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Tubular Surfaces with Galilean Darboux Frame in G_3

Dae Won Yoon and Zühal Küçükarslan Yüzbaşı

The main point of the research is to study a new approach for defining the tubular surfaces with the Galilean Darboux frame in 3-dimensional Galilean space. Also, we obtain the Gaussian and mean curvatures and derive some parametrizations for a special curve to lie on tubular surfaces with the Galilean Darboux frame.

Key words: tubular surface, Galilean Darboux frame, geodesic, asymptotic, Galilean space.

Mathematical Subject Classification 2010: 53A35, 53Z05.

1. Introduction

A tubular surface is described as the envelope of the set of spheres with radius r and with centers lying on a spine curve. The tubular surface can be characterized using the Frenet frame, and is a helpful structure in many application areas such as medicine and computer aided geometric design. Therefore, there have been many studies in detecting and characterizing the tubular surfaces and special surfaces in several spaces [3, 6, 7, 9, 10, 14, 21]. Lately, several authors have been studied in Galilean space [2, 4, 15, 22].

On the other hand, Dogan et al. [5] introduced a new method to parametrize and characterize a tubular surface with Darboux frame in Euclidean 3-space, and Kiziltug et al. [8] developed this method for Minkowski space.

The aim of this paper is to improve this new approach and to define the tubular surfaces with the Galilean Darboux frame in the Galilean 3-space. We further compute the Gaussian and mean curvatures and derive some characterizations for given curves as lying on a tubular surface in terms of the Galilean Darboux frame.

2. Preliminaries

The Galilean 3-space $\mathbf{G}_3$ is a Cayley–Klein space equipped with the projective metric of signature $(0, 0, +, +)$, given in [11]. The absolute figure of the Galilean space consists of an ordered triple $\{\omega, f, I\}$ in which ω is the ideal (absolute) plane, f is the line (absolute line) in ω and I is the fixed elliptic involution of

f. We introduce homogeneous coordinates in $\mathbf{G}_3$ in such a way that the absolute plane ω is given by $x_0 = 0$, the absolute line f, by $x_0 = x_1 = 0$, and the elliptic involution, by

$$(0 : 0 : x_2 : x_3) \to (0 : 0 : x_3 : -x_2). \tag{2.1}$$

A plane is called Euclidean if it contains f, otherwise it is called isotropic, i.e., planes $x = \text{const}$ are Euclidean, and so is the plane ω. Other planes are isotropic. In other words, an isotropic plane does not involve any isotropic direction.

Definition 2.1 ([17])**.** Let $\lambda = (\lambda_1, \lambda_2, \lambda_3)$ and $\xi = (\xi_1, \xi_2, \xi_3)$ be any two vectors in $\mathbf{G}_3$. A vector λ is called isotropic if $\lambda_1 = 0$, otherwise it is called non-isotropic. Then the Galilean scalar product of these vectors is given by

$$\langle \lambda, \xi \rangle = \begin{cases} \lambda_1 \xi_1 & \text{if } \lambda_1 \neq 0 \text{ or } \xi_1 \neq 0 \\ \lambda_2 \xi_2 + \lambda_3 \xi_3 & \text{if } \lambda_1 = 0 \text{ and } \xi_1 = 0 \end{cases}.$$

Definition 2.2 ([1, 20])**.** Let $\lambda = (\lambda_1, \lambda_2, \lambda_3)$ and $\xi = (\xi_1, \xi_2, \xi_3)$ be any two vectors in $\mathbf{G}_3$. The Galilean cross product is given as

$$\lambda \wedge \xi = \begin{cases} \begin{vmatrix} 0 & e_2 & e_3 \\ \lambda_1 & \lambda_2 & \lambda_3 \\ \xi_1 & \xi_2 & \xi_3 \end{vmatrix} & \text{if } \lambda_1 \neq 0 \text{ or } \xi_1 \neq 0 \\ \begin{vmatrix} e_1 & e_2 & e_3 \\ \lambda_1 & \lambda_2 & \lambda_3 \\ \xi_1 & \xi_2 & \xi_3 \end{vmatrix} & \text{if } \lambda_1 \neq 0 \text{ and } \xi_1 \neq 0 \end{cases},$$

where $e_1 = (1, 0, 0), e_2 = (0, 1, 0), e_3 = (0, 0, 1)$.

