

Biographical Sketch of Aleksei Vasilyevich Pogorelov

Aleksei Vasilyevich Pogorelov was born on March 3, 1919 in the town of Korocha near Belgorod, Russia. On his father's "farm" there was just a cow and a horse which were collectivized. Once his father came to the collective-farm stable and found his horse exhausted, dying from thirst, while the stableman slept drunk. Vasily Stepanovich became angry and hit the stableman, a former pauper. The incident was treated as if a kulak (rich man) beat a peasant, for which he might get a death sentence or sent to Siberia with the family. Thus Vasily Stepanovich and his wife Ekaterina Ivanovna had to escape promptly. A week later Ekaterina Ivanovna returned secretly to take the children. This is how A.V. Pogorelov moved to Kharkiv, where his father could find a job of a construction worker on the building of the tractor factory. A.V. Pogorelov told me this story only in 2000. In my opinion, these events had a strong influence on his life and on the way of his public behavior. He was always very cautious in expressions and liked to quote his mother who said that silence is gold. However, he never did the things contradicting his political views. Several times he successfully avoided joining the Communist Party (which was almost compulsory for a person of his rank in the USSR). As far as I know, he never signed either the letters of condemnation of dissidents or the letters in their support. Several times he was elected to the Supreme Council of the Ukrainian SSR (although, as he said later, against his will).

At school Aleksei was so good at mathematics that he got a nickname Pascal. He was a winner of one of the first school mathematical competitions organized by Kharkiv State University (V.N. Karazin Kharkiv National University now). Another his passion was painting. His parents did not know which profession to choose for their son and his mother went to school to ask the Maths teacher for an advice. He had a look at the paintings and said that the boy had brilliant abilities, but in the time of industrialization being an artist he would not earn enough for living. This advice determined their decision. In 1937, Aleksei became a student of the Department of Mathematics at the Faculty of Physics and Mathematics of Kharkiv State University.

His passion to mathematics immediately drew attention of the teachers. Professor P.A. Soloviev gave him the book by T. Bonnezen and V. Fenchel "Theory of convex bodies". From that moment and for the rest of his life geometry became the main interest of Aleksei Vasilyevich. His study was interrupted by the World War II. He was conscripted and sent to study at the Zhukovsky Air Force Engineering Academy. But he still thought about geometry. In August 1943, in a letter to professor Ya.P. Blank he wrote, "Very much I regret that the abstracts of Bonnezen and Fenchel on convex bodies I left in Kharkiv. There are many interesting problems in geometry 'in the large' ... Could you find an interesting

problem of geometry ‘in the large’ or of geometry in general for me? I would like to break down my head.”

Having graduated from Academy in 1945, A.V. Pogorelov started work as a designer engineer at the Central Aero-Hydrodynamic Institute. But the desire to finish university education (he completed four of five courses) and to work in geometry brought him to Lomonosov State Moscow University. A.V. Pogorelov asked the head of the Department of Mechanics and Mathematics, academician I.G. Petrovsky, how he could finish his education. On hearing that Pogorelov had already graduated from the Zhukovski Academy, Petrovsky decided that there was no need in the formal completion of the university course. When A.V. Pogorelov expressed his interest in geometry, I.G. Petrovski advised him to contact V.F. Kagan. V.F. Kagan asked what area of geometry Aleksei Vasilyevich was interested in, and the answer was: convex geometry. As it was not his field of expertise, Kagan sent him to A.D. Aleksandrov. At that time he was at B.N. Delone’s apartment in Moscow preparing to a mountain climbing expedition (A.D. Aleksandrov was master of sports in rock climbing, and B.N. Delone was the pioneer of Soviet rock climbing).

Their first meeting lasted only ten minutes. Sitting on a packed rucksack, A.D. Aleksandrov formulated the problem, “*Is it true that on a closed convex surface of the Gauss curvature $K \leq 1$ any geodesic segment of lengths at most π is minimal?*” It took A.V. Pogorelov a year to solve the problem, and the result was published in 1946. The multidimensional generalization of his theorem is a well-known theorem of Riemannian geometry proved by W. Klingenberg in 1959, “*On a complete simply connected Riemannian manifold M^{2n} of sectional curvature satisfying $0 < K_\sigma \leq \lambda$, a geodesic of the length $\leq \pi/\sqrt{\lambda}$ is minimal.*” In the odd-dimensional case, one needs a two-sided bound for the curvature to obtain the same result, namely $0 < \frac{1}{4}\lambda \leq K_\sigma \leq \lambda$ (and the inequality cannot be improved).

Some years later I asked Aleksei Vasilyevich, why the Soviet mathematicians showed not much interest to the global Riemannian geometry. His answer was that at that time they had enough interesting problems to think about. However, as V.A. Toponogov told me later, the first person who appreciated his comparison theorem for triangles in a Riemannian space was A.V. Pogorelov (in my opinion, it would be more correct to call this theorem the Aleksandrov–Toponogov theorem since A.D. Aleksandrov discovered and proved it for general convex surfaces in the three-dimensional Euclidean space).

Aleksei Vasilyevich took an extramural PhD course at Lomonosov Moscow State University under the supervision of professor N.V. Efimov. After having read the manuscript of A.D. Aleksandrov’s book “*Intrinsic geometry of convex surfaces*”, he started his work in the geometry of general convex surfaces.

N.V. Efimov thought that the main role of a supervisor was to inspire a post-graduate student to solve difficult and challenging problems. I gave numerous talks both at N.V. Efimov’s and A.V. Pogorelov’s seminars. They were very different by style. The N.V. Efimov seminar was long gathered, then the talk lasted for two hours or more, and the talk was always praised very warmly.

