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A NEW METHOD FOR GENERATING TEST CODES TO DETECT MULTIPLE STUCK-AT-FAULTS IN COMBINATIONAL CIRCUITS. Part 2

The article is devoted to a new method of generating test codes to detect multiple stuck-at-faults (0/1) type damages in digital combinational PIPO-circuits, which is based on the artificial introduction of non-essential variables and the application of the procedure of q-partition of the system minterms of a given system of functions. Due to the use of the numerical set-theoretic approach, the proposed method differs from the known ones in a relatively simpler practical implementation to detect stuck-at-faults (0/1) type damages both at one point and simultaneously at several points simultaneously of the studied circuit.

Keywords: *combinational PIPO-circuit, single and multiple stuck-at-faults (0/1) type damages, procedure of q-partition of system minterms, non-essential variables, vector of test codes.*

Introduction

The method of generating test codes for detecting multiple stuck-at-faults (0/1) in combinational circuits, proposed in the previous article (Part 1), is of particularly practical importance for digital microelectronics. In digital devices of the PIPO type that are described by the system of functions $F(X)=\{f_1(X), f_2(X),..., f_s(X)\}$, $X=\{x_1, x_2,..., x_n\}$, and compared to devices of the PISO type, the interconnectedness of system functions makes the detection of multiple stuck-at faults (0/1) considerably more difficult

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than detecting a single one. Multiple damage of the stuck-at-faults (0/1) type in the studied circuit manifests itself as the simultaneous appearance of an arbitrary combination of faults s-a-0 and s-a-1 at any of its points, and therefore, the detected damage in any one function can simultaneously occur in other functions of the system. Therefore, the usage of the search for such damage in logic networks that are based on single fault methods and the determination of the code test vectors does not provide a reliable result for the diagnosis of digital devices of the PIPO type.

A number of analytical and heuristic methods (path sensitizing, Equivalent-Normal-Form (ENF) method, Karnaugh map and tabular method, the ENF K-map method, the Boolean difference method, the SPOOF and ILP method and Genetic Algorithm method) that have appeared in the literature over the past decades is presented in [3]. There are also methods for diagnosing multiple stuck-at-faults in combinational circuits that work on the basis of modeling single faults with the procedure of repeated addition and elimination of faults [4–6] and on the analytical approach basis [7–14]. However, these methods have restrictive assumptions about the final result and they are quite complex and time-consuming in terms of finding test code vectors. Moreover, the proposed structural approaches additionally complicate the correction of design errors and impose certain practical limitations in the process of detecting multiple damage in combinational PIPO-circuits.

The paper considers a method for generating test code vectors to determine the location and of stuck-at-faults (0/1) type of single and multiple faults in combinational PIPO-circuits. It is based on the artificial introduction of one or more non-essential variables into the studied circuit and the application of the well-known procedure of *q*-partition [15, 16] of the system minterms of a given system of Boolean functions.

Theoretical Part

To determine the location and type of multiple stuck-at-faults (0/1) in combinational circuits described by a system of Boolean functions [2], the generation of test code vectors is performed using the same algorithm as for a single function *f* (Part 1), but taking into account that the objects of operations and procedures will now be system minterms $m_{s_1}, m_{s_2}, ..., m_{s_k}$ of the set $Y^1_{1,2,...,s} = \{m_{s_1}, m_{s_2}, ..., m_{s_k}\}^1$ with indices $s_i \in \{1, 2, ..., s\}$ of functions of the given system *F*(*X*), the total number of which is equal to $s \times 2^n$.

The *q*-partition procedure (operator $\Rightarrow$) is performed over the system minterms $m_{s_1}, m_{s_2}, ..., m_{s_k}$ of a given system *F*(*X*) for an arbitrary literal mask $\{l_{\alpha_1} l_{\alpha_2} \cdots l_{\alpha_{n-q}} \mid l_{\beta_1} l_{\beta_2} \cdots l_{\beta_q}\}$, where $l_i \in \{\overline{x}_i, x_i\}$ and the dash | is the *q*-partition symbol. As a result, a set of partitioned system minterms is formed, which consist of system subminterms of the (*n* – *q*)-class and *q*-class. As noted in Part 1, binary subminterms of the (*n* – *q*)-class are chosen to introduce "non-essential" variables. Therefore, the simplification of the set of partitioned system minterms is performed first by the procedure of "right union" (operator $\overset{|\cup}{\Rightarrow}$) and then (if necessary) by the procedure of "left union" (operator $\overset{\cup|}{\Rightarrow}$). Then, "non-essential" variables will be introduced for any partition mask provided that the values of the system subminterms of the $(n-q)$-class form the complete set $\mathbf{E}_2^r$, where $r = 1, 2, ..., (n-1)$ is the number of introduced "non-essential" variables. Therefore, to introduce one ($r = 1$) "non-essential" variable x_i into the system $F(X)$, it is necessary to impose a literal mask $\{l_{\alpha_i} \mid l_{\beta_1} l_{\beta_2} \cdots l_{\beta_{n-1}}\}$ on the set of system minterms $m_{s_1}, m_{s_2}, ..., m_{s_k}$ and to obtain (after the simplification procedure) two subsets ($\mathbf{E}_2^1 = 2$) of partitioned system minterms $\{0_{\alpha_i} \mid l_{\beta_1} l_{\beta_2} \cdots l_{\beta_{n-1}}\}$ and $\{1_{\alpha_i} \mid l_{\beta_1} l_{\beta_2} \cdots l_{\beta_{n-1}}\}$. The latter, similarly as in [2], will form pseudo-perfect STFs $Y^1_{s_i(x_i/0)}$ and $Y^1_{s_i(x_i/1)}$ after introducing into the values of their subminterms of the $(n-q)$-class (here 1-class) log 1 $(x_i / 0)$, forming $\{(0_{\alpha_i}, 1) \mid l_{\beta_1} l_{\beta_2} \cdots l_{\beta_{n-1}}\}$, and log 0 $(x_i / 1)$, forming $\{(0, 1_{\alpha_i}) \mid l_{\beta_1} l_{\beta_2} \cdots l_{\beta_{n-1}}\}$, and applying the concatenation procedure (operator $\overset{con}{\Rightarrow}$). The elements of the pseudo-perfect STFs $Y^1_{s_i(x_i/0)}$ and $Y^1_{s_i(x_i/1)}$ of the "damaged" system $F(X)$ are then combined with the system minterms of the perfect STF Y_s^1 of the given system $F(X)$ in a polynomial format, where a simplification procedure is performed by eliminating pairs of elements that have the same values and the same function indices, as well as by eliminating those elements of the perfect STF Y_s^1 that have a pair with the same value but different indices. After that, the test code vectors in the case of single stuck-at-faults (0/1) type damages will be formed in the form (4) and (5) (Part 1).

In the case of introducing of two ($r = 2$) "non-essential" variables x_i and x_j, that is necessary to determine the desired vectors for multiple damage at two points of the circuit, the algorithm is implemented in a similar way, starting with the imposition of a literal mask $\{l_{\alpha_i} l_{\beta_j} \mid l_{\beta_1} l_{\beta_2} \cdots l_{\beta_{n-2}}\}$ on the set of system minterms of the given system $F(X)$. At the same time, in order to form four pseudo-perfect STFs $Y^1_{s_i(x_i x_j/00)}$, $Y^1_{s_i(x_i x_j/01)}$, $Y^1_{s_i(x_i x_j/10)}$, $Y^1_{s_i(x_i x_j/11)}$, it is necessary to introduce certain logical values from the set {00, 01, 10, 11} into the values of their subminterms of the $(n-q)$-class (here of 2-class) so that the complete set $\mathbf{E}_2^2 = 4$ is formed there and so on.

We also note, that similarly to the previous article (Part 1), when $|Y_s^1| > |Y_s^0|$, it is appropriate to execute all procedures and operations taking into account the set Y^0, as well as that the operations of combining and simplifying the elements of the pseudo-perfect STF $Y^0_{x_i/\sim}$ and the elements of the perfect STF Y_s^0 will now be performed in the inverse polynomial format; in particular, in the case of one "non-essential" variable x_i this is the set $\begin{Bmatrix} Y^0_{x_i/\sim} \\ Y^0_s \end{Bmatrix}^{\overline{\oplus}}$. In order to determine the vectors of test codes of the form (4) and (5) in this simplified set (Part 1), it is necessary to perform certain operations on the elements of these sets, taking into account the priority of the elements of the pseudo-perfect STF $Y^0_{x_i/\sim}$ over the elements of the perfect STF Y^0_s. To implement the proposed approach, the initial requirement is to cancel first all the function indices in the elements of the set Y^0_s, that is, to form a set of elements without indices Y^{0^*}. Next, it is necessary to perform procedural transformations on the elements of the sets $\begin{cases} Y^0_{x_i/\sim} \\ Y^{0^*} \end{cases}$ into vectors of test codes of the form (4) and (5) $Y^0_{x_i/\sim} \to C^1_{x_i/\sim}$ and $Y^{0^*} \to C^0_{x_i/\sim}$, that is, to perform transformations $\begin{cases} Y^0_{x_i/\sim} \\ Y^{0^*} \end{cases} \overset{Y\to C}{\Rightarrow} \begin{cases} C^1_{x_i/\sim} \\ C^0_{x_i/\sim} \end{cases}$ (where $\overset{Y\to C}{\Rightarrow}$ is the transformation operator), according to the following rule:

1) move the elements of the set without indices Y^{0^*} into the set $C^1_{x_i/\sim}$, writing down all the indices s of the given system $F(X)$;

2) move the elements of the set $Y^0_{x_i/\sim}$ that have all the indices s of the given system $F(X)$ to the set $C^0_{x_i/\sim}$ by canceling all their indices, and move the elements of the set $Y^0_{x_i/\sim}$ with indices $s' < s$ to the set $C^1_{x_i/\sim}$ with indices $s - s'$.

Thus, the procedure for converting (operator $\overset{Y\to C}{\Rightarrow}$) a set from the inverse polynomial format into a set of vectors of test codes of the form (4) and (5) (*see* Part 1) looks like this (using the example of one "non-essential" variable x_i):

$$\begin{Bmatrix} Y^0_{x_i/0} \\ Y^0 \end{Bmatrix}^{\bar{\oplus}} \Rightarrow \begin{cases} Y^0_{x_i/0} \\ Y^{0^*} \end{cases} \overset{Y\to C}{\Rightarrow} \begin{cases} C^1_{x_i/0} \\ C^0_{x_i/0} \end{cases}, \tag{1}$$

$$\begin{Bmatrix} Y^0_{x_i/1} \\ Y^0 \end{Bmatrix}^{\bar{\oplus}} \Rightarrow \begin{cases} Y^0_{x_i/1} \\ Y^{0^*} \end{cases} \overset{Y\to C}{\Rightarrow} \begin{cases} C^1_{x_i/1} \\ C^0_{x_i/1} \end{cases}. \tag{2}$$

The transformation procedures (1) and (2) are illustrated by an example:

$$\Rightarrow \begin{Bmatrix} 100_{1,2}, 101_1 \\ 000_{1,2}, 001_1, 110_2 \end{Bmatrix}^{\bar{\oplus}} \Rightarrow \begin{cases} \{100_{1,2}, 101_1\}^0 \\ \{000, 001, 110\}^0 \end{cases} \overset{Y\to C}{\Rightarrow} \begin{cases} \{000_{1,2}, 001_{1,2}, 101_2, 110_{1,2}\}^1 \\ \{100\}^0 \end{cases}.$$

It is obvious that procedures (1) and (2) will be performed in the same way for a larger number of "non-essential" variables, taking into account the elements of the sets of indices of the functions of the given system.

Practical Part

Let us consider the proposed method of generating and determining test code vectors based on the introduction of one or more "non-essential"

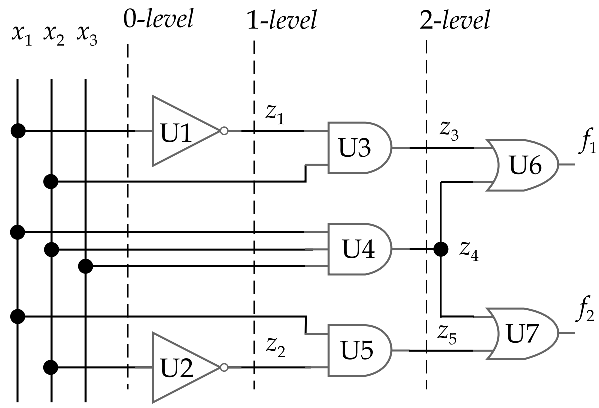

Fig. 1

variables for detecting multiple stuck-at-fault (0/1) type faults on examples of circuits described by a system of functions.

Example 1. Determine test code vectors for detecting single and multiple stuck-at-fault (0/1) type faults at different points on the 0-, 1- and 2-levels of the circuit shown in Fig. 1 (the circuit is borrowed from [2, 17, p. 87]).

Solution. The scheme in Fig. 1 is described by a system of functions $\begin{cases} f_1(x_1,x_2,x_3)=\overline{x}_1x_2 \vee x_1x_2x_3 \\ f_2(x_1,x_2,x_3)=x_1\overline{x}_2 \vee x_1x_2x_3 \end{cases}$, to which corresponds a system of perfect STFs $\{Y_1^1, Y_2^1\}$ $\begin{cases} Y_1^1=\{(01-),(111)\}^1 \equiv \{2,3,7\}^1 \\ Y_2^1=\{(10-),(111)\}^1 \equiv \{4,5,7\}^1 \end{cases}$ and a set of system minterms $Y_{1,2}^1=\{010_1, 011_1, 100_2, 101_2, 111_{1,2}\}^1$.