Let α be an admissible curve of the class C^∞ in $\mathbf{G}_3$, and parametrized by the invariant parameter s, defined by

$$\alpha(s) = (s, f(s), g(s)).$$

Then the curvatures $\kappa(s)$ and $\tau(s)$ of $\alpha(s)$ can be written as

$$\kappa(s) = \sqrt{f''(s)^2 + g''(s)^2},$$
$$\tau(s) = \frac{\det(\alpha'(s), \alpha''(s), \alpha'''(s))}{\kappa^2(s)},$$

respectively, and the Frenet formula of the curve is written as

$$T' = \kappa N,$$
$$N' = \tau B,$$
$$B' = -\tau N,$$

where T, N and B are said to be the tangent, the principal normal and the binormal vectors of $\alpha(s)$ [13].

Let the equation of a surface $\Psi = \Psi(s, \vartheta)$ in $\mathbf{G}_3$ be given by

$$\Psi(s, \vartheta) = (x(s, \vartheta), y(s, \vartheta), z(s, \vartheta)).$$

Then the unit isotropic normal vector field η on $\Psi(s, \vartheta)$ is given by

$$\eta = \frac{\Psi_{,1} \wedge \Psi_{,2}}{\|\Psi_{,1} \wedge \Psi_{,2}\|},$$

where the partial differentiations with respect to s and ϑ will be denoted by suffixes 1 and 2 respectively, that is, $\Psi_{,1} = \dfrac{\partial \Psi(s, \vartheta)}{\partial s}$ and $\Psi_{,2} = \dfrac{\partial \Psi(s, \vartheta)}{\partial \vartheta}$.

On the other hand, we get the isotropic unit vector δ in the tangent plane of the surface as

$$\delta = \frac{x_{,2} \Psi_{,1} - x_{,1} \Psi_{,2}}{w}, \tag{2.2}$$

where $x_{,1} = \dfrac{\partial x(s, \vartheta)}{\partial s}$, $x_{,2} = \dfrac{\partial x(s, \vartheta)}{\partial \vartheta}$ and $w = \|\Psi_{,1} \wedge \Psi_{,2}\|$.

Let us define

$$\begin{aligned} &g_1 = x_{,1}, && g_2 = x_{,2}, && g_{ij} = g_i g_j, && \\ &g^1 = \frac{x_{,2}}{w}, && g^2 = -\frac{x_{,1}}{w}, && g^{ij} = g^i g^j && (i, j = 1, 2), \\ &h_{11} = \langle \tilde{\Psi}_{,1}, \tilde{\Psi}_{,1} \rangle, && h_{12} = \langle \tilde{\Psi}_{,1}, \tilde{\Psi}_{,2} \rangle, && h_{22} = \langle \tilde{\Psi}_{,2}, \tilde{\Psi}_{,2} \rangle, && \end{aligned}$$

where $\tilde{\Psi}_{,1}$ and $\tilde{\Psi}_{,2}$ are the projections of the vectors $\Psi_{,1}$ and $\Psi_{,2}$ onto the yz-plane, respectively. Then the corresponding matrix of the first fundamental form ds^2 of the surface $\Psi(s, \vartheta)$ is given by (cf. [18])

$$ds^2 = \begin{pmatrix} ds_1^2 & 0 \\ 0 & ds_2^2 \end{pmatrix}, \tag{2.3}$$

where $ds_1^2 = (g_1 ds + g_2 d\vartheta)^2$ and $ds_2^2 = h_{11} ds^2 + 2h_{12}\, ds\, d\vartheta + h_{22} d\vartheta^2$. In such case, we denote the coefficients of ds^2 by g_{ij}^*. On the other hand, the function w can be represented in terms of g_i and h_{ij} as follows:

