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## STAGED PERMEABLE THERMOELECTRIC COOLING THERMOPILE

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- *A model of staged permeable thermopile for cooling liquid and gas fluxes is represented. Methods for calculation of the temperature fields and determination of the energy characteristics of cooling thermopile are described. Results of calculation for Bi<sub>2</sub>Te<sub>3</sub>-based thermoelectric materials point to the existence of optimal values of heat carrier rate in the channels and the electric current densities whereby the coefficient of performance acquires maximum values. The use of 2 – 3-staged permeable thermoelements makes it possible to improve the coefficient of performance by a factor of 1.2 to 1.7 under the boundary temperature differences and to increase the depth of cooling by 30 to 40 K.*

### Introduction

The possibility of expanding practical use of thermoelectric cooling devices is primarily dependant on the energy conversion efficiency, provided that the necessary temperature conditions are assured. The main methods of improving the energy conversion efficiency were already formulated by A.F. Ioffe [1]. They are based on improving the figure of merit parameter of thermoelectric materials by doping with active impurities of base material to achieve maximum values of  $\alpha^2\sigma$  ( $\sigma$  is thermoEMF,  $\alpha$  is electric conductivity) and isovalent substitutional impurities to reduce thermal conductivity. These methods were applied to some materials, which resulted in the figure of merit increase and, accordingly, contributed to a wide application of thermoelectricity.

In recent decades, despite numerous investigations, further increase in the figure of merit of thermoelectric materials has been insignificant. New ways of efficiency improvement should be sought for. So, attention is being increasingly focused on other lines of research where not only the near-contact thermoelectric effects, but also the entire bulk of thermoelectric material is used efficiently. Such variants include the use of permeable thermoelements [2].

In permeable thermoelements heat is supplied or rejected not only through the surfaces of the hot and cold junctions, but also through the internal surfaces of legs due to pumping of heat carrier through the channels (pores) of legs. I.V. Zorin in his certificates of authorship was one of the first to point to the possibility of improving the efficiency of thermoelectric energy conversion by using permeable thermoelements [3, 4]. Consistent research on the potential of thermoelements with permeable legs was performed in Ukraine [5 – 7]. In such thermoelements heat carrier is pumped through the channels (pores) of material and can be further cooled due to the Joule-Thomson effect generated at throttling of gas fluxes [8]. Owing to this, situations can be realized when a joint action of thermoelectric effects and the Joule-Thomson effect will allow improving the coefficient of performance.

Investigations of permeable cooling materials for Bi<sub>2</sub>Te-based materials reported in [9] suggested the possibility of improving the coefficient of performance by 40 – 60 % as compared to conventional thermocouple elements. This opens up wide opportunities for using permeable thermoelements to cool liquid and gas fluxes.

However, the minimum achieved heat carrier cooling temperature remains on the level of conventional thermoelements. Obtaining lower cooling temperatures would expand the possibilities of

practical application of a direct machine-free thermoelectric cooling method which is characterized by simple design, high operating reliability, ecological cleanliness, high precision of temperature mode control, etc.

A greater heat carrier cooling depth can be achieved through use of staged systems. Though the idea of using staged systems is not new [10], the issue of research on permeable staged systems has not been addressed in the literary sources known to the author.

Therefore, our purpose in this work is to determine the energy characteristics of a staged permeable cooling thermopile, to study their dependences on the structural and thermophysical parameters.

### **Physical model, mathematical description and method of solving the problem**

A staged cooling system based on permeable thermoelements is schematically shown in Fig. 1. It consists of a battery of permeable thermoelements 1 comprising *n*- and *p*-type legs of semiconductor thermoelectric materials with channels 2 for pumping of heat carrier 3, the thermoelements 1 being electrically connected parallel-in-series with connecting plates 4 so as to form a group (stage). Each stage is provided with a system 5 for heat rejection from the hot junctions of permeable thermoelements and a system 6 for circulation of heat carrier which is cooled. A combination of such stages forms a thermopile. For adjacent stages the system of heat carrier circulation and the system of heat rejection are connected in tandem, so that cooled heat carrier of one stage is pumped with pump 7 through heat rejection system of another stage, forming a closed loop 8 for heat carrier circulation. The system of heat rejection from the hottest stage is matched with the external heat exchange system 9 for heat rejection to the environment. The heat carrier circulation system of the coldest stage is matched with the external heat exchange system 10 for cooling of the object (not shown in Fig. 1).

