0\Rightarrow\left\{\begin{matrix}\mathbf{E}_2^4\\0000,0001,1000,1001,1111\end{matrix}\right\}^{\bar\oplus}\Rightarrow$$

$$\Rightarrow\begin{cases}(0010,0011,0100,0101,0110,0111,1010,1011,1100,1101,1110)^0\\ \{\varnothing\}^1\end{cases};$$

$$Y^0_{bc/01,10}=\{(00,\mathbf{01},\mathbf{10},11)|\varnothing\}^0\Rightarrow\{\varnothing\}^0\Rightarrow$$

$$\Rightarrow\left\{\begin{matrix}\varnothing\\0000,0001,1000,1001,1111\end{matrix}\right\}^{\bar\oplus}\Rightarrow\begin{cases}\{\varnothing\}^0\\(0000,0001,1000,1001,1111)^1\end{cases},$$

$$Y^0_{bc/11}=\{(00,01,10,\mathbf{11})|11\}^0\overset{con}{\Rightarrow}\{1001,1011,1101,1111\}^0\Rightarrow$$

$$\Rightarrow\left\{\begin{matrix}0000,0001,1000,\cancel{1001,1111}\\ \cancel{1001},1011,1101,\cancel{1111}\end{matrix}\right\}^{\bar\oplus}\Rightarrow\begin{cases}(1011,1101)^0\\(0000,0001,1000)^1\end{cases}.$$

Table 9

«10»	a b c d	f	$f_{b/\sim}$		$f_{c/\sim}$		$f_{bc/\sim}$			
			b/0	b/1	c/0	c/1	bc/00	bc/01	bc/10	bc/11
0	0 0 0 0	0	0	**1**	0	**1**	0	**1**	**1**	**1**
1	0 0 0 1	0	0	**1**	0	**1**	0	**1**	**1**	**1**
2	0 0 1 0	1	1	1	**0**	1	**0**	1	1	1
3	0 0 1 1	1	1	1	**0**	1	**0**	1	1	1
4	0 1 0 0	1	**0**	1	1	1	**0**	1	1	1
5	0 1 0 1	1	**0**	1	1	1	**0**	1	1	1
6	0 1 1 0	1	1	1	1	1	**0**	1	1	1
7	0 1 1 1	1	1	1	1	1	**0**	1	1	1
8	1 0 0 0	0	0	**1**	0	**1**	0	**1**	**1**	**1**
9	1 0 0 1	0	0	**1**	0	**1**	0	**1**	**1**	0
10	1 0 1 0	1	1	1	**0**	1	**0**	1	1	1
11	1 0 1 1	1	1	**0**	**0**	1	**0**	1	1	**0**
12	1 1 0 0	1	**0**	1	1	1	**0**	1	1	1
13	1 1 0 1	1	**0**	1	1	**0**	**0**	1	1	**0**
14	1 1 1 0	1	1	1	1	1	**0**	1	1	1
15	1 1 1 1	0	**1**	0	**1**	0	0	**1**	**1**	0

The obtained test codes for single and simultaneously both "nonessential" variables b and c of the function $f(a,b,c,d)$, which is implemented by the scheme in Fig. 2, are shown in Table 9 (*see* the bold numbers in the columns for $f_{b/\sim}$, $f_{c/\sim}$ and $f_{bc/\sim\sim}$).

Conclusion

A new method for generating test codes for detecting multiple stuck-at-faults in digital combinational circuits is proposed, which is based on the artificial introduction of nonessential variables and the application of the procedure of q-partition of minterms of a given function describing the operation of the studied circuit. Due to the application of a numerical set-theoretic approach to the execution of operations and procedures, the proposed method, compared to the known ones, is characterized by a relatively simpler practical implementation of detecting the mentioned faults both at any point and at several points of the studied circuit simultaneously.