To detect stuck-at-fault (0/1) type "damages" at individual points of the circuit, at the 0-level of the circuit, we perform a 2-partition of the system minterms of the set $Y_{1,2}^1$:

$$Y_{1,2}^1=\{010_1, 011_1, 100_2, 101_2, 111_{1,2}\}^1 \overset{P^2}{\Rightarrow}$$

$$\overset{P^2}{\Rightarrow} \begin{cases} \{l_1|l_2l_3\}=\{(0\,|\,10,11)_1, (1\,|\,00_2, 01_2, 11_{1,2})\}^1 \\ \{l_2|l_1l_3\}=\{(0\,|\,10,11)_2, (1\,|\,00_1, 01_1, 11_{1,2})\}^1 \\ \{l_3|l_1l_2\}=\{(0\,|\,01_1, 10_2), (1\,|\,01_1, 10_2, 11_{1,2})\}^1 \end{cases}.$$

From here we define the vectors of test codes at the 0-level of the scheme, introducing into the studied scheme the "non-essential" variables x_1, x_2 and x_3. As mentioned above, after the procedure of concatenation of the partitioned system minterms, the next procedure is the simplification in the polynomial format of the combined sets – pseudo-perfect STF $Y_{1,2(x_i/\sim)}^1$ and perfect STF $Y_{1,2}^1$. It is performed by eliminating pairs of elements with the same values and indices, as well as elements of perfect STF $Y_{1,2}^{1}$ having the same values but different indices, compared to the elements of the set $Y_{1,2(x_i/\sim)}^{1}$ (we will denote them by a dash with the opposite slope). Therefore, the required vectors for the "non-essential" variable x_1 are obtained as follows:

$$Y_{1,2(x_1/0)}^{1} = \{((\mathbf{0},1)\,|\,10,11)_1\}^1 \overset{con}{\Rightarrow} \{(010,011,110,111)_1\}^1 \Rightarrow$$

$$\Rightarrow \begin{Bmatrix} \cancel{010_1,011_1},110_1,111_1 \\ \cancel{010_1,011_1},100_2,101_2,\bcancel{111}_{1,2} \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{110_1,111_1\}^1 \\ \{100_2,101_2\}^0 \end{cases},$$

$$Y_{1,2(x_1/1)}^{1} = \{(0,\mathbf{1})\,|\,00_2,01_2,11_{1,2}\}^1 \overset{con}{\Rightarrow} \{000_2,001_2,011_{1,2},100_2,101_2,111_{1,2}\}^1 \Rightarrow$$

$$\Rightarrow \begin{Bmatrix} 000_2,001_2,011_{1,2},\cancel{100_2,101_2,111}_{1,2} \\ 010_1,\bcancel{011}_1,\cancel{100_2,101_2,111}_{1,2} \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{000_2,001_2,011_{1,2}\}^1 \\ \{010_1\}^0 \end{cases};$$

for "non-essential" variables x_2 and x_3 we get the same vectors as in ([2] Table 7):

$$Y_{1,2(x_2/0)}^{1} = \begin{cases} \{110_2,111_2\}^1 \\ \{010_1,011_1\}^0 \end{cases},\ Y_{1,2(x_2/1)}^{1} = \begin{cases} \{000_1,001_1,101_{1,2}\}^1 \\ \{100_2\}^0 \end{cases};$$

$$Y_{1,2(x_3/0)}^{1} = \begin{cases} \{\varnothing\}^1 \\ \{111_{1,2}\}^0 \end{cases},\ Y_{1,2(x_3/1)}^{1} = \begin{cases} \{110_{1,2}\}^1 \\ \{\varnothing\}^0 \end{cases}.$$

To determine the vectors of test codes with simultaneous "damage" at two points of 0-level of the scheme, we apply the 1-partition of the system minterms of the set $Y_{1,2}^{1}$:

$$Y_{1,2}^{1} = \{010_1,011_1,100_2,101_2,111_{1,2}\}^1 \overset{P^1}{\Rightarrow}$$

$$\overset{P^1}{\Rightarrow} \begin{cases} \{l_1 l_2 | l_3\} = \{(00\,|\,\varnothing),(01\,|\,0,1)_1,(10\,|\,0,1)_2,(11\,|\,1)_{1,2}\}^1 \\ \{l_1 l_3\,|\,l_2\} = \{(00,01\,|\,1)_1,(10\,|\,0)_2,(11\,|\,0_2,1_{1,2})\}^1 \\ \{l_2 l_3 | l_1\} = \{(00,01\,|\,1)_2,(10\,|\,0)_1,(11\,|\,0_1,1_{1,2})\}^1 \end{cases};$$

$$Y_{1,2(x_1x_2/00)}^{1} = \{(\mathbf{00},01,10,11)\,|\,\varnothing\}^1 \overset{con}{\Rightarrow} \{\varnothing\}^1 \Rightarrow$$

$$\Rightarrow \begin{Bmatrix} \varnothing \\ 010_1,011_1,100_2,101_2,111_{1,2} \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{\varnothing\}^1 \\ \{010_1,011_1,100_2,101_2,111_{1,2}\}^0 \end{cases},$$

$$Y_{1,2(x_1x_2/01)}^{1} = \{(00,\mathbf{01},10,11)\,|\,0,1)_1\}^1 \overset{con}{\Rightarrow}$$

$$\overset{con}{\Rightarrow} \{(000,001,010,011,100,101,110,111)_1\}^1 \Rightarrow$$

$$\Rightarrow \begin{cases} 000_1, 001_1, \cancel{010_1, 011_1}, 100_1, 101_1, 110_1, 111_1 \\ \cancel{010_1, 011_1}, \cancel{100_2}, \cancel{101_2}, \cancel{111_{1,2}} \end{cases}^{\oplus} \Rightarrow$$

$$\Rightarrow \begin{cases} \{000_1,001_1,100_1,101_1,110_1,111_1\}^1 \\ \{\varnothing\}^0 \end{cases},$$

$$Y^1_{1,2(x_1x_2/10)} = \{(00,01,\mathbf{10},11)\,|\,0,1)_2\}^1 \overset{con}{\Rightarrow}$$

$$\overset{con}{\Rightarrow}\{(000,001,010,011,100,101,110,111)_2\}^1 \Rightarrow$$

$$\Rightarrow \begin{cases} 000_2,001_2,010_2,011_2,\cancel{100_2,101_2},110_2,111_2 \\ \cancel{010_1},\cancel{011_1},\cancel{100_2,101_2},\cancel{111_{1,2}} \end{cases}^\oplus \Rightarrow$$

$$\Rightarrow \begin{cases}\{000_2,001_2,010_2,011_2,110_2,111_2\}^1 \\ \{\varnothing\}^0\end{cases},$$

$$Y^1_{1,2(x_1x_2/11)} = \{((00,01,10,\mathbf{11})\,|\,1)_{1,2}\}^1 \overset{con}{\Rightarrow}\{(001,011,101,111)_{1,2}\}^1 \Rightarrow$$

$$\Rightarrow \begin{cases} 001_{1,2},011_{1,2},101_{1,2},\cancel{111_{1,2}} \\ 010_1,\cancel{011_1},100_2,\cancel{101_2},\cancel{111_{1,2}} \end{cases}^\oplus \Rightarrow \begin{cases}\{001_{1,2},011_{1,2},101_{1,2}\}^1 \\ \{010_1,100_2\}^0\end{cases};$$

$$Y^1_{1,2(x_1x_3/00,01)} = \{((\mathbf{00},\mathbf{01},10,11)\,|\,1)_1\}^1 \overset{con}{\Rightarrow}\{(010,011,110,111)_1\}^1 \Rightarrow$$

$$\Rightarrow \begin{cases} \cancel{010_1,011_1},110_1,111_1 \\ \cancel{010_1,011_1},100_2,101_2,\cancel{111_{1,2}} \end{cases}^\oplus \Rightarrow \begin{cases}\{110_1,111_1\}^1 \\ \{100_2,101_2\}^0\end{cases},$$

$$Y^1_{1,2(x_1x_3/10)} = \{((00,01,\mathbf{10},11)\,|\,0)_2\}^1 \overset{con}{\Rightarrow}\{(000,001,100,101)_2\}^1 \Rightarrow$$

$$\begin{cases} 000_2,001_2,\cancel{100_2,101_2} \\ 010_1,011_1,\cancel{100_2,101_2},111_{1,2}\end{cases}^\oplus \Rightarrow \begin{cases}\{000_2,001_2\}^1\\ \{010_1,011_1,111_{1,2}\}^0\end{cases},$$

$$Y^1_{x_1x_3/11} = \{(00,01,10,\mathbf{11})\,|\,0_2,1_{1,2}\}^1 \overset{con}{\Rightarrow}$$

$$\{000_2,001_2,010_{1,2},011_{1,2},100_2,101_2,110_{1,2},111_{1,2}\}^1 \Rightarrow$$

$$\Rightarrow\begin{cases}000_2,001_2,010_{1,2},011_{1,2},\cancel{100_2,101_2},110_{1,2},\cancel{111_{1,2}} \\ \cancel{010_1},\cancel{011_1},\cancel{100_2,101_2,111_{1,2}}\end{cases}^\oplus \Rightarrow$$

$$\Rightarrow\begin{cases}\{000_2,001_2,010_{1,2},011_{1,2},110_{1,2}\}^1\\ \{\varnothing\}^0\end{cases};$$

$$Y^1_{1,2(x_2x_3/00,01)} = \{((\mathbf{00},\mathbf{01},10,11)\,|\,1)_2\}^1 \overset{con}{\Rightarrow}\{(100,101,110,111)_2\}^1 \Rightarrow$$

$$\begin{cases}\cancel{100_2,101_2},110_2,111_2 \\ 010_1,011_1,\cancel{100_2,101_2},\cancel{111_{1,2}}\end{cases}^\oplus \Rightarrow \begin{cases}\{110_2,111_2\}^1\\ \{010_1,011_1\}^0\end{cases},$$

$$Y^{1}_{1,2(x_2x_3/10)}=\{((00,01,\mathbf{10},11)\,|\,0)_1\}^1\overset{con}{\Rightarrow}\{(000,001,010,011)_1\}^1\Rightarrow$$

$$\left\{\begin{matrix}000_1,001_1,\cancel{010_1},\cancel{011_1}\\ \cancel{010_1},\cancel{011_1},100_2,101_2,111_{1,2}\end{matrix}\right\}^{\oplus}\Rightarrow\begin{cases}\{000_1,001_1\}^1\\ \{100_2,101_2,111_{1,2}\}^0\end{cases},$$

$$Y^{1}_{1,2(x_2x_3/11)}=\{(00,01,10,\mathbf{11})\,|\,0_1,1_{1,2}\}^1\overset{con}{\Rightarrow}$$

$$\overset{con}{\Rightarrow}\{000_1,001_1,010_1,011_1,100_{1,2},101_{1,2},110_{1,2},111_{1,2}\}^1\Rightarrow$$

$$\Rightarrow\left\{\begin{matrix}000_1,001_1,\cancel{010_1},\cancel{011_1},100_{1,2},101_{1,2},110_{1,2},\cancel{111_{1,2}}\\ \cancel{010_1},\cancel{011_1},\cancel{100_2},\cancel{101_2},\cancel{111_{1,2}}\end{matrix}\right\}^{\oplus}\Rightarrow$$

$$\Rightarrow\begin{cases}\{000_1,001_1,100_{1,2},101_{1,2},110_{1,2}\}^1\\ \{\varnothing\}^0\end{cases}.$$

Table 1 contains test code vectors for detecting possible stuck-at-fault (0/1) type damages at two points of the 0-level of the circuit.

Table 1

Stuck-at-fault in $f_{1,2}$	$x_1x_2/\sim\sim$	$x_1x_3/\sim\sim$	$x_2x_3/\sim\sim$
s-a-00	$\begin{pmatrix}010_1\\011_1\\100_2\\101_2\\111_{1,2}\end{pmatrix}^0$	$\begin{pmatrix}110_1\\111_1\end{pmatrix}^1,\begin{pmatrix}100_2\\101_2\end{pmatrix}^0$	$\begin{pmatrix}110_2\\111_2\end{pmatrix}^1,\begin{pmatrix}010_1\\011_1\end{pmatrix}^0$
s-a-01	$\begin{pmatrix}000_1\\001_1\\100_1\\101_1\\110_1\\111_1\end{pmatrix}^1$	$\begin{pmatrix}110_1\\111_1\end{pmatrix}^1,\begin{pmatrix}100_2\\101_2\end{pmatrix}^0$	$\begin{pmatrix}110_2\\111_2\end{pmatrix}^1,\begin{pmatrix}010_1\\011_1\end{pmatrix}^0$
s-a-10	$\begin{pmatrix}000_2\\001_2\\010_2\\011_2\\110_2\\111_2\end{pmatrix}^1$	$\begin{pmatrix}000_2\\001_2\end{pmatrix}^1,\begin{pmatrix}010_1\\011_1\\111_{1,2}\end{pmatrix}^0$	$\begin{pmatrix}000_1\\001_1\end{pmatrix}^1,\begin{pmatrix}100_2\\101_2\\111_{1,2}\end{pmatrix}^0$
s-a-11	$\begin{pmatrix}001_{1,2}\\011_{1,2}\\101_{1,2}\end{pmatrix}^1,\begin{pmatrix}010_1\\100_2\end{pmatrix}^0$	$\begin{pmatrix}000_2\\001_2\\010_{1,2}\\011_{1,2}\\110_{1,2}\end{pmatrix}^1$	$\begin{pmatrix}000_1\\001_1\\100_{1,2}\\101_{1,2}\\110_{1,2}\end{pmatrix}^1$

Similarly, we define the vectors of test codes on the 1-level of the scheme (*see* Fig. 1) for the system of functions $\begin{cases} f_1(z_1,z_2,x_3)=z_1\overline{z}_2 \vee \overline{z}_1\overline{z}_2 x_3 \\ f_2(z_1,z_2,x_3)=\overline{z}_1 z_2 \vee \overline{z}_1\overline{z}_2 x_3 \end{cases}$, where $z_1=\overline{x}_1$, and $z_2=\overline{x}_2$, to which corresponds the system of perfect STFs $\{Y_1^1,Y_2^1\}\begin{cases} Y_1^1=\{(10-),(001)\}^1 \\ Y_2^1=\{(01-),(001)\}^1 \end{cases}$, and hence $Y_{1,2}^1=\{001_{1,2},010_2,011_2,100_1,101_1\}^1$ or $Y_{1,2}^0=\{000_{1,2},010_1,011_1,100_2,101_2,110_{1,2},111_{1,2}\}^0$.