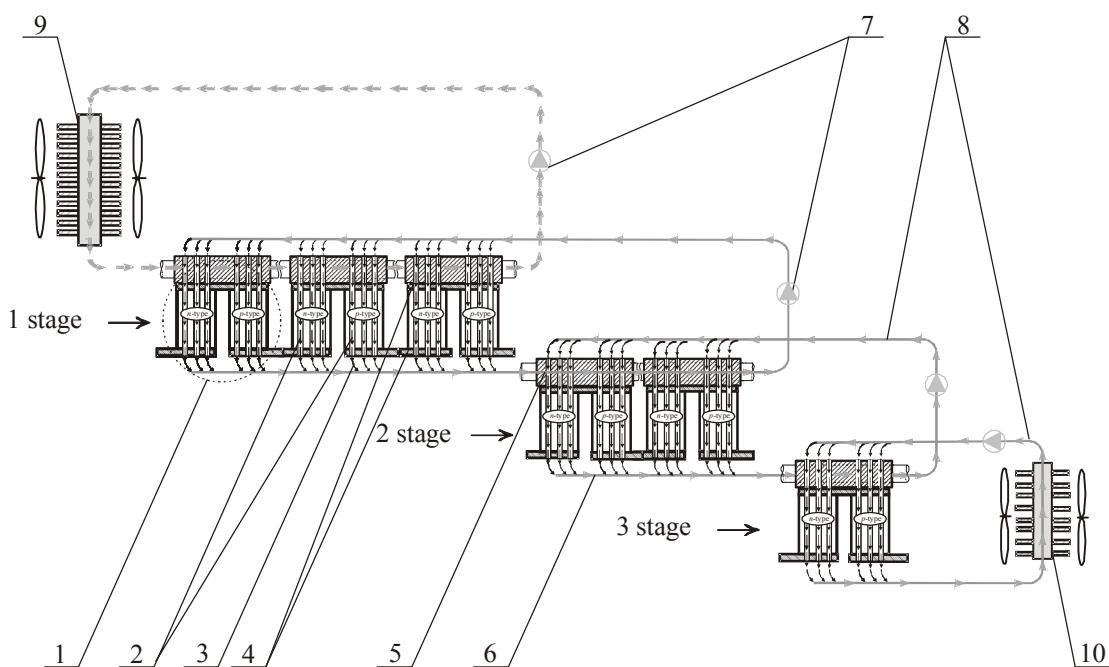


Fig. 1. Schematic of a permeable staged cooling thermopile.

Passing of electric current through each thermopile stage of respective direction results in cooling the lower parts of permeable thermoelements due to the Peltier effect. Heat carrier is pumped along the channels through the legs of permeable thermoelements toward the cold parts and is cooled due to heat exchange with the leg material. Cooled heat carrier is fed to heat rejection system of the

second stage, cooling the hot junctions of its thermoelements. This enables the second stage to work at a lower hot junction temperature and, accordingly, to cool down the heat carrier to a lower temperature level. Increasing the number of stages (Fig. 1 shows a variant of a three-staged thermopile), the heat carrier can be cooled down to even lower temperature level.

The amount of heat rejected from the hot junctions of  $k$ -th stage will be found by the change in heat carrier enthalpy

$$Q_h^{(k)} = G_h^{(k)} \cdot c_p \cdot (t_h^{(k)out} - t_h^{(k)in}), \quad (1)$$

where  $G_h^{(k)}$  is heat carrier flow rate through the heat exchanger on the hot surface of  $k$ -th stage;  $c_p$  is heat carrier heat capacity;  $t_h^{(k)in}$ ,  $t_h^{(k)out}$  are heat carrier temperatures at the inlet and outlet of  $k$ -th stage, respectively.