$$w^2 = g_1^2 h_{22} - 2 g_1 g_2 h_{12} + g_2^2 h_{11}.$$

The Gaussian curvature and the mean curvature of a surface are defined by means of the coefficients of the second fundamental form L_{ij}, which are the normal components of $\Psi_{,i,j}$ $(i, j = 1, 2)$. That is,

$$\Psi_{,i,j} = \sum_{k=1}^{2} \Gamma_{ij}^k \Psi_{,k} + L_{ij} \eta,$$

where Γ_{ij}^k is the Christoffel symbols of the surface and L_{ij} are given by

$$L_{ij} = \frac{1}{g_1} \langle g_1 \tilde{\Psi}_{,i,j} - g_{i,j} \tilde{\Psi}_{,1}, \eta \rangle = \frac{1}{g_2} \langle g_2 \tilde{\Psi}_{,i,j} - g_{i,j} \tilde{\Psi}_{,2}, \eta \rangle. \tag{2.4}$$

From this, the Gaussian curvature K and the mean curvature H of the surface are expressed as [16]

$$K = \frac{L_{11}L_{22} - L_{12}^2}{w^2},$$
$$H = \frac{g_2^2 L_{11} - 2g_1 g_2 L_{12} + g_1^2 L_{22}}{2w^2}.$$

Definition 2.3 ([19])**.** Let T be the unit tangent vector of a curve α on a surface in $\mathbf{G}_3$, and n be the unit normal vector to the surface at the point $\alpha(s)$ of α, respectively. Let $Q = n \wedge T$ be the tangential-normal. Then $\{T, Q, n\}$ is an orthonormal frame at $\alpha(s)$ in $\mathbf{G}_3$. The frame is called a Galilean Darboux frame or a tangent-normal frame and expressed as

$$T' = k_g Q + k_n n,$$
$$Q' = \tau_g n,$$
$$n' = -\tau_g Q,$$

where k_g, k_n and τ_g are the geodesic curvature, the normal curvature and the geodesic torsion, respectively.

For the curvature κ of $\alpha(s)$, $\kappa^2 = k_g^2 + k_n^2$ holds. Also, a curve $\alpha(s)$ is a geodesic or an asymptotic curve or a line of curvature if and only if k_g or k_n or τ_g vanishes, respectively.

3. Tubular surfaces with Darboux frame in Galilean space $\mathbf{G}_3$

The starting point of this section is to express a simple method for parametrization of tubular surfaces with the Galilean Darboux frame in $\mathbf{G}_3$. Let us denote by σ the vector connecting the point from $\alpha(s)$ with the point from the surface with the Galilean Darboux frame $\{T, Q, n\}$ along $\alpha(s)$. Then, we have the position vector P of a point on the surface as follows:

$$P = \alpha(s) + \sigma. \tag{3.1}$$

Thus, we can write

$$\sigma = r(\cos\vartheta Q(s) + \sin\vartheta n(s)), \tag{3.2}$$

where r is a constant radius of a Euclidean circle of the Galilean Darboux frame and ϑ is the Euclidean angle between the isotropic vectors Q and σ.

Combining (3.1) and (3.2), we can define a tubular surface with the constant radius r in terms of the Galilean Darboux frame as

$$\Psi(s, \vartheta) = \alpha(s) + r(\cos\vartheta Q(s) + \sin\vartheta n(s)), \tag{3.3}$$

where n is the unit isotropic normal vector of the surface along a curve $\alpha(s)$ parametrized by arc-length s. Then we get partial derivatives of $\Psi(s, \vartheta)$ with respect to s and ϑ as follows:

$$\Psi_s = T - r\tau_g \sin\vartheta Q + r\tau_g \cos\vartheta n \tag{3.4}$$

and

$$\Psi_\vartheta = -r\sin\vartheta Q + r\cos\vartheta n. \tag{3.5}$$

It follows that the vector cross product of these vectors is obtained by

$$\Psi_s \wedge \Psi_\vartheta = -r(\cos\vartheta Q + \sin\vartheta n), \tag{3.6}$$