As a result of the 2-partition of the system minterms of the set $Y_{1,2}^1$, we obtain the following sets:

$$Y_{1,2}^1=\{001_{1,2},010_2,011_2,100_1,101_1\}^1 \overset{p^2}{\Rightarrow}$$

$$\overset{p^2}{\Rightarrow}\begin{cases} \{l_1|l_2l_3\}=\{(0\,|\,01_{1,2},10_2,11_2),(1\,|\,00_1,01_1)\}^1 \\ \{l_2|l_1l_3\}=\{(0\,|\,01_{1,2},10_1,11_1),(1\,|\,00_2,01_2)\}^1 \\ \{l_3|l_1l_2\}=\{(0\,|\,01_2,10_1),(1\,|\,00_{1,2},01_2,10_1)\}^1 \end{cases}.$$

Let's define the vectors of test codes for the "non-essential" variable z_1:

$$Y_{1,2(z_1/0)}^1=\{(\mathbf{0},1)\,|\,01_{1,2},10_2,11_2\}^1 \overset{con}{\Rightarrow}\{001_{1,2},010_2,011_2,101_{1,2},110_2,111_2\}^1 \Rightarrow$$

$$\Rightarrow\begin{Bmatrix} \cancel{001_{1,2},010_2,011}_2,101_{1,2},110_2,111_2 \\ \cancel{001_{1,2},010_2,011}_2,100_1,\cancel{101}_1 \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{101_{1,2},110_2,111_2\}^1 \\ \{100_1\}^0 \end{cases},$$

$$Y_{1,2(z_1/1)}^1=\{(0,\mathbf{1})\,|\,00_1,01_1\}^1 \overset{con}{\Rightarrow}\{000_1,001_1,100_1,101_1\}^1 \Rightarrow$$

$$\Rightarrow\begin{Bmatrix} 000_1,001_1,\cancel{100_1,101}_1 \\ \cancel{00}1_{1,2},010_2,011_2,\cancel{100_1,101}_1 \end{Bmatrix}^{\oplus} \Rightarrow \begin{cases} \{000_1,001_1\}^1 \\ \{010_2,011_2\}^0 \end{cases}.$$

The test code vectors for z_1, as well as for the remaining "non-essential" variables z_2 and x_3, are the same as in (in [2] Table 9), namely:

$$Y_{1,2(z_2/0)}^1=\begin{cases} \{011_{1,2},110_1,111_1\}^1 \\ \{010_2\}^0 \end{cases},\ Y_{1,2(z_2/1)}^1=\begin{cases} \{000_2,001_2\}^1 \\ \{100_2,101_2\}^0 \end{cases};$$

$$Y_{1,2(x_3/0)}^1=\begin{cases} \{\varnothing\}^1 \\ \{001_{1,2}\}^0 \end{cases},\ Y_{1,2(x_3/1)}^1=\begin{cases} \{000_{1,2}\}^1 \\ \{\varnothing\}^0 \end{cases}.$$

In the case of simultaneous "damage" of the stuck-at-fault type (0/1) at two points of the 1-level scheme of test code vectors, we retain after 1-split the system minters of the set:

$$Y^1_{1,2} = \{001_{1,2},010_2,011_2,100_1,101_1\}^1 \overset{P^1}{\Rightarrow}$$

$$\begin{cases} \{l_1 l_2 | l_3\} = \{(00|1_{1,2}),(01|0_2,1_2),(10|0_1,1_1),(11|\varnothing)\}^1 \\ \{l_1 l_3 | l_2\} = \{(00|1_2),(01|0_{1,2},1_2),(10,11|0_1)\}^1 \\ \{l_2 l_3 | l_1\} = \{(00|1_1),(01|0_{1,2},1_1),(10,11|0_2)\}^1 \end{cases};$$

$$Y^1_{1,2(z_1z_2/00)} = \{(\mathbf{00},01,10,11)|1_{1,2}\}^1 \overset{con}{\Rightarrow} \{001_{1,2},011_{1,2},101_{1,2},111_{1,2}\}^1 \Rightarrow$$

$$\left\{\begin{matrix} \cancel{001}_{1,2},011_{1,2},101_{1,2},111_{1,2} \\ \cancel{001}_{1,2},010_2,\cancel{011}_2,100_1,\cancel{101}_1 \end{matrix}\right\}^{\oplus} \Rightarrow \begin{cases} \{011_{1,2},101_{1,2},111_{1,2}\}^1 \\ \{010_2,100_1\}^0 \end{cases},$$

$$Y^1_{1,2(z_1z_2/01)} = \{(00,\mathbf{01},10,11)|0_2,1_2\}^1 \overset{con}{\Rightarrow}$$

$$\overset{con}{\Rightarrow} \{000_2,001_2,010_2,011_2,100_2,101_2,110_2,111_2\}^1 \Rightarrow$$

$$\Rightarrow \left\{\begin{matrix} 000_2,001_2,\cancel{010_2,011_2},100_2,101_2,110_2,111_2 \\ \cancel{001}_{1,2},\cancel{010_2,011_2},\cancel{100}_1,\cancel{101}_1 \end{matrix}\right\}^{\oplus} \Rightarrow$$

$$\Rightarrow \begin{cases} \{000_2,001_2,100_2,101_2,110_2,111_2\}^1 \\ \{\varnothing\}^0 \end{cases},$$

$$Y^1_{1,2(z_1z_2/10)} = \{(00,01,\mathbf{10},11)|0_1,1_1\}^1 \overset{con}{\Rightarrow}$$

$$\overset{con}{\Rightarrow} \{000_1,001_1,010_1,011_1,100_1,101_1,110_1,111_1\}^1 \Rightarrow$$

$$\Rightarrow \left\{\begin{matrix} 000_1,001_1,010_1,011_1,\cancel{100_1,101_1},110_1,111_1 \\ \cancel{001}_{1,2},\cancel{010}_2,\cancel{011}_2,\cancel{100_1,101_1} \end{matrix}\right\}^{\oplus} \Rightarrow$$

$$\Rightarrow \begin{cases} \{000_1,001_1,010_1,011_1,110_1,111_1\}^1 \\ \{\varnothing\}^0 \end{cases},$$

$$Y^1_{1,2(z_1z_2/11)} = \{(00,01,10,\mathbf{11})|\varnothing\}^1 \overset{con}{\Rightarrow} \{\varnothing\}^1 \Rightarrow$$

$$\Rightarrow \left\{\begin{matrix} \varnothing \\ 001_{1,2},010_2,011_2,100_1,101_1 \end{matrix}\right\}^{\oplus} \Rightarrow \begin{cases} \{\varnothing\}^1 \\ \{001_{1,2},010_2,011_2,100_1,101_1\}^0 \end{cases};$$

$$Y^1_{1,2(z_1x_3/00)} = \{(\mathbf{00},01,10,11)|1_2\}^1 \overset{con}{\Rightarrow} \{010_2,011_2,110_2,111_2\}^1 \Rightarrow$$

$$\Rightarrow \left\{\begin{matrix} \cancel{010_2,011_2},110_2,111_2 \\ 001_{1,2},\cancel{010_2,011_2},100_1,101_1 \end{matrix}\right\}^{\oplus} \Rightarrow \begin{cases} \{110_2,111_2\}^1 \\ \{001_{1,2},100_1,101_1\}^0 \end{cases},$$

$$Y^1_{1,2(z_1x_3/01)} = \{(00,\mathbf{01},10,11)|0_{1,2},1_2\}^1 \overset{con}{\Rightarrow}$$

$$\overset{con}{\Rightarrow} \{000_{1,2},001_{1,2},010_2,011_2,100_{1,2},101_{1,2},110_2,111_2\}^1 \Rightarrow$$

$$\Rightarrow \left\{\begin{matrix} 000_{1,2}, \cancel{001_{1,2}, 010_2, 011_2}, 100_{1,2}, 101_{1,2}, 110_2, 111_2 \\ 001_{1,2}, 010_2, 011_2, 100_1, \cancel{101_1} \end{matrix}\right\}^{\oplus} \Rightarrow$$

$$\Rightarrow \begin{cases} \{000_{1,2}, 100_{1,2}, 101_{1,2}, 110_2, 111_2\}^1 \\ \{\varnothing\}^0 \end{cases},$$

$$Y^1_{1,2(z_1x_3/10,11)} = \{(00,01,\mathbf{10},\mathbf{11}) \,|\, 0_1\}^1 \overset{con}{\Rightarrow} \{000_1, 001_1, 100_1, 101_1\}^1 \Rightarrow$$

$$\Rightarrow \left\{\begin{matrix} 000_1, 001_1, \cancel{100_1, 101_1} \\ \cancel{001}_{1,2}, 010_2, 011_2, \cancel{100_1, 101_1} \end{matrix}\right\}^{\oplus} \Rightarrow \begin{cases} \{000_1, 001_1\}^1 \\ \{010_2, 011_2\}^0 \end{cases};$$

$$Y^1_{1,2(z_2x_3/00)} = \{(\mathbf{00},01,10,11) \,|\, 1_1\}^1 \overset{con}{\Rightarrow} \{100_1, 101_1, 110_1, 111_1\}^1 \Rightarrow$$

$$\left\{\begin{matrix} \cancel{100_1, 101_1}, 110_1, 111_1 \\ 001_{1,2}, 010_2, 011_2, \cancel{100_1, 101_1} \end{matrix}\right\}^{\oplus} \Rightarrow \begin{cases} \{110_1, 111_1\}^1 \\ \{001_{1,2}, 010_2, 011_2\}^0 \end{cases},$$

$$Y^1_{1,2(z_2x_3/01)} = \{(00,\mathbf{01},10,11) \,|\, 0_{1,2}, 1_1\}^1 \overset{con}{\Rightarrow}$$

$$\overset{con}{\Rightarrow} \{000_{1,2}, 001_{1,2}, 010_{1,2}, 011_{1,2}, 100_1, 101_1, 110_1, 111_1\}^1 \Rightarrow$$

$$\Rightarrow \left\{\begin{matrix} 000_{1,2}, 001_{1,2}, 010_{1,2}, 011_{1,2}, \cancel{100_1, 101_1}, 110_1, 111_1 \\ \cancel{001}_{1,2}, \cancel{010}_2, \cancel{011}_2, \cancel{100_1, 101_1} \end{matrix}\right\}^{\oplus} \Rightarrow$$

$$\Rightarrow \begin{cases} \{000_{1,2}, 010_{1,2}, 011_{1,2}, 110_1, 111_1\}^1 \\ \{\varnothing\}^0 \end{cases},$$

$$Y^1_{1,2(z_2x_3/10,11)} = \{(00,01,\mathbf{10},\mathbf{11}) \,|\, 0_2\}^1 \overset{con}{\Rightarrow} \{000_2, 001_2, 010_2, 011_2\}^1 \Rightarrow$$

$$\left\{\begin{matrix} 000_2, 001_2, \cancel{010_2, 011_2} \\ \cancel{001}_{1,2}, \cancel{010_2, 011}_2, 100_1, 101_1 \end{matrix}\right\}^{\oplus} \Rightarrow \begin{cases} \{000_2, 001_2\}^1 \\ \{100_1, 101_1\}^0 \end{cases}.$$

The determined vectors of test codes at 1-level to detect “damages” at two points of the circuit simultaneously, obtained by introducing two non-essential variables of the system of functions $\begin{cases} f_1(z_1, z_2, x_3) \\ f_2(z_1, z_2, x_3) \end{cases}$, are presented in Table 2.

At the 2-level of the scheme (*see* Fig. 1) we have a system of functions $\begin{cases} f_1(z_3, z_4, z_5) = z_3 \vee z_4 \\ f_2(z_3, z_4, z_5) = z_4 \vee z_5 \end{cases}$, where $z_3 = \overline{x}_1 x_2$, $z_4 = x_1 x_2 x_3$, $z_5 = x_1 \overline{x}_2$, to which corresponds the system of perfect STFs $\{Y_1^1, Y_2^1\}$ $\begin{cases} Y_1^1 = \{(1--),(-1-)\}^1 \\ Y_2^1 = \{(-1-),(--1)\}^1 \end{cases}$, as well as the sets of system minterms

$Y_{1,2}^1 = \{001_2, 010_{1,2}, 011_{1,2}, 100_1, 101_{1,2}, 110_{1,2}, 111_{1,2}\}^1$ or $Y_{1,2}^0 = \{000_{1,2}, 001_1, 100_2\}^0$.

Since $|Y_{1,2}^1| > |Y_{1,2}^0|$, then on the 2-level of the scheme it is advisable to apply the perfect STF $Y_{1,2}^0$.