- amount of heat rejected from heat carrier in the channels of  $k$ -th thermopile stage

$$Q_C^{(k)} = G_k \cdot c_p \cdot (t_k^h - t_k^c), \quad (2)$$

where  $G_k$  is heat carrier flow rate through the stage;  $c_p$  is heat carrier heat capacity;  $t_k^h$ ,  $t_k^c$  are heat carrier temperatures at the inlet and outlet of the first thermopile stage.

The coefficient of performance will be found from the relation:

$$\varepsilon = \frac{Q_C^{(N)}}{W} = \frac{Q_C^{(N)}}{\sum_{k=1}^N W^{(k)}}, \quad (3)$$

where  $W^{(k)} = Q_h^{(k)} - Q_C^{(k)}$  is power consumed by  $k$ -th stage.

To determine the thermophysical characteristics of a staged thermopile, let us consider its physical model (Fig. 2). It is comprised of parallel-in-series connected permeable thermoelements 1 that are formed by permeable *n*- and *p*-type legs 2 joined by connecting plates 4. Through the thermoelements along the channels of legs 2 heat carrier 3 is pumped to be cooled. The heat carrier is cooled due to heat exchange with the legs material where the temperature gradient is established due to the Peltier and Thomson thermoelectric effects in the presence of electric current. Heat from the hot junctions of thermoelements is rejected by heat exchanger 6 due to pumping of heat carrier, the rest of the surface is adiabatically isolated 5.

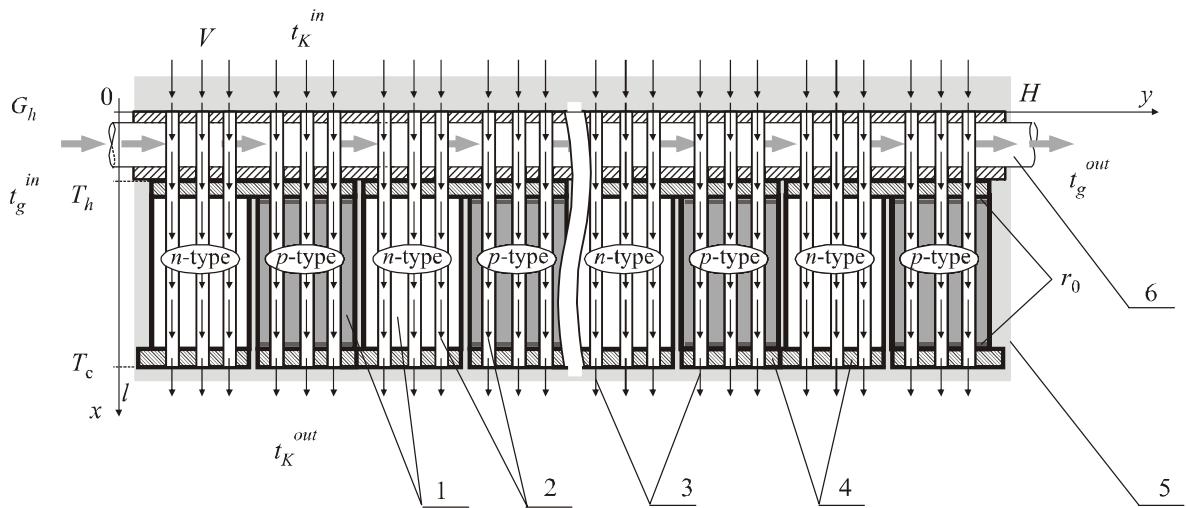


Fig. 2. Physical model of a permeable stage

To calculate the energy characteristics of the device, we can write a system of balance equations for the steady-state case in the approximation when the properties of parameters of *n*- and *p*- type thermoelement legs are identical.