REFERENCES

1. Rytsar B. Ye. A Simple Stuck-at-faults Detection Method in Digital Combinational Circuits. *Control Systems and Computers*, 2023, Vol 1 (301), 5–17. https://doi.org/10.15407/csc.2023.01.005
2. Rytsar B. Ye. A Simple Stuck-at-faults Detection Method in Digital Combinational Circuits. II. *Control Systems and Computers*, 2024, Vol 1 (301), 3–17. https://doi.org/10.15407/csc.2024.01.003
3. Azam Beg. A framework for finding minimal test vectors for stuck-at-faults. *3rd Inter. Conf. ICICT'2009 – Aug 2009,*Karachi, Pakistan, 259–262.
4. Jong Chang Kim, Vishvani D. Agraval, Kewal K. Saluja. Multiple Faults: Modeling, Simulation and Test. *7th ASPDAC and 15th Int'l Conf. on VLSI Design*, 2002, pp. 592–597.
5. Jutman A., Ubar R. Design error diagnostic in digital circuits with stuck-at-fault model. *Microelectronics Reliability*, 28 Febr. 2000, Vol. 40 (2), 307–320.
6. Koundinya P., Reddy S., Deepak V., Rutwesh K., Deshpande A.. Test Set Generation for Multiple Faults in Boolean Systems using SAT Solver. *12th Inter. Conf. ICCCNT* – 06-08 July 2021, 329–340.
7. Parag K. Lala. *An Introduction to Logic Circuit Testing*. Morgan & Claypool, 2009, 111 p.
8. Kohavi Z., Jha *N*. Switching and Finite Automata Theory. *Cambridge University Press,* 2010, 206—250.
9. Leila Malihi, Razieh Malihi. Single stuck-at-faults detection using test generation vector and deep stacked-sparse-auto-encoder. *SN Applied Sciences*, 2020, Vol. 2 (1715). https://doi.org/10.1007/s42452-020-03460-0
10. Fujiwara H. Logic testing and design for testability. In *Comp. Syst. Series. Cambridge*, MA: Mass. Inst. Tech, 1986.
11. Karkouri Y, Aboulhamid. Multiple Stuck-at Fault in Logic Circuits. URL: http://www.iro.umontreal.ca/~aboulham/pdfs_sources/KCCVLSI.pdf
12. Rytsar B.Ye. Dekompozytsija bulovykh funktsij metodom q-rozbyttja. *UsiM,* 1999, Issue 6, 29–42. [In Ukrainian: Рицар Б.Є. Декомпозиція булових функцій методом q-розбиття. 1]
13. Rytsar B.Ye. *Teoretyko-mnozhynni optymizatsijni metody lohikovoho syntezu kombinatsijnykh merezh*. Dissertation DSc (Engineering), Lviv, 2004, 138–142. [In Ukrainian: Рицар Б.Є. Теоретико-множинні оптимізаційні методи логікового синтезу комбінаційних мереж: дис. доктора техн.наук. Львів, 2004,138–142.]

14. Rytsar B., Romanowski P., Shvay A. Set-theoretical Constructions of Boolean Functions and theirs Applications in Logic Synthesis. *Fundamenta Informatica*, 2010, Vol. 99, №3, 339–354.

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НОВИЙ МЕТОД ГЕНЕРУВАННЯ ТЕСТОВИХ КОДІВ
ДЛЯ ВИЯВЛЕННЯ МНОЖИННИХ ПОШКОДЖЕНЬ *STUCK-AT-FAULTS*
У КОМБІНАЦІЙНИХ СХЕМАХ. ЧАСТИНА 1

Вступ. Важливим розділом логікового проєктування цифрових пристроїв є технічна діагностика, в межах якої розробляються методи перевірки технічного стану пристроїв для забезпечення надійності їх роботи. Виявити несправність у схемі пристрою можна послідовністю контрольних тестів (генеруванням векторів тестових кодів) на її входах та спостереження результатів на її виходах. На практиці проєктування мікросхем часто трапляються ситуації, коли пошкодження типу *stuck-at-faults* (0/1) можуть виникати як в одній точці схеми, так і в кількох різних взаємопов'язаних точках схеми одночасно, які складно виявляти. Відомі методи діагностики множинних пошкоджень такого типу, які ґрунтуються на моделюванні одиночних помилок та символьних методах, не забезпечують переконливі докази достовірності результату, що знижує надійність процесу проєктування.

Мета статті. Запропонувати метод генерування векторів тестових кодів для виявлення як одиночних, так і множинних пошкоджень типу *stuck-at-faults* (0/1) у комбінаційних пристроях, який порівняно з відомими методами може забезпечувати достовірні результати з допомогою реалізації простих операцій і процедур.

Методи. Запропонований метод генерування тестових кодів ґрунтується на числовому теоретико-множинному підході до реалізації всіх операцій і процедур, а саме: штучного впровадження у буловий простір повної функції $f(x_1, x_2, ..., x_ї)$, що описує роботу схеми досліджуваного комбінаційного пристрою, одної або більше (до $n-1$) неістотних змінних та застосуванні процедури q-розбиття мінтермів досконалої ТМФ Y^1 функції f.

Результати. За допомогою згенерованих запропонованим методом векторів тестових кодів можна визначити в схемі пристрою як місце пошкодження, так і тип одиночного та множинного *stuck-at-faults* (0/1) пошкодження. Показано застосування процедури q-розбиття двійкових мінтермів, на основі якої реалізується впровадження одної неістотної змінної та формування псевдодосконалої ТМФ $Y^1_{x_i/\sim}$ функції f для визначення одиночних пошкоджень, а також більше (від двох до n–1) неістотних змінних та формування відповідних псевдодосконалих ТМФ функції f для визначення множинних пошкоджень.

Висновки. Завдяки застосуванню числового теоретико-множинного підходу для виконання операцій і процедур пропонований метод, порівняно з відомими, відрізняється відносно простішою практичною реалізацією виявлення згаданих несправностей як в будь якій одній точці, так і в одночасно кількох точках досліджуваної схеми. Зазначені переваги методу ілюструють наведені в статті приклади визначення можливих пошкоджень у реальних схемах комбінаційних пристроїв.

Ключові слова: *комбінаційна схема, одиничне та множинне пошкодження stuck-at-faults (0/1), q-розбиття мінтермів, неістотні змінні, вектор тестових кодів.*