Over the system minterms of the set $Y_{1,2}^0$, we perform the 2-partition procedure:

$$Y_{1,2}^0 = \{000_{1,2}, 001_1, 100_2\}^0 \overset{P^2}{\Rightarrow} \begin{cases} \{l_1 \mid l_2 l_3\} = \{(0 \mid 00_{1,2}, 01_1), (1 \mid 00_2)\}^0 \\ \{l_2 \mid l_1 l_3\} = \{(0 \mid 00_{1,2}, 01_1, 10_2), (1 \mid \varnothing)\}^0 \\ \{l_3 \mid l_1 l_2\} = \{(0 \mid 00_{1,2}, 10_2), (1 \mid 00_1)\}^0 \end{cases}.$$

From here we define the vectors of test codes at individual points of the 2-level scheme taking into account the transformation procedure (operator $\overset{Y \to C}{\Rightarrow}$):

$$Y_{1,2(z_3/0)}^0 = \{(\mathbf{0},1) \mid 00_{1,2}, 01_1\}^0 \overset{con}{\Rightarrow} \{000_{1,2}, 001_1, 100_{1,2}, 101_1\}^0 \Rightarrow$$

$$\Rightarrow \begin{Bmatrix} \cancel{000_{1,2}}, \cancel{001_1}, 100_{1,2}, 101_1 \\ \cancel{000_{1,2}}, \cancel{001_1}, \cancel{100_2} \end{Bmatrix}^{\overline{\oplus}} \Rightarrow \begin{cases} \{100_{1,2}, 101_1\}^0 \\ \{\varnothing\}^0 \end{cases} \overset{Y \to C}{\Rightarrow} \begin{cases} \{101_2\}^1 \\ \{100\}^0 \end{cases},$$

Table 2

$f_{1,2}$	$x_1 x_2 / \sim\sim$	$x_1 x_3 / \sim\sim$	$x_2 x_3 / \sim\sim$
s-a-00	$\begin{pmatrix} 011 \\ 101 \\ 111 \end{pmatrix}_{1,2}^{1}, \begin{pmatrix} 010_2 \\ 100_1 \end{pmatrix}^{0}$	$\begin{pmatrix} 110 \\ 111 \end{pmatrix}_{2}^{1}, \begin{pmatrix} 001_{1,2} \\ 100_1 \\ 101_1 \end{pmatrix}^{0}$	$\begin{pmatrix} 110 \\ 111 \end{pmatrix}_{1}^{1}, \begin{pmatrix} 001_{1,2} \\ 010_2 \\ 011_2 \end{pmatrix}^{0}$
s-a-01	$\begin{pmatrix} 000 \\ 001 \\ 100 \\ 101 \\ 110 \\ 111 \end{pmatrix}_{2}^{1}$	$\begin{pmatrix} 000_{1,2} \\ 100_{1,2} \\ 101_{1,2} \\ 110_2 \\ 111_2 \end{pmatrix}^{1}$	$\begin{pmatrix} 000_{1,2} \\ 010_{1,2} \\ 011_{1,2} \\ 110_1 \\ 111_1 \end{pmatrix}^{1}$
s-a-10	$\begin{pmatrix} 000 \\ 001 \\ 010 \\ 011 \\ 110 \\ 111 \end{pmatrix}_{1}^{1}$	$\begin{pmatrix} 000 \\ 001 \end{pmatrix}_{1}^{1} \begin{pmatrix} 010 \\ 011 \end{pmatrix}_{2}^{0}$	$\begin{pmatrix} 000 \\ 001 \end{pmatrix}_{2}^{1} \begin{pmatrix} 100 \\ 101 \end{pmatrix}_{1}^{0}$
s-a-11	$\begin{pmatrix} 001_{1,2} \\ 010_2 \\ 011_2 \\ 100_1 \\ 101_1 \end{pmatrix}^{0}$	$\begin{pmatrix} 000 \\ 001 \end{pmatrix}_{1}^{1} \begin{pmatrix} 010 \\ 011 \end{pmatrix}_{2}^{0}$	$\begin{pmatrix} 000 \\ 001 \end{pmatrix}_{2}^{1} \begin{pmatrix} 100 \\ 101 \end{pmatrix}_{1}^{0}$

$$Y^0_{1,2(z_3/1)}=\{(0,\mathbf{1})\,|\,00_2\}^0\overset{con}{\Rightarrow}\{000_2,100_2\}^0\Rightarrow$$

$$\Rightarrow\begin{Bmatrix}000_2,\cancel{100_2}\\ \cancel{000}_{1,2},001_1,\cancel{100_2}\end{Bmatrix}^{\overline{\oplus}}\Rightarrow\begin{cases}\{000_2\}^0\\ \{001\}^0\end{cases}\overset{Y\to C}{\Rightarrow}\begin{cases}\{000_1,001_{1,2}\}^1\\ \{\varnothing\}^0\end{cases};$$

$$Y^0_{1,2(z_4/0)}=\{(\mathbf{0},1)\,|\,00_{1,2},01_1,10_2\}^0\overset{con}{\Rightarrow}\{000_{1,2},001_1,010_{1,2},011_1,100_2,110_2\}^0\Rightarrow$$

$$\Rightarrow\begin{Bmatrix}\cancel{000_{1,2}},\cancel{001_1},010_{1,2},011_1,\cancel{100_2},110_2\\ \cancel{000_{1,2}},\cancel{001_1},\cancel{100_2}\end{Bmatrix}^{\overline{\oplus}}\Rightarrow$$

$$\Rightarrow\begin{cases}\{010_{1,2},011_1,110_2\}^0\\ \{\varnothing\}^0\end{cases}\overset{Y\to C}{\Rightarrow}\begin{cases}\{011_2,110_1\}^1\\ \{010\}^0\end{cases},$$

$$Y^0_{1,2(z_4/1)}=\{(0,\mathbf{1})\,|\,\varnothing\}^0\overset{con}{\Rightarrow}\{\varnothing\}^0\Rightarrow$$

$$\Rightarrow\begin{Bmatrix}\varnothing\\ 000_{1,2},001_1,100_2\end{Bmatrix}^{\overline{\oplus}}\Rightarrow\begin{cases}\{\varnothing\}^0\\ \{000,001,100\}^0\end{cases}\overset{Y\to C}{\Rightarrow}\begin{cases}\{000_{1,2},001_{1,2},100_{1,2}\}^1\\ \{\varnothing\}^0\end{cases};$$

$$Y^0_{1,2(z_5/0)}=\{(\mathbf{0},1)\,|\,00_{1,2},10_2\}^0\overset{con}{\Rightarrow}\{000_{1,2},001_{1,2},100_2,101_2\}^0\Rightarrow$$

$$\Rightarrow\begin{Bmatrix}\cancel{000}_{1,2},001_{1,2},\cancel{100_2},101_2\\ \cancel{000}_{1,2},\cancel{001_1},\cancel{100_2}\end{Bmatrix}^{\oplus}\Rightarrow\begin{cases}\{001_{1,2},101_2\}^0\\ \{\varnothing\}^0\end{cases}\overset{Y\to C}{\Rightarrow}\begin{cases}\{101_1\}^1\\ \{001\}^0\end{cases},$$

$$Y^0_{1,2(z_5/1)}=\{(0,\mathbf{1})\,|\,00_1\}^0\overset{con}{\Rightarrow}\{000_1,001_1\}^0\Rightarrow$$

$$\Rightarrow\begin{Bmatrix}000_1,\cancel{001_1}\\ \cancel{000}_{1,2},\cancel{001_1},100_2\end{Bmatrix}^{\oplus}\Rightarrow\begin{cases}\{000_1\}^0\\ \{100\}^0\end{cases}\overset{Y\to C}{\Rightarrow}\begin{cases}\{000_2,100_{1,2}\}^1\\ \{\varnothing\}^0\end{cases}.$$

Table 3 contains the truth table of a given system of functions at 2-level of the scheme and its "damaged" variants in the case of stuck-at-

Table 3

«10»	$z_3z_4z_5$	$f_{1,2}$	$f_{1,2z_3}/\sim$		$f_{1,2z_4}/\sim$		$f_{1,2z_5}/\sim$	
			$z_3/0$	$z_3/1$	$z_4/0$	$z_4/1$	$z_5/0$	$z_5/1$
0	0 0 0	0	0	$\mathbf{1_1}$	0	$\mathbf{1_{1,2}}$	0	$\mathbf{1_2}$
1	0 0 1	1_2	1_2	$\mathbf{1_{1,2}}$	1_2	$\mathbf{1_{1,2}}$	$\mathbf{0}$	1_2
2	0 1 0	$1_{1,2}$	$1_{1,2}$	$1_{1,2}$	$\mathbf{0}$	$1_{1,2}$	$1_{1,2}$	$1_{1,2}$
3	0 1 1	$1_{1,2}$	$1_{1,2}$	$1_{1,2}$	$\mathbf{1_2}$	$1_{1,2}$	$1_{1,2}$	$1_{1,2}$
4	1 0 0	1_1	$\mathbf{0}$	1_1	1_1	$\mathbf{1_{1,2}}$	1_1	$\mathbf{1_{1,2}}$
5	1 0 1	$1_{1,2}$	$\mathbf{1_2}$	$1_{1,2}$	$1_{1,2}$	$1_{1,2}$	$\mathbf{1_1}$	$1_{1,2}$
6	1 1 0	$1_{1,2}$	$1_{1,2}$	$1_{1,2}$	$\mathbf{1_1}$	$1_{1,2}$	$1_{1,2}$	$1_{1,2}$
7	1 1 1	$1_{1,2}$	$1_{1,2}$	$1_{1,2}$	$1_{1,2}$	$1_{1,2}$	$1_{1,2}$	$1_{1,2}$

faults (0/1) at one point, caused by the introduction of one specific "non-essential" variable, where the values of the "damaged" functions $f_{1,2z_3/\sim}$, $f_{1,2z_4/\sim}$ and $f_{1,2z_5/\sim}$, which differ from the values of the functions $f_{1,2}$ of the given system, are highlighted in bold.

This table also illustrates how the proposed method "works" in the event of damage at a single point on the 2-level of the studied circuit, which is shown in Fig. 1. For example, if at the input code $\langle 0,1,0\rangle$, both outputs of the circuit are observed to be log. 0, at the input code $\langle 0,1,1\rangle$ we have log 1 only at the output f_2, and at the input code $\langle 1,1,0\rangle$ we have log 1 only at the output f_1, then this indicates that the circuit has damage at a point z_4 of the stuck-at-faults type $z_4/0$ (the 6th column in Table 3).

In the case of "damage" stuck-at-faults (0/1) on the 2-level at two points of the scheme, we perform the procedure of 1-partition of the system minterms of the set $Y^0_{1,2}$:

$$Y^0_{1,2}=\{000_{1,2},001_1,100_2\}^0\overset{P^1}{\Rightarrow}\begin{cases}\{l_1l_2\,|\,l_3\}=\{(00\,|\,0_{1,2},1_1),(01,11\,|\,\varnothing),(10\,|\,0_2)\}^0\\ \{l_1l_3\,|\,l_2\}=\{(00\,|\,0_{1,2}),(01\,|\,0_1),(10\,|\,0_2),(11\,|\,\varnothing)\}^0\\ \{l_2l_3\,|\,l_1\}=\{(00\,|\,0_{1,2},1_2),(01\,|\,0_1),(10,11\,|\,\varnothing)\}^0\end{cases}.$$

The vectors of test codes in the case of introducing two "non-essential" variables are obtained as follows:

$$Y^0_{1,2(z_3z_4/00)}=\{(\mathbf{00},01,10,11)\,|\,0_{1,2},1_1\}^0\overset{con}{\Rightarrow}$$

$$\overset{con}{\Rightarrow}\{000_{1,2},001_1,010_{1,2},011_1,100_{1,2},101_1,110_{1,2},111_1\}^0\Rightarrow$$

$$\Rightarrow\begin{Bmatrix}\cancel{000_{1,2},001_1},010_{1,2},011_1,100_{1,2},101_1,110_{1,2},111_1\\ \cancel{000_{1,2},001_1},\cancel{100_2}\end{Bmatrix}^{\overline{\oplus}}\Rightarrow$$

$$\Rightarrow\begin{cases}\{010_{1,2},011_1,100_{1,2},101_1,110_{1,2},111_1\}^0\\ \{\varnothing\}^0\end{cases}\overset{Y\to C}{\Rightarrow}\begin{cases}\{011_2,101_2,111_2\}^1\\ \{010,100,110\}^0\end{cases},$$

$$Y^0_{1,2(z_3z_4/01,11)}=\{(00,\mathbf{01},10,\mathbf{11})\,|\,\varnothing\}^0\overset{con}{\Rightarrow}\{\varnothing\}^0\Rightarrow\begin{Bmatrix}\varnothing\\ 000_{1,2},001_1,100_2\end{Bmatrix}^{\overline{\oplus}}\Rightarrow$$

$$\Rightarrow\begin{cases}\{\varnothing\}^0\\ \{000,001,100\}^0\end{cases}\overset{Y\to C}{\Rightarrow}\begin{cases}\{000_{1,2},001_{1,2},100_{1,2}\}^1\\ \{\varnothing\}^0\end{cases},$$

$$Y^0_{1,2(z_3z_4/10)}=\{(00,01,\mathbf{10},11)\,|\,0_2\}^0\overset{con}{\Rightarrow}\{000_2,010_2,100_2,110_2\}^0\Rightarrow$$

$$\Rightarrow\begin{Bmatrix}000_2,010_2,\cancel{100_2},110_2\\ \cancel{000_{1,2}},001_1,\cancel{100_2}\end{Bmatrix}^{\overline{\oplus}}\Rightarrow\begin{cases}\{000_2,010_2,110_2\}^0\\ \{001\}^0\end{cases}\overset{Y\to C}{\Rightarrow}\begin{cases}\{000_1,001_{1,2},010_1,110_1\}^1\\ \{\varnothing\}^0\end{cases};$$

$$Y^0_{1,2(z_3z_5/00)}=\{(\mathbf{00},01,10,11)\,|\,0_{1,2}\}^0\overset{con}{\Rightarrow}\{000_{1,2},001_{1,2},100_{1,2},101_{1,2}\}^0\Rightarrow$$