Heat carrier which is pumped through the hot heat exchanger changes its temperature

$$\frac{dt_g}{dy} = \frac{1}{G_g c_p} q_h(T_h, T_c, j, \dots), \quad (4)$$

where  $t_g$  is heat carrier temperature at point  $y$ ;  $G_g$  is heat carrier mass flow rate;  $c_p$  is heat carrier heat capacity;  $q_h(T_h, T_c, j, \dots)$  is specific heat flux on the hot surface of thermoelements which is a function of the  $T_h$  and cold  $T_c$  surface temperatures of junctions, the electric current density  $j$  and other structural and thermophysical parameters of a permeable thermoelement.

The specific heat flux  $q_h(T_h, T_c, j, \dots)$  will be found from the system of differential equations of thermal and electric conductivity written for one-dimensional steady-state case [9]

$$\left. \begin{aligned} \frac{dT}{dx} &= -\frac{\alpha j}{\kappa} T - \frac{q}{\kappa}, \\ \frac{dq}{dx} &= \frac{\alpha^2 j}{\kappa} T + \frac{\alpha j}{\kappa} q + i^2 \rho - \frac{\alpha_T \Pi_K N_K l^2}{(S - S_K) j} (T - t_K), \\ \frac{dt_K}{dx} &= \frac{\alpha_T \Pi_K N_K l}{V c_p S_K} (T - t_K). \end{aligned} \right\} \quad (5)$$

where  $t_K$  is temperature of heat carrier in the channels of thermoelement legs at point  $x$ ;  $T$  is temperature of leg material at point  $x$ ;  $j = il$ ;  $l$  is the height of thermoelement legs;  $i$  is the electric current density ( $i = \frac{I}{S - S_K}$ );  $q = \frac{1}{j} \left( \alpha(T, \xi(x)) j T - \kappa(T, \xi(x)) \frac{dT}{dx} \right)$ ,  $q$  is reduced specific heat flux ( $q_h(T_h, T_c, j, \dots) = (q \cdot j)|_{x=0}$ );  $x = \frac{x}{l}$  is dimensionless coordinate;  $I$  is current strength;  $S_K$  is cross-section area of all channels;  $S$  is the section of leg and the channels;  $\Pi_K$  is channel perimeter,  $N_K$  is the number of channels;  $V$  is mass rate of heat carrier in the channels;  $c_p$  is heat carrier heat capacity;  $\alpha_T$  is coefficient of heat transfer in a channel;  $\alpha(T)$ ,  $\kappa(T)$ ,  $\rho(T)$  - the Seebeck coefficient, thermal conductivity and resistivity of material are functions of temperature  $T$ .

The system of differential equations (4) and (5) will be supplemented by the boundary conditions

$$t_g|_{y=0} = t_g^{in}; \quad q_h(T_h, T_c, j, \dots) = (q \cdot j)|_{x=0} = \frac{1}{R_{ef}} (T_h - t_g); \quad t_K|_{x=0} = t_K^{in}. \quad (6)$$

$R_{ef}$  is the effective coefficient of heat transfer between the hot junctions of thermoelements and the heat carrier in the heat sink;  $t_g^{in}$  is the initial temperature of heat carrier in the heat sink;  $t_K^{in}$  is the temperature of heat carrier at the inlet of a permeable thermoelement.

Based on relations (4) – (6) we can make thermal calculation of each stage of a permeable staged thermopile. For the calculation of the entire staged thermopile we should take into account the interconnection between stages which is given by the relations

$$(V c_p S_K)_k = (G_g c_p)_{k+1} \quad (7)$$

$$\left( t_K \Big|_{x=l} \right)_k = \left( t_g^{in} \right)_{k+1} \quad (8)$$

where index  $k = 1, \dots, N$ ;  $N$  is the number of stages in a thermopile.