from above equation, for small $r > 0$, we have

$$\|\Psi_s \wedge \Psi_\vartheta\| = r. \tag{3.7}$$

Therefore, by using (3.6) and (3.7), the unit isotropic normal vector η of $\Psi(s, \vartheta)$ is found as

$$\eta = -\cos\vartheta Q - \sin\vartheta n. \tag{3.8}$$

On the other hand, from (3.8) and (2.2), it is easy to show that

$$\delta = \sin\vartheta Q - \cos\vartheta n.$$

Since Q and n are the isotropic vectors, using the Galilean Darboux frame, we can obtain

$$g_1 = 1, \ g_2 = 0. \tag{3.9}$$

By taking account of the projection of Ψ_s and Ψ_ϑ onto the Euclidean yz-plane, we get

$$h_{22} = r^2. \tag{3.10}$$

If we substitute (3.9) and (3.10) into (2.3), the coefficients of the first fundamental form of the tubular surface with the Galilean Darboux frame in Galilean space are obtained as

$$g^*_{11} = 1, \quad g^*_{12} = 0, \quad g^*_{22} = r^2.$$

To compute the second fundamental form of $\Psi(s, \vartheta)$, we have to calculate the following:

$$\begin{aligned}
\Psi_{ss} &= \left(k_g - r\sin\vartheta\tau'_g - r\cos\vartheta\tau_g^2\right)Q + \left(k_n + r\cos\vartheta\tau'_g - r\sin\vartheta\tau_g^2\right)n, \qquad (3.11)\\
\Psi_{\vartheta s} &= -r\tau_g\cos\vartheta Q - r\tau_g\sin\vartheta n,\\
\Psi_{\vartheta\vartheta} &= -r\cos\vartheta Q - r\sin\vartheta n.
\end{aligned}$$

From (2.4) and (3.11), the coefficients of the second fundamental form are computed as

$$L_{11} = -k_g\cos\vartheta - k_n\sin\vartheta + r\tau_g^2, \quad L_{12} = r\tau_g, \quad L_{22} = r. \tag{3.12}$$

Thus, the Gaussian curvature K and the mean curvature H are expressed as

$$K = -\frac{k_g\cos\vartheta + k_n\sin\vartheta}{r}, \tag{3.13}$$

$$H = \frac{1}{2r}. \tag{3.14}$$

4. Some characterizations for given curves as lying on tubular surfaces

We will give the conditions for parameter curves being a geodesic, an asymptotic curve, and a line of curvature on the tubular surface $\Psi(s,\vartheta)$.

Theorem 4.1. *For a tubular surface of $\Psi(s,\vartheta)$ given by (3.3),*

1) *ϑ-parameter curves are geodesics.*

2) *A necessary and sufficient condition that s-parameter curves are also geodesics is that k_g, k_n and τ_g hold the system*

$$-k_n\cos\vartheta + k_g\sin\vartheta - r\tau_g' = 0. \tag{4.1}$$

Proof. It is well known that a curve lying on a surface is a geodesic if and only if the acceleration vector α'' and the surface normal vector η are linearly dependent, that is, $\eta\wedge\alpha'' = 0$. Then, for the ϑ- and s-parameter curves, we have

$$\begin{aligned}\eta\wedge\Psi_{\vartheta\vartheta} &= (r\sin\vartheta\cos\vartheta - r\cos\vartheta\sin\vartheta)\,T = 0,\\ \eta\wedge\Psi_{ss} &= \left(-k_n\cos\vartheta + k_g\sin\vartheta - r\tau_g'\right)T.\end{aligned}$$

1) Since $\eta\wedge\Psi_{\vartheta\vartheta} = 0$, we can easily say that ϑ-parameter curves are geodesics.
2) s-parameter curves are geodesics iff

$$\eta\wedge\Psi_{ss} = \left(-k_n\cos\vartheta + k_g\sin\vartheta - r\tau_g'\right)T = 0.$$