$$\Rightarrow\left\{\begin{matrix}\cancel{000}_{1,2},001_{1,2},100_{1,2},101_{1,2}\\ \cancel{000}_{1,2},\cancel{001}_1,\cancel{100}_2\end{matrix}\right\}^{\overline{\oplus}}\Rightarrow\left\{\begin{matrix}\{001_{1,2},100_{1,2},101_{1,2}\}^0\\ \{\varnothing\}^0\end{matrix}\right.\overset{Y\to C}{\Rightarrow}\left\{\begin{matrix}\{\varnothing\}^1\\ \{001,100,101\}^0\end{matrix}\right.,$$

$$Y^0_{1,2(z_3z_5/01)}=\{(00,\mathbf{01},10,11)\,|\,0_1\}^0\overset{con}{\Rightarrow}\{000_1,001_1,100_1,101_1\}^0\Rightarrow$$

$$\Rightarrow\left\{\begin{matrix}000_1,\cancel{001}_1,100_1,101_1\\ \cancel{000}_{1,2},\cancel{001}_1,\cancel{100}_2\end{matrix}\right\}^{\overline{\oplus}}\Rightarrow\left\{\begin{matrix}\{001_1,100_1,101_1\}^0\\ \{\varnothing\}^0\end{matrix}\right.\overset{Y\to C}{\Rightarrow}\left\{\begin{matrix}\{001_2,100_2,101_2\}^1\\ \{\varnothing\}^0\end{matrix}\right.,$$

$$Y^0_{1,2(z_3z_5/10)}=\{(00,01,\mathbf{10},11)\,|\,0_2\}^0\overset{con}{\Rightarrow}\{000_2,001_2,100_2,101_2\}^0\Rightarrow$$

$$\Rightarrow\left\{\begin{matrix}000_2,001_2,\cancel{100}_2,101_2\\ \cancel{000}_{1,2},\cancel{001}_1,\cancel{100}_2\end{matrix}\right\}^{\overline{\oplus}}\Rightarrow\left\{\begin{matrix}\{000_2,001_2,101_2\}^0\\ \{\varnothing\}^0\end{matrix}\right.\overset{Y\to C}{\Rightarrow}\left\{\begin{matrix}\{000_1,001_1,101_1\}^1\\ \{\varnothing\}^0\end{matrix}\right.,$$

$$Y^0_{1,2(z_3z_5/11)}=\{(00,01,10,\mathbf{11})\,|\,\varnothing\}^0\overset{con}{\Rightarrow}\{\varnothing\}^0\Rightarrow\left\{\begin{matrix}\varnothing\\ 000_{1,2},001_1,100_2\end{matrix}\right\}^{\overline{\oplus}}\Rightarrow$$

$$\Rightarrow\left\{\begin{matrix}\{\varnothing\}^0\\ \{000,001,100\}^0\end{matrix}\right.\overset{Y\to C}{\Rightarrow}\left\{\begin{matrix}\{000_{1,2},001_{1,2},100_{1,2}\}^1\\ \{\varnothing\}^0\end{matrix}\right.;$$

$$Y^0_{1,2(z_4z_5/00)}=\{(\mathbf{00},01,10,11)\,|\,0_{1,2},1_2\}^0\overset{con}{\Rightarrow}$$

$$\overset{con}{\Rightarrow}\{000_{1,2},001_{1,2},010_{1,2},011_{1,2},100_2,101_2,110_2,111_2\}^0\Rightarrow$$

$$\Rightarrow\left\{\begin{matrix}\cancel{000}_{1,2},001_{1,2},010_{1,2},011_{1,2},\cancel{100}_2,101_2,110_2,111_2\\ \cancel{000}_{1,2},\cancel{001}_1,\cancel{100}_2\end{matrix}\right\}^{\overline{\oplus}}\Rightarrow$$

$$\Rightarrow\left\{\begin{matrix}\{001_{1,2},010_{1,2},011_{1,2},101_2,110_2,111_2\}^0\\ \{\varnothing\}^0\end{matrix}\right.\overset{Y\to C}{\Rightarrow}\left\{\begin{matrix}\{101_1,110_1,111_1\}^1\\ \{001,010,011\}^0\end{matrix}\right.,$$

$$Y^0_{1,2(z_4z_5/01)}=\{(00,\mathbf{01},10,11)\,|\,0_1\}^0\overset{con}{\Rightarrow}\{000_1,001_1,010_1,011_1\}^0\Rightarrow$$

$$\Rightarrow\left\{\begin{matrix}000_1,\cancel{001}_1,010_1,011_1\\ \cancel{000}_{1,2},\cancel{001}_1,100_2\end{matrix}\right\}^{\overline{\oplus}}\Rightarrow\left\{\begin{matrix}\{000_1,010_1,011_1\}^0\\ \{100\}^0\end{matrix}\right.\overset{Y\to C}{\Rightarrow}\left\{\begin{matrix}\{000_2,010_2,011_2,100_{1,2}\}^1\\ \{\varnothing\}^0\end{matrix}\right.,$$

$$Y^0_{1,2(z_4z_5/10,11)}=\{(00,01,\mathbf{10},\mathbf{11})\,|\,\varnothing\}^0\overset{con}{\Rightarrow}\{\varnothing\}^0\Rightarrow$$

$$\Rightarrow\left\{\begin{matrix}\varnothing\\ 000_{1,2},001_1,100_2\end{matrix}\right\}^{\overline{\oplus}}\Rightarrow\left\{\begin{matrix}\{\varnothing\}^0\\ \{000,001,100\}^0\end{matrix}\right.\overset{Y\to C}{\Rightarrow}\left\{\begin{matrix}\{000_{1,2},001_{1,2},100_{1,2}\}^1\\ \{\varnothing\}^0\end{matrix}\right..$$

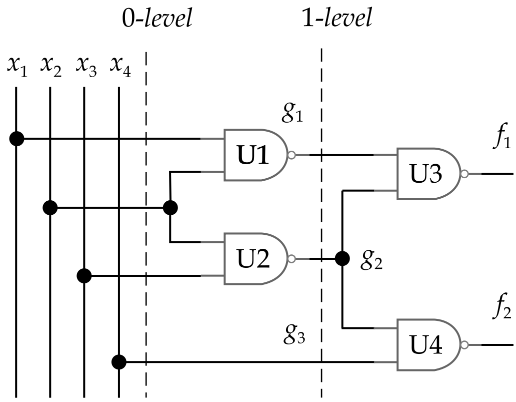

Fig. 2

Table 4

Stuck-at-fault in $f_{1,2}$	$x_3x_4/\sim\sim$	$x_3x_5/\sim\sim$	$x_4x_5/\sim\sim$
s-a-00	$\begin{pmatrix}011\\101\\111\end{pmatrix}^{1}_{2},\begin{pmatrix}010\\100\\110\end{pmatrix}^{0}$	$\begin{pmatrix}001\\100\\101\end{pmatrix}^{0}$	$\begin{pmatrix}101\\110\\111\end{pmatrix}^{1}_{1},\begin{pmatrix}001\\010\\011\end{pmatrix}^{0}$
s-a-01	$\begin{pmatrix}000\\001\\100\end{pmatrix}^{1}_{1,2}$	$\begin{pmatrix}001\\100\\101\end{pmatrix}^{1}_{2}$	$\begin{pmatrix}000_2\\010_2\\011_2\\100_{1,2}\end{pmatrix}^{1}$
s-a-10	$\begin{pmatrix}000_1\\001_{1,2}\\010_1\\110_1\end{pmatrix}^{1}$	$\begin{pmatrix}000\\001\\101\end{pmatrix}^{1}_{1}$	$\begin{pmatrix}000\\001\\100\end{pmatrix}^{1}_{1,2}$
s-a-11	$\begin{pmatrix}000\\001\\100\end{pmatrix}^{1}_{1,2}$	$\begin{pmatrix}000\\001\\100\end{pmatrix}^{1}_{1,2}$	$\begin{pmatrix}000\\001\\100\end{pmatrix}^{1}_{1,2}$

Table 4 contains vectors of test codes determined by the proposed method for the case of stuck-at-fault (0/1) type "damages" at arbitrary two points of the 2-level of the circuit.

Example 2. Determine the vectors of test codes to detect multiple stuck-at-faults (0/1) type in the logic circuit shown in Fig. 2, which implements the system of functions (*the circuit is borrowed from* [18, p.217]).

Solution. In the set-theoretic format, the given system

$$\begin{cases} f_1(x_1,x_2,x_3,x_4)=\overline{\overline{x_1x_2}\,\overline{x_2x_3}} \\ f_2(x_1,x_2,x_3,x_4)=\overline{\overline{x_2x_3}\,x_4} \end{cases}$$

corresponds to a system of perfect STFs

$$\{Y_1^1, Y_2^1\}\ \begin{cases} Y_1^1 = \{6,7,12,13,14,15\}^1 \\ Y_2^1 = \{0,2,4,6,7,8,10,12,14,15\}^1 \end{cases},$$

which can also be represented by a set of system minterms

$$Y_{1,2}^1 = \{0_2, 2_2, 4_2, 6_{1,2}, 7_{1,2}, 8_2, 10_2, 12_{1,2}, 13_1, 14_{1,2}, 15_{1,2}\}^1$$

or

$$Y_{1,2}^0 = \{0_1, 1_{1,2}, 2_1, 3_{1,2}, 4_1, 5_{1,2}, 8_1, 9_{1,2}, 10_1, 11_{1,2}, 13_2\}^0 .$$

Since both sets have the same power, we will use the set $Y_{1,2}^1$ to solve the problem at the 0-level of the scheme.

First, we define vectors of test codes for detecting stuck-at-fault (0/1) damage at separate points on the 0-level of the circuit (Fig. 2), introducing one "non-essential" variable into the given system. To do this, we use the procedure of 3-partition to system minters of the set $Y_{1,2}^1$:

$$Y_{1,2}^1 = \{0000_2, 0010_2, 0100_2, 0110_{1,2}, 0111_{1,2}, 1000_2, 1010_2, 1100_{1,2}, 1101_1, 1110_{1,2}, 1111_{1,2}\}^1 \overset{P^3}{\Rightarrow}$$

$$\overset{P^3}{\Rightarrow}\begin{cases} \{l_1 \mid l_2l_3l_4\} = \{(0 \mid 000_2, 010_2, 100_2, 110_{1,2}, 111_{1,2}), (1 \mid 000_2, 010_2, 100_{1,2}, 101_1, 110_{1,2}, 111_{1,2})\}^1 \\ \{l_2 \mid l_1l_3l_4\} = \{0 \mid 000_2, 010_2, 100_2, 110_2), (1 \mid 000_2, 010_{1,2}, 011_{1,2}, 100_{1,2}, 101_1, 110_{1,2}, 111_{1,2})\}^1 \\ \{l_3 \mid l_1l_2l_4\} = \{0 \mid 000_2, 010_2, 100_2, 110_{1,2}, 111_1), (1 \mid 000_2, 010_{1,2}, 011_{1,2}, 100_2, 110_{1,2}, 111_{1,2})\}^1 \\ \{l_4 \mid l_1l_2l_3\} = \{0 \mid 000_2, 001_2, 010_2, 011_{1,2}, 100_2, 101_2, 110_{1,2}, 111_{1,2}), (1 \mid 011_{1,2}, 110_1, 111_{1,2})\}^1 \end{cases}$$

For the introduced "non-essential" variable x_1, we obtain the following vectors of test codes:

$$Y_{1,2(x_1/0)}^1 = \{(\mathbf{0},1) \mid 000_2, 010_2, 100_2, 110_{1,2}, 111_{1,2}\}^1 \overset{con}{\Rightarrow}$$

$$\overset{con}{\Rightarrow}\{0000_2, 0010_2, 0100_2, 0110_{1,2}, 0111_{1,2}, 1000_2, 1010_2, 1100_2, 1110_{1,2}, 1111_{1,2}\}^1 \Rightarrow$$

$$\Rightarrow\left\{\begin{matrix} \cancel{0000_2, 0010_2, 0100_2, 0110_{1,2}, 0111_{1,2}, 1000_2, 1010_2}, 1100_2, \cancel{1110_{1,2}, 1111_{1,2}} \\ \cancel{0000_2, 0010_2, 0100_2, 0110_{1,2}, 0111_{1,2}, 1000_2, 1010_2}, \cancel{1100_{1,2}}, 1101_1, \cancel{1110_{1,2}, 1111_{1,2}} \end{matrix}\right\}^{\oplus} \Rightarrow$$

$$\Rightarrow\begin{cases} \{1100_2\}^1 \\ \{1101_1\}^0 \end{cases},$$

$$Y_{1,2(x_1/1)}^1 = \{(0,\mathbf{1}) \mid 000_2, 010_2, 100_{1,2}, 101_1, 110_{1,2}, 111_{1,2}\}^1 \overset{con}{\Rightarrow}$$

$$\overset{con}{\Rightarrow}\{0000_2, 0010_2, 0100_{1,2}, 0101_1, 0110_{1,2}, 0111_{1,2}, 1000_2, 1010_2, 1100_{1,2}, 1101_1, 1110_{1,2}, 1111_{1,2}\}^1 \Rightarrow$$

$$\Rightarrow\left\{\begin{matrix} \cancel{0000_2, 0010_2}, 0100_{1,2}, 0101_1, \cancel{0110_{1,2}, 0111_{1,2}, 1000_2, 1010_2, 1100_{1,2}, 1101_1, 1110_{1,2}, 1111_{1,2}} \\ \cancel{0000_2, 0010_2}, \cancel{0100_2}, \cancel{0110_{1,2}, 0111_{1,2}, 1000_2, 1010_2, 1100_{1,2}, 1101_1, 1110_{1,2}, 1111_{1,2}} \end{matrix}\right\}^{\oplus} \Rightarrow$$

$$\Rightarrow\begin{cases} \{0100_{1,2}, 0101_1\}^1 \\ \{\varnothing\}^0 \end{cases}.$$

Performing similar procedures and operations on the system minterms of the set $Y_{1,2}^{1}$, after the introducing of "non-essential" variables x_2, x_3, x_4, we obtain the following vectors of test codes:

$$Y_{1,2(x_2/0)}^{1} \Rightarrow \begin{cases} \{0110_2, 1100_2, 1110_2\}^1 \\ \{0111_{1,2}, 1101_1, 1111_{1,2}\}^0 \end{cases},$$

$$Y_{1,2(x_2/1)}^{1} \Rightarrow \begin{cases} \{0010_{1,2}, 0011_{1,2}, 1000_{1,2}, 1001_1, 1010_{1,2}, 1011_{1,2}\}^1 \\ \{\varnothing\}^0 \end{cases},$$

$$Y_{1,2(x_3/0)}^{1} \Rightarrow \begin{cases} \{0110_2, 1111_1\}^1 \\ \{0111_{1,2}\}^0 \end{cases}, \quad Y_{1,2(x_3/1)}^{1} \Rightarrow \begin{cases} \{0100_{1,2}, 0101_{1,2}, 1101_{1,2}\}^1 \\ \{\varnothing\}^0 \end{cases},$$

$$Y_{1,2(x_4/0)}^{1} \Rightarrow \begin{cases} \{0001_2, 0011_2, 0101_2, 1001_2, 1011_2, 1101_{1,2}\}^1 \\ \{\varnothing\}^0 \end{cases},$$

$$Y_{1,2(x_4/1)}^{1} \Rightarrow \begin{cases} \{1100_1\}^1 \\ \{0000_2, 0010_2, 0100_2, 1000_2, 1010_2\}^0 \end{cases}.$$

The test code vectors obtained at the 0-level of the scheme are listed in Table 5.