Based on the resulting relations, through use of successive approximations method, numerical methods for solving systems of differential equations (4), (5) with the boundary conditions (6), (7), (8), a computer program was elaborated for investigation of the energy characteristics of a staged permeable thermopile.

### **Results of computer investigation of the energy characteristics of a staged permeable thermopile**

Computer simulation of permeable thermoelement stages was done for  $Bi_2Te_3$ -based material [11]. The temperature dependences of parameters  $\alpha$ ,  $\kappa$ ,  $\sigma$  of these materials were approximated by least-squares method with a relative error not more than 1% and this data was used in the calculations. The temperature dependences of material parameters  $\alpha$ ,  $\kappa$ ,  $\sigma$  are given in Fig. 3.

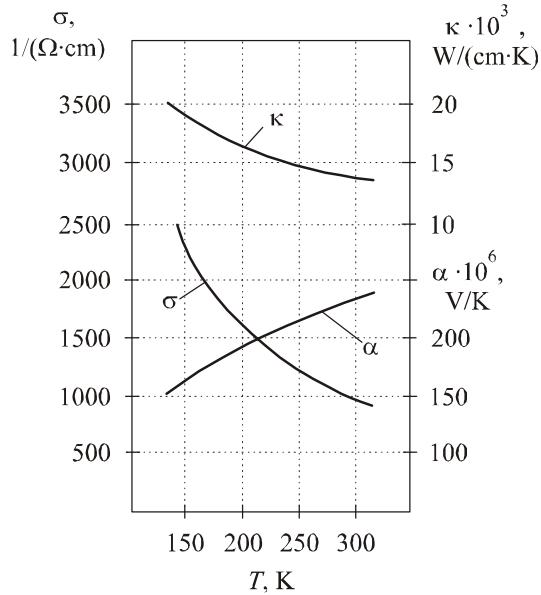


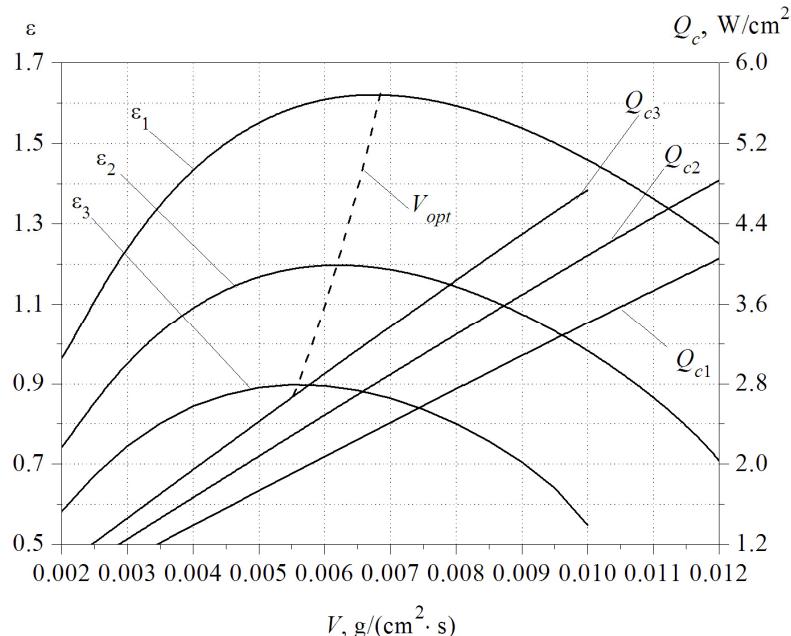
Fig. 3. Temperature dependences of leg material parameters.

The energy characteristics of a staged thermopile are determined through performance parameters of each stage. The results of studying the energy characteristics of one stage are given in Fig. 4. Here, the values of coefficient of performance  $\varepsilon$  and specific cooling capacity  $Q_c$  are represented as a function of heat carrier mass rate  $V$  in the channels under the optimal electric current density.