Thus, we get

$$\left(-k_n\cos\vartheta + k_g\sin\vartheta - r\tau_g'\right) = 0.$$

□

Corollary 4.2. *Let $\alpha(s)$ be a geodesic on the tubular surface $\Psi(s,\vartheta)$ given by (3.3). If s-parameter curves are geodesics on $\Psi(s,\vartheta)$, then the curvatures κ and τ of $\alpha(s)$ satisfy the relation*

$$\tau' = -\frac{\kappa\cos\vartheta}{r}. \tag{4.2}$$

Proof. If the center curve $\alpha(s)$ is a geodesic, then $k_g = 0$. Thus, replacing $k_g = 0$ in (4.1), we simply get (4.2). □

Corollary 4.3. *Let $\alpha(s)$ be an asymptotic curve on the tubular surface $\Psi(s,\vartheta)$ given by (3.3). If s-parameter curves are asymptotic curves on $\Psi(s,\vartheta)$, then the curvatures κ and τ of $\alpha(s)$ satisfy the relation*

$$\tau' = \frac{\kappa\sin\vartheta}{r}. \tag{4.3}$$

Proof. If the center curve $\alpha(s)$ is an asymptotic curve, then $k_n = 0$. Thus, replacing $k_n = 0$ in (4.1), we simply get (4.3). □

Theorem 4.4. *For a tubular surface of* $\Psi(s,\vartheta)$ *given by* (3.3),

1) ϑ*-parameter curves cannot be asymptotic curves.*

2) *A necessary and sufficient condition that* s*-parameter curves are also asymptotic curves is that* $\Psi(s,\vartheta)$ *is produced by a moving sphere with the radius function*

$$r = \frac{k_g \cos\vartheta + k_n \sin\vartheta}{\tau_g^2} = c \tag{4.4}$$

for some constant c.

Proof. A curve $\alpha(s)$ lying on the tubular surface $\Psi(s,\vartheta)$ is an asymptotic curve iff $\langle \eta, \alpha'' \rangle = 0$. Then, for the ϑ- and s-parameter curves, we have

1) Since $\langle \eta, \Psi_{\vartheta\vartheta} \rangle = r \neq 0$, ϑ-parameter curves cannot be asymptotic curves.

2) s-parameter curves are asymptotic curves iff

$$\langle \eta, \Psi_{ss} \rangle = -k_g \cos\vartheta - k_n \sin\vartheta + r\tau_g^2 = 0.$$

From the above equation, we obtain the radius function

$$r = \frac{k_g \cos\vartheta + k_n \sin\vartheta}{\tau_g^2} = c$$

as a constant. □

Corollary 4.5. *Let* s*-parameter curves be asymptotic curves on the tubular surface* $\Psi(s,\vartheta)$*. Then, for the center curve* $\alpha(s)$*, we have the following conditions:*

1) *If* $\alpha(s)$ *is a geodesic on* $\Psi(s,\vartheta)$*, then*

$$r = \frac{\kappa \sin\vartheta}{\tau^2} = c. \tag{4.5}$$

2) *If* $\alpha(s)$ *is an asymptotic curve on* $\Psi(s,\vartheta)$*, then*

$$r = \frac{\kappa \cos\vartheta}{\tau^2} = c. \tag{4.6}$$

3) $\alpha(s)$ *cannot be a line of curvature on* $\Psi(s,\vartheta)$*.*

Proof. 1) Since $\alpha(s)$ is a geodesic, $k_g = 0$. So, from the Galilean Darboux frame, we can write $k_n = \kappa$ and $\tau_g = \tau$. By replacing these in (4.4), we can easily get (4.5).

2) Since $\alpha(s)$ is asymptotic, $k_n = 0$. So, from the Galilean Darboux frame, we can write $k_g = \kappa$ and $\tau_g = \tau$. By replacing these in (4.4), we can easily obtain (4.6).

3) Since s-parameter curves are also asymptotic curves, they satisfy (4.4). From this, τ_g cannot be zero. □

Example 4.6. We can give some examples to verify the above Corollary.