By introducing two "non-essential" variables at the 0-level of the scheme, we will be able to determine the stuck-at-fault damage (0/1) in it at two of its points and the values of the test code vectors. To do this, we will apply the 2-partition procedure to the system minterms of the set $Y_{1,2}^{1}$:

Table 5

Stuck-at-fault in $f_{1,2}$	$x_1/\sim$	$x_2/\sim$	$x_3/\sim$	$x_4/\sim$
s-a-0	$(1100_2)^1$ $(1101_1)^0$	$\begin{pmatrix} 0110_2 \\ 1100_2 \\ 1110_2 \end{pmatrix}^1$, $\begin{pmatrix} 0111_{1,2} \\ 1101_1 \\ 1111_{1,2} \end{pmatrix}^0$	$\begin{pmatrix} 0110_2 \\ 1111_1 \end{pmatrix}^1$ $(0111_{1,2})^0$	$\begin{pmatrix} 0001_2 \\ 0011_2 \\ 0101_2 \\ 1001_2 \\ 1011_2 \\ 1101_{1,2} \end{pmatrix}^1$
s-a-1	$\begin{pmatrix} 0100_{1,2} \\ 0101_1 \end{pmatrix}^1$	$\begin{pmatrix} 0010_{1,2} \\ 0011_{1,2} \\ 1000_{1,2} \\ 1001_1 \\ 1010_{1,2} \\ 1011_{1,2} \end{pmatrix}^1$	$\begin{pmatrix} 0100_{1,2} \\ 0101_{1,2} \\ 1101_{1,2} \end{pmatrix}^1$	$(1100_1)^1$ $\begin{pmatrix} 0000_2 \\ 0010_2 \\ 0100_2 \\ 1000_2 \\ 1010_2 \end{pmatrix}^0$

$$Y_{1,2}^{1}=\{0000_{2},0010_{2},0100_{2},0110_{1,2},0111_{1,2},1000_{2},1010_{2},1100_{1,2},1101_{1},1110_{1,2},1111_{1,2}\}^{1}\overset{P^2}{\Rightarrow}$$

$$\overset{P^2}{\Rightarrow}\begin{cases}\{l_1l_2\,|\,l_3l_4\}=\{(00\,|\,00_2,10_2),(01\,|\,00_2,10_{1,2},11_{1,2}),(10\,|\,00_2,10_2),(11\,|\,00_{1,2},01_1,10_{1,2},11_{1,2})\}^1\\ \{l_1l_3|\,l_2l_4\}=\{(00\,|\,00_2,10_2),(01\,|\,00_2,10_{1,2},11_{1,2}),(10\,|\,00_2,10_{1,2},11_1),(11\,|\,00_2,10_{1,2},11_{1,2})\}^1\\ \{l_1l_4|\,l_2l_3\}=\{(00\,|\,00_2,01_2,10_2,11_{1,2}),(01\,|\,11_{1,2}),(10\,|\,00_2,01_2,10_{1,2},11_{1,2}),(11\,|\,10_1,11_{1,2})\}^1\\ \{l_2l_3|\,l_1l_4\}=\{00\,|\,00_2,10_2),(01\,|\,00_2,10_2),(10\,|\,00_2,10_{1,2},11_1),(11\,|\,00_{1,2},01_{1,2},10_{1,2},11_{1,2})\}^1\\ \{l_2l_4|\,l_1l_3\}=\{00\,|\,00_2,01_2,10_2,11_2),(01\,|\,\varnothing),(10\,|\,00_2,01_{1,2},10_{1,2},11_{1,2}),(11\,|\,01_{1,2},10_1,11_{1,2})\}^1\\ \{l_3l_4|\,l_1l_2\}=\{00\,|\,00_2,01_2,10_2,11_{1,2}),(01\,|\,11_1),(10\,|\,00_2,01_{1,2},10_2,11_{1,2}),(11\,|\,01_{1,2},11_{1,2})\}^1\end{cases}\overset{|\cup}{\Rightarrow}$$

$$\overset{|\cup}{\Rightarrow}\begin{cases}\{l_1l_2\,|\,l_3l_4\}=\{(00,10\,|\,00_2,10_2),(01\,|\,00_2,10_{1,2},11_{1,2}),(11\,|\,00_{1,2},01_1,10_{1,2},11_{1,2})\}^1\\ \{l_1l_3|\,l_2l_4\}=\{(00\,|\,00_2,10_2),(01,11\,|\,00_2,10_{1,2},11_{1,2}),(10\,|\,00_2,10_{1,2},11_1)\}^1\\ \{l_1l_4|\,l_2l_3\}=\{(00\,|\,00_2,01_2,10_2,11_{1,2}),(01\,|\,11_{1,2}),(10\,|\,00_2,01_2,10_{1,2},11_{1,2}),(11\,|\,10_1,11_{1,2})\}^1\\ \{l_2l_3|\,l_1l_4\}=\{00,01\,|\,00_2,10_2),(10\,|\,00_2,10_{1,2},11_1),(11\,|\,00_{1,2},01_{1,2},10_{1,2},11_{1,2})\}^1\\ \{l_2l_4|\,l_1l_3\}=\{00\,|\,00_2,01_2,10_2,11_2),(01\,|\,\varnothing),(10\,|\,00_2,01_{1,2},10_{1,2},11_{1,2}),(11\,|\,01_{1,2},10_1,11_{1,2})\}^1\\ \{l_3l_4|\,l_1l_2\}=\{00\,|\,00_2,01_2,10_2,11_{1,2}),(01\,|\,11_1),(10\,|\,00_2,01_{1,2},10_2,11_{1,2}),(11\,|\,01_{1,2},11_{1,2})\}^1\end{cases}$$

Using the example of introducing "non-essential" variables x_1 and x_2, which reflects the literal mask $\{l_1l_2\,|\,l_3l_4\}$, we will show the procedure for determining test code vectors for the stuck-at-fault (0/1) damage case at any two points on the 0-level of the scheme:

$$Y^1_{1,2(x_1x_2/00,10)}=\{(\mathbf{00},01,\mathbf{10},11)\,|\,00_2,10_2\}^1\overset{con}{\Rightarrow}$$

$$\overset{con}{\Rightarrow}\{0000_2,0010_2,0100_2,0110_2,1000_2,1010_2,1100_2,1110_2\}^1\Rightarrow$$

$$\Rightarrow\left\{\begin{matrix}\cancel{0000_2},\cancel{0010_2},\cancel{0100_2},0110_2,\cancel{1000_2},\cancel{1010_2},1100_2,1110_2\\ \cancel{0000_2},\cancel{0010_2},\cancel{0100_2},\cancel{0110_{1,2}},0111_{1,2},\cancel{1000_2},\cancel{1010_2},\cancel{1100_{1,2}},1101_1,\cancel{1110_{1,2}},1111_{1,2}\end{matrix}\right\}^{\oplus}\Rightarrow$$

$$\Rightarrow\begin{cases}\{0110_2,1100_2,1110_2\}^1\\ \{0111_{1,2},1101_1,1111_{1,2}\}^0\end{cases},$$

$$Y^1_{1,2(x_1x_2/01)}=\{(00,\mathbf{01},10,11)\,|\,00_2,10_{1,2},11_{1,2}\}^1\overset{con}{\Rightarrow}$$

$$\overset{con}{\Rightarrow}\{0000_2,0010_{1,2},0011_{1,2},0100_2,0110_{1,2},0111_{1,2},1000_2,1010_{1,2},1011_{1,2},1100_2,1110_{1,2}1111_{1,2}\}^1\Rightarrow$$

$$\Rightarrow\left\{\begin{matrix}\cancel{0000_2},0010_{1,2},0011_{1,2},\cancel{0100_2},\cancel{0110_{1,2}},\cancel{0111_{1,2}},\cancel{1000_2},1010_{1,2},1011_{1,2},1100_2,\cancel{1110_{1,2}}\cancel{1111_{1,2}}\\ \cancel{0000_2},\cancel{0010_2},\cancel{0100_2},\cancel{0110_{1,2}},\cancel{0111_{1,2}},\cancel{1000_2},\cancel{1010_2},\cancel{1100_{1,2}},1101_1,\cancel{1110_{1,2}},\cancel{1111_{1,2}}\end{matrix}\right\}^{\oplus}\Rightarrow$$

$$\Rightarrow\begin{cases}\{0010_{1,2},0011_{1,2},1010_{1,2},1011_{1,2},1100_2\}^1\\ \{1101_1\}^0\end{cases},$$

$$Y^1_{1,2(x_1x_2/11)}=\{(00,01,10,\mathbf{11})\,|\,00_{1,2},01_1,10_{1,2},11_{1,2}\}^1\overset{con}{\Rightarrow}$$

$$\overset{con}{\Rightarrow}\{0000_{1,2},0001_1,0010_{1,2},0011_{1,2},0100_{1,2},0101_1,0110_{1,2},0111_{1,2},$$
$$1000_{1,2},1001_1,1010_{1,2},1011_{1,2},1100_{1,2},1101_1,1110_{1,2}1111_{1,2}\}^1\Rightarrow$$

$$\Rightarrow \left\{\begin{matrix} 0000_{1,2},0001_{1},0010_{1,2},0011_{1,2},0100_{1,2},0101_{1},\cancel{0110_{1,2},0111_{1,2}},1000_{1,2},1001_{1}, \\ 1010_{1,2},1011_{1,2},\cancel{1100_{1,2},1101_{1},1110_{1,2}1111_{1,2}} \\ \cancel{0000_{2}},\cancel{0010_{2}},\cancel{0100_{2}},\cancel{0110_{1,2},0111_{1,2}},\cancel{1000_{2}},\cancel{1010_{2}},\cancel{1100_{1,2},1101_{1},1110_{1,2},1111_{1,2}} \end{matrix}\right\}^{\oplus} \Rightarrow$$

$$\Rightarrow \begin{cases} \{0000_{1,2},0001_{1},0010_{1,2},0011_{1,2},0100_{1,2},0101_{1},1000_{1,2},1001_{1},1010_{1,2},1011_{1,2}\}^{1} \\ \{\varnothing\}^{0} \end{cases}.$$

The vectors of test codes for the remaining pairs of introduced "non-essential" variables, namely $\langle x_1, x_4 \rangle$, $\langle x_2, x_3 \rangle$, $\langle x_2, x_4 \rangle$ and $\langle x_3, x_4 \rangle$, are easily determined in a similar way, but not given here due to the cumbersomeness of the notation.