The data is given for the case when water is used as a heat carrier, the heat carrier temperature at the inlet of thermoelement channels is 310 K, the height of legs is 1 cm, the channel diameter is 0.12 cm, the cross-section area of legs together with the channels is  $0.36 \text{ cm}^2$ , the contact resistance at points of connection of thermoelements legs is  $5 \cdot 10^{-5} \Omega \cdot \text{cm}^2$ , the heat carrier temperature at the inlet of the hot junction heat exchanger is 310 K. Dependences of the energy characteristics are given for different temperatures of thermoelement cold junctions – index 1 corresponds to  $T_c = 285$  K, 2 –  $T_c = 280$  K, 3 –  $T_c = 275$  K. In so doing, the resulting water cooling temperature corresponds to values 286 K, 280.7 and 285.5 K, respectively.

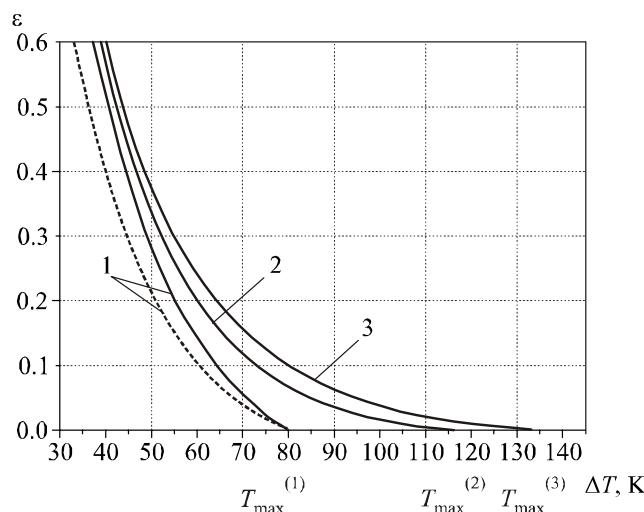
From the figure it is evident that specific cooling capacity monotonously grows with increasing heat carrier rate. Coefficient of performance of a permeable stage has maximum heat carrier rate (optimal

value is designated as  $V_{opt}$ ) which depends on the temperature operating conditions. Therefore, Newton's numerical method for solving extreme problems was used to generalize a computer program for calculation of a permeable staged cooling thermopile for the case of looking for the optimal values of heat carrier rate in a channel.



*Fig. 4. Dependences of the energy characteristics of a permeable stage on heat carrier mass rate in a channel under the optimal electric current density.  
(index 1 corresponds to  $T_c = 285\text{ K}$ , 2 –  $T_c = 280\text{ K}$ , 3 –  $T_c = 275\text{ K}$ )*

Fig. 5 represents the estimated values of the coefficient of performance of single- (dependence 1), two- (dependence 2) and three- (dependence 3) staged permeable coolers versus the temperature difference and the number of stages. The dotted line corresponds to the value of coefficient of performance of a single-stage impermeable thermopile, and solid lines – to an optimized permeable staged cooling thermopile. It is seen that the use of staging in permeable cooling systems allows improving the coefficient of performance by a factor of 1.2 – 1.7 at the boundary temperature differences and improve liquid cooling depth by 30 – 40 K.



*Fig. 5. Coefficient of performance versus the temperature difference and the number of stages.*

## **Conclusions**

1. A model of staged permeable thermopile for cooling liquid and gas fluxes is represented. A system of differential equations is used to describe a method for calculation of the temperature fields and determination of the energy characteristics of cooling thermopile.
2. A model of permeable staged thermopile for cooling  $Bi_2Te_3$ -based materials is investigated. Results of calculations point to the existence of optimal values of heat carrier rate in the channels and the electric current density whereby coefficient of performance acquires maximum values.
3. Results of calculation of 2 – 3-staged permeable thermopiles suggest that the coefficient of performance can be improved by a factor of 1.2 to 1.7 at the boundary temperature differences and the depth of cooling can be increased by 30 to 40 K.

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