Actually, you could not understand whether the obtained result was good or not. Aleksei Vasilyevich was very punctual and always started seminars on time. The report lasted for at most an hour. Pogorelov did not like to go through the details of the proof (probably because very often as soon as the theorem was stated, he could prove it immediately).

In the estimation of the results he was strict and even severe. For example, in 1968, three applicants for the doctor's degree presented their theses at the Pogorelov seminar in Kharkiv. He supported only one of them, V.A. Toponogov, and rejected the other two. They went to Novosibirsk to A.D. Aleksandrov and later the theses were successfully defended.

Aleksei Vasilyevich rarely praised anyone, but when he did—that meant that the result was really good. He had a very fast thinking brain, an enormous geometric intuition and grasped the essence of the result very fast. Many seminar participants hesitated to ask questions not to look foolish.

In 1947, A.V. Pogorelov defended his candidate (PhD) thesis. The main result of his thesis was the theorem where he proved that every general closed convex surface has at least three closed quasi-geodesics. This theorem generalizes the Lusternik–Shnirelman theorem on the existence of three closed geodesics on a closed regular convex surface (a quasi-geodesic is a generalization of geodesics; both the left and the right turns of a quasi-geodesic are nonnegative; for instance, the union of two generatrices of a round cone dividing the cone angle into two halves is a quasi-geodesic).

After defending his candidate thesis, Aleksei Vasilyevich left the military service and moved to Kharkiv. At that time it was not easy to leave the Army. It is known that Pogorelov was discharged by the same Order of the Defence Minister as the son of M.M. Litvinov, the former Minister of Foreign Affairs of the USSR. In a year he gained doctoral degree submitting the thesis on the unique determination of a convex surface of bounded relative curvature. Subsequently he proved the theorem on the unique determination in the most general settings.

Until 1970, A.V. Pogorelov lectured at Kharkiv State University. Basing on his lectures notes, he published a series of brilliant textbooks on analytic and differential geometry and the foundation of geometry. Sometimes, during routine lectures, he was thinking about his research and made mistakes in his speech. Then he opened the textbook with the words, “What does the author say on the topic? Oh, yes, it is obvious . . .” In contrast, when lecturing on a topic interesting to him, A.V. Pogorelov was very enthusiastic and inspired (I remember one of his topology courses for the 4th year students). But perhaps the best of his lecturing brilliance was seen when he was presenting his own results. His talks were real fine art performances. In his opinion, one of the most valuable qualities of a mathematical result is its beauty and naturalness. That is why he usually omitted technicalities, and for the sake of simplicity and beauty was ready to sacrifice the generality.

A.V. Pogorelov was the author of one of the most popular school textbooks in geometry. This is how it began. He was an Education Commission member of school education headed by A.N. Kolmogorov. Pogorelov disagreed with

the textbook written by A.N. Kolmogorov and his coauthors and wrote his own manual for teachers on elementary geometry in which he built the whole school geometry course starting with a set of natural and intuitive axioms. The manual was published in 1969 and became a basis for his school textbook. Pogorelov liked to say, “My textbook is the Kiselyov improved textbook” (“Elementary geometry” by A.P. Kiselyov is probably the most well-known Russian-language school geometry textbook; it was first published in 1892 with the last edition in 2002; many generations of students studied this textbook). The first version of A.V. Pogorelov’s textbook sparked sharp criticism from A.D. Aleksandrov who Pogorelov deeply respected. This criticism was based on implementing the axiomatic approach at school as early as in the six grade, “What is the point to prove ‘obvious’ statements (from the student’s point of view)?” After reworking of the textbook these disagreements were resolved, and they remained in strong friendship till the last days of A.D. Aleksandrov’s life.

Aleksei Vasilyevich was a person of the highest decency. When a five-year contract with the “Prosveshcheniye Publishers” was coming to an end, another publisher offered a very tempting contract to him. He refused on the only reason that it would be unfair to the editor of the textbook. It should be noted that the money for the school textbook republishing was the main source of his living in the middle of the 90th.

A.V. Pogorelov told me that I.G. Petrovsky invited him to Lomonosov Moscow State University, I.M. Vinogradov invited to Steklov Mathematical Institute (Moscow), A.D. Aleksandrov invited to Leningrad several times. He even spent one year (1955–1956) in Leningrad, but then came back to Kharkiv. He preferred to live in Kharkiv far from fuss and noise of the capitals. In Kharkiv he proved his theorems, and to Moscow and Leningrad he went to shine.

Aleksei Vasilyevich Pogorelov was a person blessed by an incredible natural talent combined with a constant tireless labor [1–4].

References

- [1] A.A. Borisenko, *Aleksei Vasilyevich Pogorelov, the mathematician of an incredible power*, Zh. Mat. Fiz. Anal. Geom. **2** (2006), No. 3, 231–267 (Russian); Engl. transl.: <https://arxiv.org/abs/0810.3844>.
- [2] A.A. Borisenko, *About Kharkiv Mathematicians, Members of the Academy of Sciences of Ukraine*, Zh. Mat. Fiz. Anal. Geom. **15** (2019), No. 2, 288–303 (Russian).
- [3] A.A. Borisenko, A.Yu. Vesnin, and N. M. Ivochkina, *On the 100th anniversary of the birth of Aleksei Vasil’evich Pogorelov*, Uspekhi Mat. Nauk **74** (2019), No. 6(450), 171–193 (Russian); Engl. transl.: Russian Math. Surveys, **74** (2019), 1135–1157.
- [4] A.A. Borisenko, *The nugget faceted by restless work*, Visn. Nats. Akad. Nauk of Ukraine, (2019) No. 3, 85–96 (Ukrainian).

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* A significant part of this biographical sketch have been earlier published in [1].