To determine the vectors of test codes in the case of three introduced "non-essential" variables at the 0-level of the scheme, which, according to the proposed method, allow to recognize the type and location of damage simultaneously at three points of the studied scheme, we apply the 1-partition procedure to the system minterms of the set $Y_{1,2}^{1}$:

$$Y_{1,2}^{1} = \{0000_{2},0010_{2},0100_{2},0110_{1,2},0111_{1,2},1000_{2},1010_{2},1100_{1,2},1101_{1},1110_{1,2},1111_{1,2}\}^{1} \overset{P^1}{\Rightarrow}$$

$$\overset{P^1}{\Rightarrow} \begin{cases} \{l_1 l_2 l_3 \mid l_4\} = \{(000,001,010,100,101 \mid 0_2),(011,111 \mid 0_{1,2},1_{1,2}),(110 \mid 0_{1,2},1_{1})\}^{1} \\ \{l_1 l_2 l_4 \mid l_3\} = \{(000,100 \mid 0_2,1_2),(001,101 \mid \varnothing),(010 \mid 0_2,1_{1,2}),(011 \mid 1_{1,2}),(110 \mid 0_{1,2},1_{1,2}),(111 \mid 0_1,1_{1,2})\}^{1} \\ \{l_1 l_3 l_4 \mid l_2\} = \{(000 \mid 0_2,1_2),(001 \mid \varnothing),(010,100,110 \mid 0_2,1_{1,2}),(011,111 \mid 1_{1,2}),(101 \mid 1_1)\}^{1} \\ \{l_2 l_3 l_4 \mid l_1\} = \{000,010 \mid 0_2,1_2),(001,011 \mid \varnothing),(100 \mid 0_2,1_{1,2}),(101 \mid 1_1),(110,111 \mid 0_{1,2},1_{1,2})\}^{1} \end{cases} \overset{\models\cup}{\Rightarrow}$$

Let us show the procedure for determining test code vectors using the example of introducing three "non-essential" variables x_1, x_2 and x_3, by using a literal mask $\{l_1 l_2 l_3 \mid l_4\}$:

$$Y_{1,2(x_1x_2x_3/000,001,010,100,101)}^{1} = \{(\mathbf{000},\mathbf{001},\mathbf{010},011,\mathbf{100},\mathbf{101},110,111) \mid 0_2\}^{1} \overset{con}{\Rightarrow}$$

$$\overset{con}{\Rightarrow} \{0000_{2},0010_{2},0100_{2},0110_{2},1000_{2},1010_{2},1100_{2},1110_{2}\}^{1} \Rightarrow$$

$$\Rightarrow \left\{\begin{matrix} \cancel{0000_{2},0010_{2},0100_{2}},0110_{2},\cancel{1000_{2},1010_{2}},1100_{2},1110_{2} \\ \cancel{0000_{2},0010_{2},0100_{2}},\cancel{0110_{1,2}},0111_{1,2},\cancel{1000_{2},1010_{2}},\cancel{1100_{1,2}},1101_{1},\cancel{1110_{1,2}},1111_{1,2} \end{matrix}\right\}^{\oplus} \Rightarrow$$

$$\Rightarrow \begin{cases} \{0110_{2},1100_{2},1110_{2}\}^{1} \\ \{0111_{1,2},1101_{1},1111_{1,2}\}^{0} \end{cases},$$

$$Y_{1,2(x_1x_2x_3/011,111)}^{1} = \{(000,001,010,\mathbf{011},100,101,110,\mathbf{111}) \mid 0_{1,2},1_{1,2}\}^{1} \overset{con}{\Rightarrow}$$

$$\overset{con}{\Rightarrow} \{0000_{1,2},0001_{1,2},0010_{1,2},0011_{1,2},0100_{1,2},0101_{1,2},0110_{1,2},0111_{1,2},1000_{1,2},$$

$$1001_{1,2},1010_{1,2},1011_{1,2},1100_{1,2},1101_{1,2},1110_{1,2},1111_{1,2}\}^{1} \Rightarrow$$

$$\Rightarrow \left\{\begin{matrix} 0000_{1,2},0001_{1,2},0010_{1,2},0011_{1,2},0100_{1,2},0101_{1,2},\cancel{0110_{1,2},0111_{1,2}},1000_{1,2},1001_{1,2}, \\ 1010_{1,2},1011_{1,2},\cancel{1100_{1,2}},1101_{1,2},\cancel{1110_{1,2},1111_{1,2}} \\ \cancel{0000_{2}},\cancel{0010_{2}},\cancel{0100_{2}},\cancel{0110_{1,2},0111_{1,2}},\cancel{1000_{2}},\cancel{1010_{2}},\cancel{1100_{1,2}},\cancel{1101_{1}},\cancel{1110_{1,2},1111_{1,2}} \end{matrix}\right\}^{\oplus} \Rightarrow$$

$$\Rightarrow \begin{cases} \{0000_{1,2},0001_{1,2},0010_{1,2},0011_{1,2},0100_{1,2},0101_{1,2},1000_{1,2},1001_{1,2},1010_{1,2},1011_{1,2},1101_{1,2}\}^1 \\ \{\varnothing\}^0 \end{cases},$$

$$Y^1_{1,2(x_1x_2x_3/110)} = \{(000,001,010,011,100,101,\mathbf{110},111)\,|\,0_{1,2},1_1\}^1 \overset{con}{\Rightarrow} \{0000_{1,2},0001_1,0010_{1,2},0011_1,$$
$$0100_{1,2},0101_1,0110_{1,2},0111_1,1000_{1,2},1001_1,1010_{1,2},1011_1,1100_{1,2},1101_1,1110_{1,2},1111_1\}^1 \Rightarrow$$
$$\Rightarrow \left\{ \begin{matrix} 0000_{1,2},0001_1,0010_{1,2},0011_1,0100_{1,2},0101_1,\cancel{0110}_{1,2},0111_1,1000_{1,2},1001_1, \\ 1010_{1,2},1011_1,\cancel{1100_{1,2},1101_1,1110}_{1,2},1111_1 \\ \cancel{0000_2},\cancel{0010_2},\cancel{0100_2},\cancel{0110}_{1,2},\cancel{0111_{1,2}},\cancel{1000_2},\cancel{1010_2},\cancel{1100_{1,2},1101_1,1110_{1,2}},\cancel{1111_{1,2}} \end{matrix} \right\}^{\oplus} \Rightarrow$$
$$\Rightarrow \begin{cases} \{0000_{1,2},0001_1,0010_{1,2},0011_1,0100_{1,2},0101_1,0111_1,1000_{1,2},1001_1,1010_{1,2},1011_1,1111_1\}^1 \\ \{\varnothing\}^0 \end{cases}.$$

Introducing in a similar way the remaining "non-essential" variables for $\langle x_1,x_2,x_4\rangle$, $\langle x_1,x_3,x_4\rangle$ and $\langle x_2,x_3,x_4\rangle$, we obtain vectors of test codes, which are not given here due to the cumbersomeness of the notation.

At the 1-level of the scheme in Fig. 2, we have a system of functions $\begin{cases} f_1(g_1,g_2,g_3)=\overline{g_1g_2} \\ f_2(g_1,g_2,g_3)=\overline{g_2g_3} \end{cases}$, where $g_1=\overline{x_1x_2}$, $g_2=\overline{x_2x_3}$, $g_3=x_4$. In the set-theoretic format, the given system corresponds to the system of perfect STFs $\{Y^1_1,Y^1_2\}$ $\begin{cases} Y^1_1=\{(0--),(-0-)\}^1=\{0,1,2,3,4,5\}^1 \\ Y^1_2=\{(-0-),(--0)\}^1=\{0,1,2,4,5,6\}^1 \end{cases}$, as well as the sets of system minterms $Y^1_{1,2}=\{0_{1,2},1_{1,2},2_{1,2},3_1,4_{1,2},5_{1,2},6_2\}^1$ and $Y^0_{1,2}=\{3_2,6_1,7_{1,2}\}^1$. Since $|Y^1_{1,2}|>|Y^0_{1,2}|$, it is advisable to use the set of system minterms $Y^0_{1,2}$ at 1-level of the scheme.

We define the vectors of test codes at individual points of the 1-level of the scheme by applying the 2-partition of the system minterms of the set $Y^0_{1,2}$:

$$Y^0_{1,2}=\{011_2,110_1,111_{1,2}\}^0 \overset{P^2}{\Rightarrow} \begin{cases} \{l_1\,|\,l_2l_3\}=\{(0\,|\,11_2),(1\,|\,10_1,11_{1,2})\}^0 \\ \{l_2\,|\,l_1l_3\}=\{(0\,|\,\varnothing),(1\,|\,01_2,10_1,11_{1,2})\}^0 \\ \{l_3\,|\,l_1l_2\}=\{(0\,|\,11_1),(1\,|\,01_2,11_{1,2})\}^0 \end{cases};$$

$$Y^0_{1,2(g_1/0)}=\{(\mathbf{0},1)\,|\,11_2\}^0 \overset{con}{\Rightarrow} \{011_2,111_2\}^0 \Rightarrow$$
$$\Rightarrow \left\{ \begin{matrix} \cancel{011}_2,111_2 \\ \cancel{011}_2,110_1,\cancel{111}_{1,2} \end{matrix} \right\}^{\bar{\oplus}} \Rightarrow \begin{cases} \{111_2\}^0 \\ \{110\}^0 \end{cases} \overset{Y\to C}{\Rightarrow} \begin{cases} \{110_{1,2},111_1\}^1 \\ \{\varnothing\}^0 \end{cases},$$

$$Y^0_{1,2(g_1/1)}=\{(0,\mathbf{1})\,|\,10_1,11_{1,2}\}^0 \overset{con}{\Rightarrow} \{010_1,011_{1,2},110_1,111_{1,2}\}^0 \Rightarrow$$
$$\Rightarrow \left\{ \begin{matrix} 010_1,011_{1,2},\cancel{110_1,111}_{1,2} \\ \cancel{011}_2,\cancel{110_1,111}_{1,2} \end{matrix} \right\}^{\bar{\oplus}} \Rightarrow \begin{cases} \{010_1,011_{1,2}\}^0 \\ \{\varnothing\}^0 \end{cases} \overset{Y\to C}{\Rightarrow} \begin{cases} \{010_2\}^1 \\ \{011\}^0 \end{cases},$$

$$Y^0_{1,2(g_2/0)} = \{(\mathbf{0},1)\,|\,\varnothing\}^0 \overset{con}{\Rightarrow} \{\varnothing\}^0 \Rightarrow \begin{Bmatrix} \varnothing \\ 011_2, 110_1, 111_{1,2} \end{Bmatrix}^{\overline{\oplus}} \Rightarrow$$

$$\Rightarrow \begin{cases} \{\varnothing\}^0 \\ \{011, 110, 111\}^0 \end{cases} \overset{Y \to C}{\Rightarrow} \begin{cases} \{011_{1,2}, 110_{1,2}, 111_{1,2}\}^1 \\ \{\varnothing\}^0 \end{cases},$$

$$Y^0_{1,2(g_2/1)} = \{(0,\mathbf{1})\,|\,01_2, 10_1, 11_{1,2}\}^0 \overset{con}{\Rightarrow} \{001_2, 011_2, 100_1, 110_1, 101_{1,2}, 111_{1,2}\}^0 \Rightarrow$$

$$\Rightarrow \begin{Bmatrix} 001_2, \cancel{011_2}, 100_1, \cancel{110_1}, 101_{1,2}, \cancel{111_{1,2}} \\ \cancel{011_2, 110_1, 111_{1,2}} \end{Bmatrix}^{\overline{\oplus}} \Rightarrow$$

$$\Rightarrow \begin{cases} \{001_2, 100_1, 101_{1,2}\}^0 \\ \{\varnothing\}^0 \end{cases} \overset{Y \to C}{\Rightarrow} \begin{cases} \{001_1, 100_2\}^1 \\ \{101\}^0 \end{cases},$$

$$Y^0_{1,2(g_3/0)} = \{(\mathbf{0},1)\,|\,11_1\}^0 \overset{con}{\Rightarrow} \{110_1, 111_1\}^0 \Rightarrow$$

$$\Rightarrow \begin{Bmatrix} \cancel{110_1}, 111_1 \\ 011_2, \cancel{110_1}, \cancel{111_{1,2}} \end{Bmatrix}^{\overline{\oplus}} \Rightarrow \begin{cases} \{111_1\}^0 \\ \{011\}^0 \end{cases} \overset{Y \to C}{\Rightarrow} \begin{cases} \{011_{1,2}, 111_2\}^1 \\ \{\varnothing\}^0 \end{cases},$$

$$Y^0_{1,2(g_3/1)} = \{(0,\mathbf{1})\,|\,01_2, 11_{1,2}\}^0 \overset{con}{\Rightarrow} \{010_2, 011_2, 110_{1,2}, 111_{1,2}\}^0 \Rightarrow$$

$$\Rightarrow \begin{Bmatrix} 010_2, \cancel{011_2}, 110_{1,2}, \cancel{111_{1,2}} \\ \cancel{011_2}, \cancel{110_1}, \cancel{111_{1,2}} \end{Bmatrix}^{\overline{\oplus}} \Rightarrow \begin{cases} \{010_2, 110_{1,2}\}^0 \\ \{\varnothing\}^0 \end{cases} \overset{Y \to C}{\Rightarrow} \begin{cases} \{010_1\}^1 \\ \{110\}^0 \end{cases}.$$

To determine the vectors of test codes in the case of stuck-at-fault (0/1) damage at two points on the 1-level of the circuit, we apply the procedure of 1-partition of the system minterms of the set $Y^0_{1,2}$:

$$Y^0_{1,2} = \{011_2, 110_1, 111_{1,2}\}^0 \overset{P^1}{\Rightarrow} \begin{cases} \{l_1 l_2 \,|\, l_3\} = \{(00,10\,|\,\varnothing),(01\,|\,1_2),(11\,|\,0_1,1_{1,2})\}^0 \\ \{l_1 l_3 \,|\, l_2\} = \{(00\,|\,\varnothing),(01\,|\,1_2),(10\,|\,1_1),(11\,|\,1_{1,2})\}^0 \\ \{l_2 l_3 \,|\, l_1\} = \{(00,01\,|\,\varnothing),(10\,|\,1_1),(11\,|\,0_2,1_{1,2})\}^0 \end{cases}.$$