For $\vartheta = \frac{\pi}{2}$, it follows that $r = \frac{\kappa}{\tau^2}$ is a constant. Thus $\alpha(s)$ becomes a Mannheim curve (for further information see [12]). For this, as $\alpha(s)$ is a Mannheim curve, the s-parameter curve at $\vartheta = \frac{\pi}{2}$,

$$\Psi\left(s, \frac{\pi}{2}\right) = \alpha(s) + rn(s),$$

is a geodesic on $\Psi(s, \vartheta)$.

The same processes can be done for $\vartheta = 0$, then we get $r = \frac{\kappa}{\tau^2}$ is a constant. Thus $\alpha(s)$ becomes a Mannheim curve and the s-parameter curve at $\vartheta = 0$,

$$\Psi(s, 0) = \alpha(s) + rQ(s)$$

is an asymptotic curve on $\Psi(s, \vartheta)$.

Theorem 4.7. *A necessary and sufficient condition that s-parameter curves are also lines of curvature is that the center curve $\alpha(s)$ is a line of curvature on the tubular surface $\Psi(s, \vartheta)$.*

Proof. It is well known that the parameter curves on a surface are lines of curvature if and only if $g^*_{12} = 0$ and $L_{12} = 0$. Since $g^*_{12} = 0$ and $L_{12} = r\tau_g$ in the surface, we can get $\tau_g = 0$ for a line of curvature, it means that $\alpha(s)$ is a line of curvature on $\Psi(s, \vartheta)$. □

Theorem 4.8. *For the center curve $\alpha(s)$ on a tubular surface $\Psi(s, \vartheta)$,*

1) *If $\alpha(s)$ is a geodesic on $\Psi(s, \vartheta)$, then the Gaussian curvature of the surface is obtained as*
$$K = -\frac{\kappa \sin \vartheta}{r}. \tag{4.7}$$

2) *If $\alpha(s)$ is an asymptotic curve on $\Psi(s, \vartheta)$, then the Gaussian curvature is obtained as*
$$K = -\frac{\kappa \cos \vartheta}{r}. \tag{4.8}$$

Proof. The proof can be done taking $k_g = 0$, $k_n = \kappa$, $\tau_g = \tau$ and $k_n = 0$, $k_g = \kappa$, $\tau_g = \tau$ into (3.13) and (3.14), respectively. □

Remark 4.9. We consider a tubular surface given as

$$\Phi(s, \vartheta) = \beta(s) + r(\cos \vartheta N(s) + \sin \vartheta B(s)), \tag{4.9}$$

where $\{T, N, B\}$ is the Frenet frame of a curve $\beta(s)$ in $\mathbf{G}_3$. In this case, the Gaussian curvature K and the mean curvature H are given by [3]

$$K = -\frac{\kappa \cos \vartheta}{r}, \quad H = \frac{1}{2r},$$

where κ is the curvature of the curve $\beta(s)$.

In the sense, a tubular surface $\Psi(s, \vartheta)$ (3.3) generated by an asymptotic curve $\alpha(s)$ is isometric to a tubular surface $\Phi(s, \vartheta)$ (4.9) generated by an arbitrary curve $\beta(s)$.

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Dae Won Yoon,
Department of Mathematics Education and RINS, Gyeongsang National University, Jinju 52828, Republic of Korea,
E-mail: dwyoon@gnu.ac.kr

Zühal Küçükarslan Yüzbaşı,
Department of Mathematics, Firat University, 23119 Elazig, Turkey,
E-mail: zuhal2387@yahoo.com.tr

Трубчасті поверхні з галілеєвим репером Дарбу в G_3

Dae Won Yoon and Zühal Küçükarslan Yüzbaşı

Суть цього дослідження полягає у вивченні нового підходу до визначення трубчастих поверхонь з галілеєвим репером Дарбу у тривимірному просторі Галілея. Отримано також гаусову та середню кривизни і виведено параметризацію для спеціальної кривої, що лежить на трубчастих поверхнях з галілеєвим репером Дарбу.

Ключові слова: трубчаста поверхня, галілеєвий репер Дарбу, геодезична лінія, асимптотична лінія, простір Галілея.