Thus, we obtain the vectors of test codes at two "damaged" points on the 1-level of the scheme:

$$Y^0_{1,2(g_1 g_2/00,10)} = \{(\mathbf{00},01,\mathbf{10},11)\,|\,\varnothing\}^0 \overset{con}{\Rightarrow} \{\varnothing\}^0 \Rightarrow \begin{Bmatrix} \varnothing \\ 011_2, 110_1, 111_{1,2} \end{Bmatrix}^{\overline{\oplus}}$$

$$\Rightarrow \begin{cases} \{\varnothing\}^0 \\ \{011, 110, 111\}^0 \end{cases} \overset{Y \to C}{\Rightarrow} \begin{cases} \{011_{1,2}, 110_{1,2}, 111_{1,2}\}^1 \\ \{\varnothing\}^0 \end{cases},$$

$$Y^0_{1,2(g_1g_2/01)} = \{(00,\mathbf{01},10,11)\,|\,1_2\}^0 \overset{con}{\Rightarrow} \{001_2,011_2,101_2,111_2\}^0 \Rightarrow$$

$$\Rightarrow \left\{\begin{matrix} 001_2,\cancel{011_2},101_2,111_2 \\ \cancel{011_2},110_1,\cancel{111_{1,2}} \end{matrix}\right\}^{\bar{\oplus}} \Rightarrow$$

$$\Rightarrow \begin{cases} \{001_2,101_2,111_2\}^0 \\ \{110\}^0 \end{cases} \overset{Y \to C}{\Rightarrow} \begin{cases} \{001_1,101_1,110_{1,2},111_1\}^1 \\ \{\varnothing\}^0 \end{cases},$$

$$Y^0_{1,2(g_1g_2/11)} = \{(00,01,10,\mathbf{11})\,|\,0_1,1_{1,2}\}^0 \overset{con}{\Rightarrow}$$

$$\overset{con}{\Rightarrow} \{000_1,001_{1,2},010_1,011_{1,2},100_1,101_{1,2},110_1,111_{1,2}\}^0 \Rightarrow$$

$$\Rightarrow \left\{\begin{matrix} 000_1,001_{1,2},010_1,011_{1,2},100_1,101_{1,2},\cancel{110_1},\cancel{111_{1,2}} \\ \cancel{011_2},\cancel{110_1},\cancel{111_{1,2}} \end{matrix}\right\}^{\bar{\oplus}} \Rightarrow$$

$$\Rightarrow \begin{cases} \{000_1,001_{1,2},010_1,011_{1,2},100_1,101_{1,2}\}^0 \\ \{\varnothing\}^0 \end{cases} \overset{Y \to C}{\Rightarrow} \begin{cases} \{000_2,010_2,100_2\}^1 \\ \{001,011,101\}^0 \end{cases};$$

$$Y^0_{1,2(g_1g_3/00)} = \{(\mathbf{00},01,10,11)\,|\,\varnothing\}^0 \overset{con}{\Rightarrow} \{\varnothing\}^0 \Rightarrow$$

$$\Rightarrow \left\{\begin{matrix} \varnothing \\ 011_2,110_1,111_{1,2} \end{matrix}\right\}^{\bar{\oplus}} \Rightarrow \begin{cases} \{\varnothing\}^0 \\ \{011,110,111\}^0 \end{cases} \overset{Y \to C}{\Rightarrow} \begin{cases} \{011_{1,2},110_{1,2},111_{1,2}\}^1 \\ \{\varnothing\}^0 \end{cases},$$

$$Y^0_{1,2(g_1g_3/01)} = \{(00,\mathbf{01},10,11)\,|\,1_2\}^0 \overset{con}{\Rightarrow} \{010_2,011_2,110_2,111_2\}^0 \Rightarrow$$

$$\Rightarrow \left\{\begin{matrix} 010_2,\cancel{011_2},110_2,111_2 \\ \cancel{011_2},\cancel{110_1},\cancel{111_{1,2}} \end{matrix}\right\}^{\bar{\oplus}} \Rightarrow \begin{cases} \{010_2,110_2,111_2\}^0 \\ \{\varnothing\}^0 \end{cases} \overset{Y \to C}{\Rightarrow} \begin{cases} \{010_1,110_1,111_1\}^1 \\ \{\varnothing\}^0 \end{cases},$$

$$Y^0_{1,2(g_1g_3/10)} = \{(00,01,\mathbf{10},11)\,|\,1_1\}^0 \overset{con}{\Rightarrow} \{010_1,011_1,110_1,111_1\}^0 \Rightarrow$$

$$\Rightarrow \left\{\begin{matrix} 010_1,011_1,\cancel{110_1},111_1 \\ \cancel{011_2},\cancel{110_1},\cancel{111_{1,2}} \end{matrix}\right\}^{\bar{\oplus}} \Rightarrow \begin{cases} \{010_1,011_1,111_1\}^0 \\ \{\varnothing\}^0 \end{cases} \overset{Y \to C}{\Rightarrow} \begin{cases} \{010_2,011_2,111_2\}^1 \\ \{\varnothing\}^0 \end{cases},$$

$$Y^0_{1,2(g_1g_3/11)} = \{(00,01,10,\mathbf{11})\,|\,1_{1,2}\}^0 \overset{con}{\Rightarrow} \{010_{1,2},011_{1,2},110_{1,2},111_{1,2}\}^0 \Rightarrow$$

$$\Rightarrow \left\{\begin{matrix} 010_{1,2},011_{1,2},110_{1,2},\cancel{111_{1,2}} \\ \cancel{011_2},\cancel{110_1},\cancel{111_{1,2}} \end{matrix}\right\}^{\bar{\oplus}} \Rightarrow$$

$$\Rightarrow \begin{cases} \{010_{1,2},011_{1,2},110_{1,2}\}^0 \\ \{\varnothing\}^0 \end{cases} \overset{Y \to C}{\Rightarrow} \begin{cases} \{\varnothing\}^1 \\ \{010,011,110\}^0 \end{cases};$$

$$Y^0_{1,2(g_2g_3/00,01)} = \{(\mathbf{00},\mathbf{01},10,11)\,|\,\varnothing\}^0 \overset{con}{\Rightarrow} \{\varnothing\}^0 \Rightarrow$$

$$\overset{con}{\Rightarrow}\{\varnothing\}^0 \Rightarrow \left\{\begin{matrix}\varnothing \\ 011_2,110_1,111_{1,2}\end{matrix}\right\}^{\bar{\oplus}} \Rightarrow \begin{cases}\{\varnothing\}^0 \\ \{011,110,111\}^0\end{cases} \overset{Y\to C}{\Rightarrow} \begin{cases}\{011_{1,2},110_{1,2},111_{1,2}\}^1 \\ \{\varnothing\}^0\end{cases},$$

$$Y^0_{1,2(g_2g_3/10)} = \{(00,01,\mathbf{10},11)\,|\,1_1\}^0 \overset{con}{\Rightarrow} \{100_1,101_1,110_1,111_1\}^0 \Rightarrow$$

$$\Rightarrow \left\{\begin{matrix}100_1,101_1,\cancel{110}_1,111_1 \\ 011_2,\cancel{110}_1,\cancel{111}_{1,2}\end{matrix}\right\}^{\bar{\oplus}} \Rightarrow$$

$$\Rightarrow \begin{cases}\{100_1,101_1,111_1\}^0 \\ \{011\}^0\end{cases} \overset{Y\to C}{\Rightarrow} \begin{cases}\{011_{1,2},100_2,101_2,111_2\}^1 \\ \{\varnothing\}^0\end{cases},$$

$$Y^0_{1,2(g_2g_3/11)} = \{(00,01,10,\mathbf{11})\,|\,0_2,1_{1,2}\}^0 \overset{con}{\Rightarrow}$$

$$\overset{con}{\Rightarrow}\{000_2,001_2,010_2,011_2,100_{1,2},101_{1,2},110_{1,2},111_{1,2}\}^0 \Rightarrow$$

$$\Rightarrow \left\{\begin{matrix}000_2,001_2,010_2,\cancel{011}_2,100_{1,2},101_{1,2},110_{1,2},\cancel{111}_{1,2} \\ \cancel{011}_2,\cancel{110}_1,\cancel{111}_{1,2}\end{matrix}\right\}^{\bar{\oplus}} \Rightarrow$$

$$\Rightarrow \begin{cases}\{000_2,001_2,010_2,100_{1,2},101_{1,2},110_{1,2}\}^0 \\ \{\varnothing\}^0\end{cases} \overset{Y\to C}{\Rightarrow} \begin{cases}\{000_1,001_1,010_1\}^1 \\ \{100,101,110\}^0\end{cases}.$$

Table 6

Stuck-at-fault in $f_{1,2}$	$g_1g_2/\sim\sim$	$g_1g_3/\sim\sim$	$g_2g_3/\sim\sim$
s-a-00	$\begin{pmatrix}011_{1,2}\\110_{1,2}\\111_{1,2}\end{pmatrix}^1$	$\begin{pmatrix}011_{1,2}\\110_{1,2}\\111_{1,2}\end{pmatrix}^1$	$\begin{pmatrix}011_{1,2}\\110_{1,2}\\111_{1,2}\end{pmatrix}^1$
s-a-01	$\begin{pmatrix}001_1\\101_1\\110_{1,2}\\111_1\end{pmatrix}^1$	$\begin{pmatrix}010_1\\110_1\\111_1\end{pmatrix}^1$	$\begin{pmatrix}011_{1,2}\\110_{1,2}\\111_{1,2}\end{pmatrix}^1$
s-a-10	$\begin{pmatrix}011_{1,2}\\110_{1,2}\\111_{1,2}\end{pmatrix}^1$	$\begin{pmatrix}010_2\\011_2\\111_2\end{pmatrix}^1$	$\begin{pmatrix}011_{1,2}\\100_2\\101_2\\111_2\end{pmatrix}^1$
s-a-11	$\begin{pmatrix}000_2\\010_2\\100_2\end{pmatrix}^1,\begin{pmatrix}001\\011\\101\end{pmatrix}^0$	$\begin{pmatrix}010\\011\\110\end{pmatrix}^0$	$\begin{pmatrix}000_1\\001_1\\010_1\end{pmatrix}^1,\begin{pmatrix}100\\101\\110\end{pmatrix}^0$

Table 6 contains vectors of test codes to detect stuck-at-faults at arbitrary two points on the 1-level of the circuit.

Conclusion

A new method of generating test code vectors for determining the location and type of stuck-at-faults (0/1) of single and multiple faults in combinational PIPO circuits is proposed. It is based on the artificial introduction of one or more non-essential variables into the studied circuit and the application of the procedure of *q*-partition of system minterms of a given system of Boolean functions. Compared to known methods and algorithms, the proposed method is characterized by a relatively simpler implementation and reliability of the final results of practical diagnostics without the use of additional tools and restrictions. The examples given in the article of determining the location and type of stuck-at-faults (0/1) of single and multiple damages illustrate the effectiveness of the proposed method.

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НОВИЙ МЕТОД ГЕНЕРУВАННЯ ТЕСТОВИХ КОДІВ ДЛЯ ВИЯВЛЕННЯ МНОЖИННИХ ПОШКОДЖЕНЬ *STUCK-AT-FAULTS* У КОМБІНАЦІЙНИХ СХЕМАХ. Частина 2

Вступ. Діагностувати множинні несправності *stuck-at-faults* (0/1) у цифрових схемах типу *PIPO* значно складніше, ніж у пристроях типу *PISO* (частина 1). Функції системи переважно взаємопов'язані між собою, а отже, виявлене пошкодження в якійсь одній функції схеми може передатися інших функцій системи, що описує роботу досліджуваної схеми. Відповідно, методи генерування тестових кодів на основі одиночних несправностей не працюють для схем типу *PIPO*, а ті методи й алгоритми діагностики, що використовують моделювання одиночних несправностей, ускладнені додатковими процедурами, не дають надійного результату. Аналогічний висновок про певні практичні обмеження можна також зробити і до аналітичних підходів до розв'язання зазначеної проблеми виявлення множинних несправностей *stuck-at-faults* (0/1) у цифрових схемах типу *PIPO*.

Мета статті. Запропонувати метод генерування векторів тестових кодів для виявлення як одиночних, так і множинних пошкоджень типу *stuck-at-faults* (0/1) у комбінаційних пристроях типу *PIPO*, який порівняно з відомими методами й алгоритмами може забезпечувати достовірні результати з допомогою реалізації простих операцій і процедур.

Методи. Запропонований метод генерування тестових кодів ґрунтується на числовому теоретико-множинному підході до реалізації всіх операцій і процедур, а саме: штучного впровадження у буловий простір заданої системи повних функцій $F(X)$, $X=\{x_1, x_2, ..., x_n\}$, що описує роботу досліджуваної *PIPO*-схеми, одної або більше (до $n-1$) неістотних змінних та застосуванні процедури *q*-розбиття до системних мінтермів з урахуванням індексів функцій заданої системи $F(X)$.

Результати. Завдяки застосуванню процедури *q*-розбиття системних мінтермів впровадження «неістотних» змінних у буловий простір заданої системи забезпечує виявлення всіх можливих як одиночних, так і множинних пошкоджень типу *stuck-at-faults* (0/1) у досліджуваній схемі. Унаслідок цього формуються 2^r $(r = 1, 2, ..., n-1)$ псевдодосконалих ТМФ «пошкодженої» системи $F(X)$, на підставі яких після виконання простих операцій спрощення одержуються шукані вектори тестових кодів, з допомогою яких можна визначити в схемі як місце пошкодження, так і тип одиночного та множинного *stuck-at-faults* (0/1) пошкодження.

Висновки. Запропоновано новий метод генерування векторів тестових кодів для визначення місця і типу *stuck-at-faults* (0/1) одиночних і множинних пошкоджень у комбінаційних *PIPO*-схемах, що ґрунтується на штучному впровадженні в досліджувану схему одної і більше неістотних змінних та застосуванні процедури *q*-розбиття системних мінтермів заданої системи булових функцій. Порівняно з відомими методами та алгоритмами метод відрізняється відносно простішою реалізацією та надійністю отриманих остаточних результатів практичної діагностики без застосування додаткових засобів та обмежень. Наведені в статті приклади визначення місця і типу *stuck-at-faults* (0/1) одиночних і множинних пошкоджень ілюструють ефективність пропонованого методу.

Ключові слова: *комбінаційна PIPO-схема, одиночне та множинне пошкодження типу stuck-at-faults (0/1), процедура q-розбиття системних мінтермів, неістотні змінні, вектор тестових кодів.*